Elliptic reconstruction and *a posteriori* error estimates for the parabolic partial differential equations with small random input data – ADMOS 2023

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ABSTRACT

Parabolic partial differential equations (PDEs) with small random input data appear in a wide range of physical and real-world applications, for instance, in glaciology. In this work, we propose and analyze residual-based *a posteriori* error estimates for such equations in the $L^2_P(\Omega; L^\infty(0,T; L^2(D)))$-norm, where $(\Omega, \mathcal{F}, P)$ is a complete probability space, D is the physical domain, $T > 0$ is the final time. To this end, we apply the perturbation technique to deal with uncertainty [2019, Arch. Comput. Methods Eng., 26, pp. 1313-1377]. In view of this technique, solving a PDE with small random input data is equivalent to solving decoupled deterministic problems. To approximate solution for these problems, we employ finite element method for the physical space approximation and backward Euler time-stepping scheme for time discretization. To obtain optