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SIMPLIFIED METHOD FOR THE LATERAL STRENGTHENING OF EARTHEN CHURCHES

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Abstract. *The evaluation of the seismic safety of Andean colonial churches is of high importance as those buildings represent part of the identity of the society and are historical emblems for the communities. Most of these buildings are composed of elongated naves with adobe masonry walls with flexible (or nonexistent) horizontal diaphragms, which generates potential of out-of-plane failures. In the last decades, different methodologies using advanced numerical analyses have been developed that allow evaluating the structural behavior of historical constructions at the expense of an arduous computational effort. In the present paper, a simplified tool is proposed for the design of walls lateral reinforcement using buttresses. The tool uses limit analysis and provides an adequate buttress design according to the configuration of the wall and seismicity of the area where the church is located. The results of the application of the methodology showed that the developed tool provides fast and accurate alternatives for the seismic strengthening of Andean adobe churches. The use of buttresses as structural reinforcement control the development of out-of-plane failure mechanisms, and provide lateral stability and resistance to the structure.*

1 INTRODUCTION

Unreinforced masonry constructions are a significant percentage in different urban areas around the world, with heritage buildings being the most representative. However, in the case of seismic events these buildings do not behave satisfactorily due to their lack of structural capacity, which is why they are known as the most vulnerable constructions to suffer irreparable damage [1-2]. In recent decades, there has been an increase in interest in the conservation of heritage buildings as a way of preserving the history of a region. Currently, there are numerous methods of analysis and computational tools available for the evaluation of the mechanical behavior of historic structures that can be successfully used in the study of masonry structures [2-3]. However, despite the great progress made in the study of historical masonry structures, important challenges still have to be faced, such as the characterization of generally complex geometries, the difficulty of developing a good analysis and the high computational effort [4]. The limit analysis is a simplified, fast and effective method that allows the analysis of the seismic behavior of heritage buildings from the study of the possible failure mechanisms that may occur. In case of the buildings with flexible diaphragms the most critical mechanisms are the ones associated to the out-of-plane overturning of walls [1-5]. The guidelines for the development of the limit analysis are detailed in the Italian construction standard, which considers certain essential principles for its application in existing non-reinforced constructions [6-7]. In the present study, a first stage of the methodology based on the use of limit analysis was apply to understand the seismic vulnerability of the church of Sacsamarca located in Ayacucho, Southern Peru. A second stage of the methodology was then applied for dimensioning the buttress solution which was adopted to increase the lateral stability of the nave walls.

2 METHODOLOGY FOR THE EVALUATION SEISMIC CAPACITY AND DIMENSIONING OF BUTTRESSES

The methodology proposed for the design of buttresses consists of two phases described in Figure 1. As a first step, the structural capacity of the longitudinal wall is calculated using limit analysis, for which it is necessary to know certain data such as the materials, loads, geometry of the wall, and seismic parameters of the site. If the structural capacity does not exceed the seismic demand, the wall must be reinforced by adding buttresses. Phase 2 is the design of the buttresses considering a series of possible design configurations that were defined based on the guidelines proposed by the Indian [8] and Peruvian [9] standards. The structural capacity of the reinforced system is calculated, taking into account the configurations established in order to determine those that offer greater lateral stability to the wall.

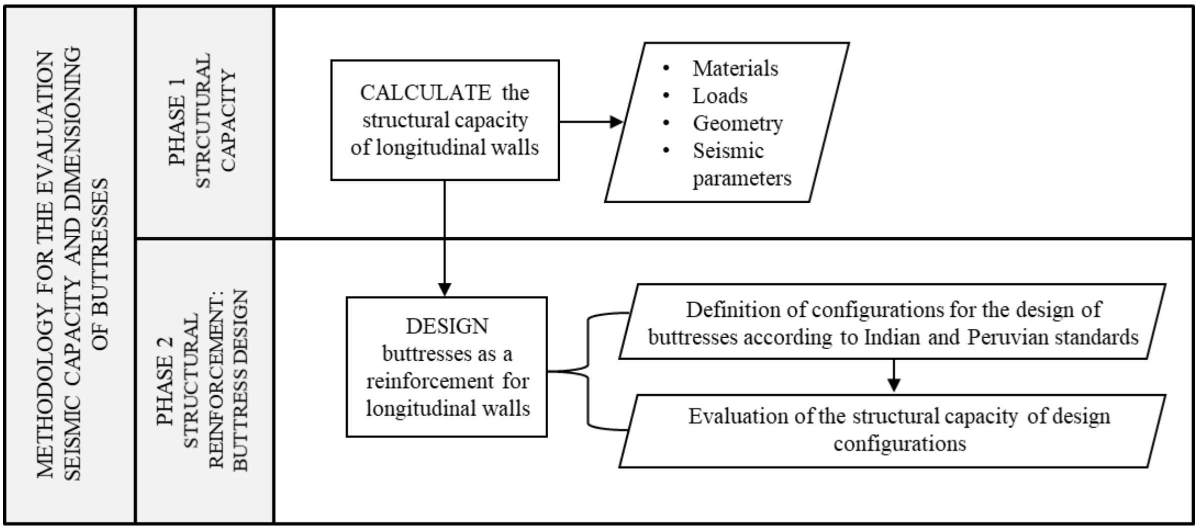


Figure 1: Methodology of analysis and design of buttresses for Andean colonial churches

3 EVALUATE THE SEISMIC SAFETY OF THE LONGITUDINAL WALLS OF THE SACSAMARCA CHURCH

3.1 Description of the case study

The church of the Virgin of the Assumption of Sacsamarca is located in the department of Ayacucho and its construction began during the last decades of the sixteenth century. It has adobe walls, stone foundations, and a wooden roofing system (Figure 2-a and 2-b). The church has a central nave that is 50 m long and approximately 12 m wide. The longitudinal walls have "arrimos" which are elements made of unconsolidated stone masonry and are a type of buttress whose function is to provide stability to the walls (Figure 2-c). The walls are made of adobe with a thickness of 1.5 m and a height of approximately 9.0 m. It has a roofing system known as "pair and knuckle" (Figure 2-d).

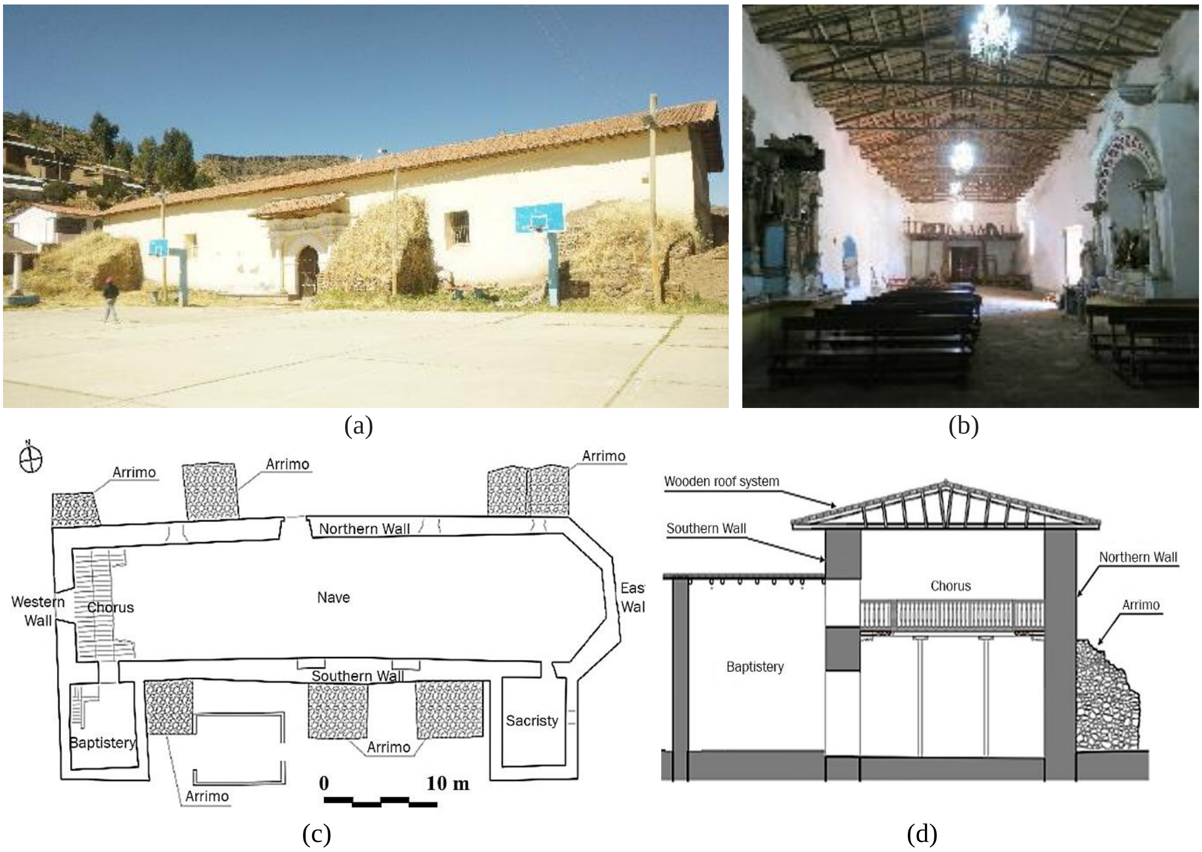
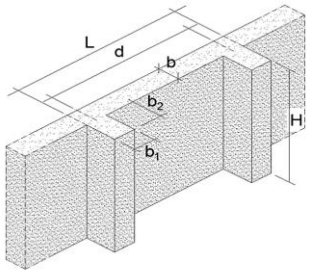


Figure 2: General views of the Sacsamarca church: (a) exterior view, (b) interior view and architectural plans: (c) plan view of the height of the base of the wall and (d) cross section

3.2 Dimensioning of the buttress

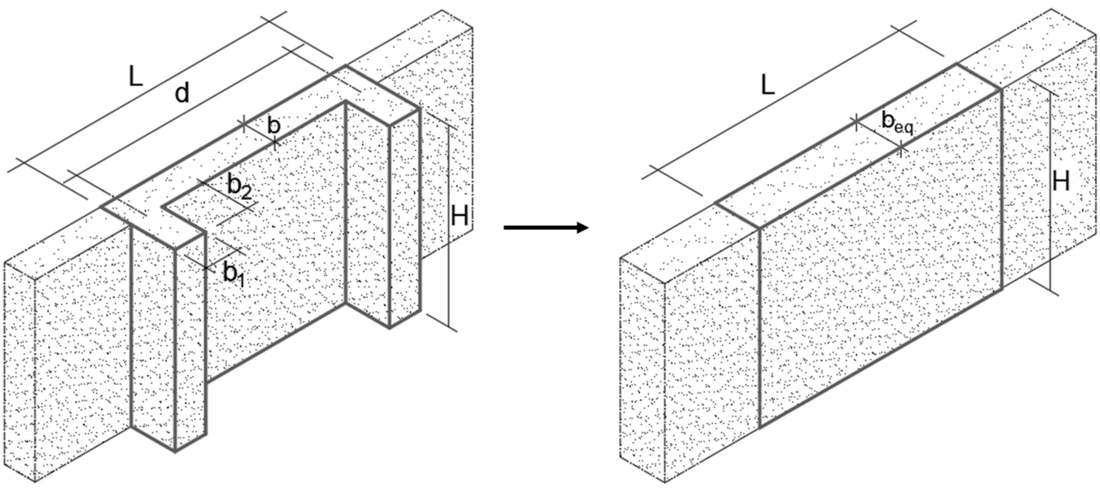
Based on the design guidelines suggested by the Indian [8] and Peruvian [9] standards for the reinforcement of walls using buttresses, a set of configurations was determined, grouped into cases I and II. It should be noted that the variable parameters are in function of b and H, which are the thickness and height of the reinforced wall. The variables b_1 and b_2 are the dimensions of the buttresses (thickness and depth) and d the distance between the reinforcements (see Table 1). The property has a stone overhang of 1.3 m and an adobe wall height of 6.6 m, however, the wall section was considered to be made only of adobe in its entire height since it is the most predominant material compared to stone, therefore, the total height of the wall was considered to be 9.0 m.

Table 1: Section values of the buttressed wall for Case I and Case II



Case I				Case II			
b_1=b, b_2=1.5b y d=[3-13]b				b_1=b, b_2=2.0b y d=[3-13]b			
b (m)	b_1	b_2	d	b (m)	b_1	b_2	d
1.5	1.5	2.25	4.5	1.5	1.5	3.0	4.5
1.5	1.5	2.25	6.0	1.5	1.5	3.0	6.0
1.5	1.5	2.25	7.5	1.5	1.5	3.0	7.5
1.5	1.5	2.25	9.0	1.5	1.5	3.0	9.0
1.5	1.5	2.25	10.5	1.5	1.5	3.0	10.5
1.5	1.5	2.25	12.0	1.5	1.5	3.0	12.0
1.5	1.5	2.25	13.5	1.5	1.5	3.0	13.5
1.5	1.5	2.25	15.0	1.5	1.5	3.0	15.0

In addition, for the development of the limit analysis methodology, a geometric conversion was performed from the original section (wall plus buttresses) called type C section (Figure 4-a) to an element of constant thickness b_{eq} (Figure 4-b). This proposed methodology for section conversion is intended to perform in a simpler and faster manner the calculation of the factor that activates the tipping failure mechanism in the limit analysis from the system load equilibrium. To calculate the equivalent thickness, a non-linear static pushover analysis using DIANA-FX software was used to evaluate the conversion of sections by generating models with displacement and rotation restrictions at the base. A horizontal force was applied to the highest node of the element to obtain the maximum displacement. The equivalent section was modeled repeatedly until obtaining a similar stiffness value to the C-type wall, applying the same force and obtaining the same displacement for both walls.



(a) (b)

Figure 4: Diagram of the reinforced wall section and conversion of (a) a type C wall to a typology of (b) wall with equivalent thickness

From the results obtained from the modeling of the two types of walls for each configuration, an equation was constructed that allows the value of the equivalent thickness to be obtained.

This equation (Eq. 1) is based on the relationship between the cross-sectional areas of type C sections and equivalent sections, considering a parameter K that represents the stiffness contribution of the buttresses to the walls. Since in cases I and II there are different dimensions for the buttresses, the values of parameter K vary resulting in K=1.8 for Case I and K=1.5 for Case II.

$$b_{eq} = b_{muro} + \mathrm{K}\frac{(2 * b1 * b2)}{d + b1} \tag{1}$$

3.3 Verification of the structural contribution of the buttress system using numerical analisis

According to the proposed methodology, for phase 1 the structural capacity of the longitudinal walls with a tendency to develop an out-of-plane tipping fault mechanism must be calculated from the development of the limit analysis. The guidelines of the Italian standard and its derivatives [6-10] for limit analysis were adopted, and the limit state relating to safety of life (SLV) was applied. Additionally, it was necessary to know the zoning and soil type parameters of the area to calculate the seismic demand. These parameters are extracted from the Peruvian seismic-resistant standard [11] and are detailed in Table 2.

Table 2: Zoning and soil type parameters for the case study

Description	Code	Value	
Maximum horizontal site acceleration: Z3	NTE 030	a_g	0.35g
Category coefficient of the subsoil and topographic conditions: Considered a bad soil in the absence of specific information	NTE 030	S	1.20
Period corresponding to the beginning of the section at constant speed of the spectrum: Tc = Tp	NTE 030	T_P	1.0 s
Period corresponding to the beginning of the section at constant spectrum shift: $T_D = T_L$	NTE 030	T_L	1.6 s

The mechanical properties of the materials were taken from the structural survey report of the Sacsamarca church [12], where the average specific weight of the masonry is 2130 kg/m3, modulus of elasticity E=271x106 N/m2 and a ductility of 1.6 mm. Table 3 details the structural parameters required for the linear and non-linear verifications of the limit analysis of the case study. The partial safety coefficient to be used for the seismic design of masonry structures q_e is equal to two according to the Italian standard (chapter 7.8) [6]. On the other hand, the Italian standard recommends the use of a factor q for the confidence level of the structure from the determination of the level of knowledge of the property. In this study, it was decided to consider conservative values and for this reason a knowledge level LC2 was chosen, related to a factor $q = 1.2$, because previous studies were carried out on the properties of the building materials [6].

Table 3: Structural parameters to calculate the accelerations and spectral displacements

Description	Value	
q_e: Structure factor	2	-
Z': Height at which the center of masses is located	4.5	m

H: Total height of the wall	9.0	m
Ψ(Z'): First form of vibration in the direction of analysis: Z'/H	0.5	-
N: Number of levels in the structure	1	-
γ: Modal participation coefficient: 3N/(2N+1)	1	-

In addition, the virtual working principle was used to calculate the factor of the turning fault (α_0). The linear safety check in terms of acceleration (Eq-2) is performed to ensure safety in the damage limit state (SLD) and the ultimate limit state (SLU). This equation makes it possible to calculate the spectral acceleration demand of the earthquake according to the Peruvian earthquake-resistant standard [11]. On the other hand, for the verification of the SLU state, the properties of the structure and the soil are used, and the structural capacity expressed in terms of acceleration (Eq-3) is calculated. The factor S is the soil amplification factor, Z is the zoning coefficient whose values are shown in Table 2, while the factor q governs the behavior of the structure and is indicated in Table 3 [6].

$$a_0^* \geq \frac{Z \cdot S \cdot g}{q} \tag{2}$$

$$a_0^* \geq \frac{Z}{q} \cdot \min\left(2.5\frac{Tp}{T1}, 2.5\right) \cdot S \cdot g \cdot \varphi(z) \cdot \gamma \tag{3}$$

The factor T_p is the period corresponding to the final part of the plateau (constant zone) in the elastic acceleration response spectrum and T_1 is the period of fundamental vibration of the structure. The factor $\varphi(z)$ is the first normalized vibration mode of the structure and is estimated as the ratio of the point of the center of masses relative to the ground and the total height of the element relative to the ground Z'/H. Finally, γ is the modal participation factor which is calculated as $3N/(2N+1)$, where N is the number of levels of the structure [4-13].

Furthermore, it is necessary to verify the system by means of a non-linear analysis in terms of the displacements. The ultimate spectral displacement ${d_u}^*$ corresponds to the limit state of the structure's service and is equivalent to $0.4\, {d_0}^*$. It is compared ${d_u}^*$ with the demand of spectral displacement ${\Delta_d}^*$ which is calculated from the secant period T_s defined for the system of a degree of freedom [6-14]. The seismic displacement demand $\Delta_d^*(Ts)$ is calculated from equations Eq-4, Eq-5 or Eq-6. Finally, the safety verification for the ultimate limit state is guaranteed when the ultimate displacement satisfies the following relation: $\Delta_d \leq d_u^*$ [13].

$$Ts < 1.5T_1 \qquad \Delta_d^*(Ts) = a_g S \frac{T_s^2}{4\pi^2}\left(\frac{3(1+Z'/H)}{1+(1-Ts/T1)^2} - 0.5\right) \tag{4}$$

$$1.5Ts \leq Ts < T_D \qquad \Delta_d^*(Ts) = a_g S \frac{1.5T1Ts}{4\pi^2}\left(1.9 + 2.4\frac{Z'}{H}\right) \tag{5}$$

$$T_D \leq T_s \qquad \Delta_d^*(Ts) = a_g S \frac{1.5T1T_D}{4\pi^2}\left(1.9 + 2.4\frac{Z'}{H}\right) \tag{6}$$

With the detailed information on the development of the limit analysis, the seismic safety of one of the longitudinal walls of the case study was evaluated. The geometry of the wall according to the architectural plans consists of a thickness of 1.5 m, a height of 9.0 m and a length of 43.3 m. Table 4 shows the results obtained for the structural capacity of the wall, resulting in a value of 0.139g compared to a seismic demand of 0.263g. Although the non-linear verification by displacement is satisfactorily fulfilled, it is necessary to propose a reinforcement to the wall until both verifications of the limit analysis are satisfied.

Table 4: Structural capacity of the longitudinal wall of the case study

Structural capacity			Seismic Demand		SF_a (g/g)	SF_d (mm/mm)	Safety
α_0	a_0*	d_u*	a_d*	Δ_d*			
0.167	0.139	296	0.263	21	0.53	14.10	Unsafe

For Phase 2 of the methodology, the reinforcement of the wall from the design of the buttresses is evaluated using the methodology of the limit analysis, the configurations set out in Table 1 and equation Eq-1 for section conversion. Table 5 shows the results of the verifications of the limit analysis for cases I and II. A comparison between the values of structural capacity and seismic demand gives the safety factors (SF) as a function of accelerations (g) and displacements (mm). If for a configuration both factors are greater than the unit, the structure is considered to be safe, otherwise it is sufficient that one factor does not satisfy the requirement to be considered an unsafe structure that does not verify the safety of the damage limit state (SLD).

Table 5: Linear and non-linear checks of the limit analysis in cases I and II

Case	Config. "d"	Structural capacity			Seismic Demand		SF_a (g/g)	SF_d (mm/mm)	Safety
		α_0	a_0*	d_u*	a_d*	Δ_d*			
I	3b	0.392	0.326	656	0.263	21	1.24	31.24	Safe
	4b	0.347	0.289	590	0.263	21	1.10	28.10	Safe
	5b	0.317	0.264	543	0.263	21	1.00	25.86	Safe
	6b	0.295	0.246	510	0.263	21	0.94	24.29	Unsafe
	7b	0.279	0.233	484	0.263	21	0.89	23.05	Unsafe
	8b	0.267	0.222	464	0.263	21	0.84	22.10	Unsafe
	9b	0.257	0.214	447	0.263	21	0.81	21.29	Unsafe
	10b	0.248	0.207	434	0.263	21	0.79	20.67	Unsafe
II	3b	0.417	0.347	692	0.263	21	1.32	32.95	Safe
	4b	0.367	0.306	620	0.263	21	1.16	29.52	Safe
	5b	0.333	0.278	569	0.263	21	1.06	27.10	Safe
	6b	0.310	0.258	532	0.263	21	0.98	25.33	Unsafe
	7b	0.292	0.243	504	0.263	21	0.92	24.00	Unsafe
	8b	0.278	0.231	482	0.263	21	0.88	22.95	Unsafe
	9b	0.267	0.222	464	0.263	21	0.84	22.10	Unsafe
	10b	0.258	0.215	449	0.263	21	0.82	21.38	Unsafe

It should be noted that the structural capacity of the original longitudinal wall is 0.139g, so adding buttresses as a reinforcement system improves the stability of the wall and can increase lateral resistance by more than 50%.

For the validation of the limit analysis methodology developed, the results obtained for the design of buttresses were compared with the results of applying the pushover analysis in the same configurations. The model of the section in study is built with finite elements (FEM) using tetrahedral elements (TE12L) with an average size of 0.5 m. The numerical model was obtained using the software DIANA-FX. The vertical forces were considered to be loads from the own weight and the seismic forces as horizontal thrust forces whose direction of analysis was +Y axis. The iteration method used to analyze gravity and seismic loads was the Modified Newton-Raphson method combined with the arc length method ARC LENGHT [4], only in the cases of seismic load analysis. The control points in each configuration were located in the highest part of the reinforced walls in order to quantify the maximum displacement [15]. Convergence was controlled from the energy parameter using a tolerance of 0.001. According to [16], a non-linear behaviour for masonry should be considered by means of a constitutive model based on total deformation (Total Strain Crack Model), since this model provides good stability in crack opening control. In addition, a crack model called the Rotating Crack Model was used and the condition of embedding in the base was absolute (FIXED).

According to the study by [17], the behaviour developed by the masonry in traction follows a model of post-peak exponential softening, while in compression it adopts parabolic hardening, followed by post-peak parabolic softening [17,18,19]. The elastic and non-linear properties of Table 6 were used to perform the pushover analysis.

Table 6: Elastic and non-linear properties of adobe material

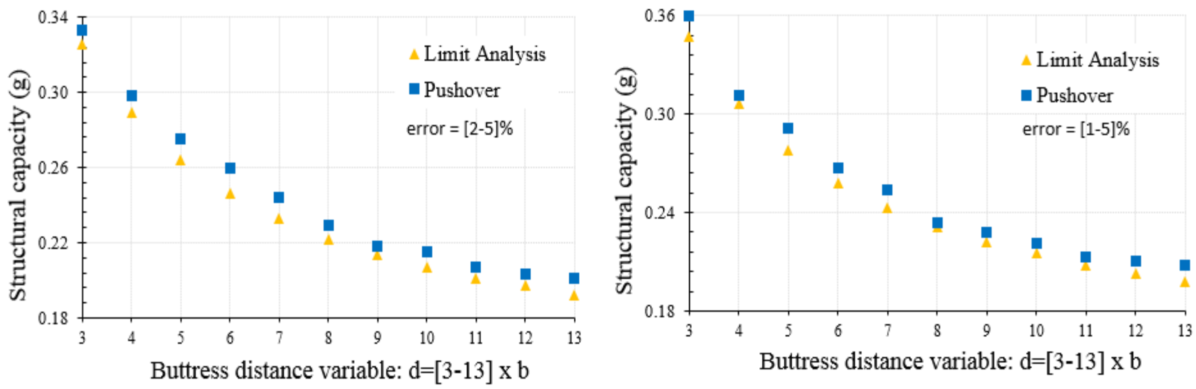
Tensile modulus (E)	Specific weight (γ)	Compressive strength (fm)	Tensile strength (ft)	Compression fracture energy (Gm)	Tensile fracture energy (Gt)	Coefficient of ductility (μ)	Poisson Module (ν)
MPa	kg/m^3	MPa	MPa	N/m	N/m	mm	-
271	2130	0.6775	0.06775	1084	10	1.6	0.24

After performing the pushover analysis, Table 7 shows the structural capacity values which are compared to the seismic demand of the area.

Table 7: Results of the maximum structural capacity as a function of acceleration

a_0* (g)	Design configurations							
	3b	4b	5b	6b	7b	8b	9b	10b
Case 1	0.333	0.298	0.275	0.257	0.244	0.229	0.218	0.215
Case II	0.360	0.311	0.291	0.267	0.254	0.234	0.228	0.221

The comparison of the structural capacity results of the configurations for both methods of analysis is shown in Figure 5-a and Figure 5-b for cases I and II, respectively. From the graphics it can be seen that the limit analysis gives clearly similar values to those obtained from the pushover analysis with an error between the results not exceeding 5% with respect to the results of the pushover analysis. It is demonstrated that using limit analysis to calculate the structural capacity of existing masonry constructions, in this case Andean colonial churches, in the face of the occurrence of seismic phenomena provides reliable results that are significantly close to the real behavior of these buildings.



(a) (b)

Figure 5: Comparison of results between limit and pushover analysis for cases (a) I and (b) II

4 PROPOSAL OF A SIMPLIFIED METHODOLOGY FOR THE DESIGN OF BUTTRESSES IN ADOBE CHURCHES

A proposal was elaborated to design buttresses to reinforce the longitudinal walls of Andean colonial churches through the creation of a computer program using MatLab software. The program implicitly develops the limit analysis and verifications for each of the proposed configurations. The objective is to obtain reinforcement designs that provide additional resistance to the wall and thus overcome the seismic demand of the area. On the other hand, the architecture of Andean colonial churches has a common pattern for such structures. They have one or more bell towers, apse, chapels, longitudinal walls on which the roof, facade and in some cases buttresses or “arrimos”. It must be taken into account that these constructions have longitudinal walls whose dimensions vary from one church to another. However, it is possible to consider typical ranges for the dimensions of the wall. In addition, because the presence of seismic events generates a variety of fault mechanisms, in the case of churches the prevailing fault is that of out-of-plane turning due to their extensive walls [20]. Table 8 shows a group of Andean colonial churches considered to be the most representative for this study.

Table 8: Dimensions of longitudinal walls in the most representative Andean colonial churches

Name	Location	Thickness (m)	Height (m)
San Juan Bautista of Huaro Church	Cusco	1.6	11
San Pedro Apóstol of Andahuaylillas Church	Cusco	1.8 - 2.0	10
Canincunca Chapel	Cusco	1.3	8
San Blas Church	Cusco	1.5	11
Temple of San Sebastián	Cusco	1.5 - 2.0	12
Kuño Tambo Church	Cusco	1.6 - 1.9	8
Sacsamarca Church	Ayacucho	1.5	9

Design configurations were generated taking as reference the Indian standard [8] and the Peruvian adobe standard [9] for masonry wall reinforcement. Table 9 shows the ranges of values to be considered for the dimensions and distances of the buttresses.

Table 9: Configurations for the design of buttresses that the program evaluates

Wall thickness b (m)	Wall height H (m)	Distance between buttresses d (m)	Buttress thickness b_1 (m)	buttress length b_2 (m)
1.0, 1.5 and 2.0	[8-12]	[3b-10b]	Equal to b	1.5b and 2b

Equation Eq-1 is also used to calculate the coefficient K with which the equivalent thickness (b_{eq}) can be determined in each of the configurations (see Table 10). In total, the program must analyze 240 design configurations by applying limit analyses and evaluating conformity to safety checks where the input parameters are the dimensions of the wall (thickness and height), the soil parameters and zoning of the site according to the Peruvian seismic-resistant standard [11].

Table 10: Values for the coefficient K to find the equivalent thickness

K	$1.0 \leq b \leq 1.2$	$1.2 < b \leq 1.5$	$1.5 < b \leq 2.0$
b_2=1.5b	2.1	1.8	1.6
b_2=2b	1.9	1.5	1.4

The program executes the limit analysis for each design configuration of buttresses in order to calculate the structural capacity of the reinforced system. Each of these results is compared to the seismic demand of the zone and then the program shows those configurations that comply satisfactorily with the linear and non-linear verifications of the limit analysis. If the program shows more than one design configuration, the professional in charge must choose the one that best matches the geometry of the church.

4 CONCLUSIONS

This work proposes a methodology for the design of buttresses with the aim of strengthening those Andean colonial churches that are vulnerable to earthquakes. Likewise, this study contributes substantially to the field of conservation and reinforcement of Andean colonial churches in Peru since, to date, there is no known simplified, fast and effective methodology based on the limit analysis that allows to verify whether or not the seismic demand of the area is exceeded. In the case that more than one design configuration is presented, the responsible professional (PR) should consider that the churches have an architecture with openings in the walls (windows and doors), as well as the presence of chapels that somehow provide stability to the walls. This means that the PR must choose the configuration that can satisfy the seismic demand of the area and that, additionally, is compatible with the architectural distribution of the building. Finally, it was possible to prove that limit analysis can be a powerful tool to evaluate the seismic vulnerability of historical adobe masonry structures.

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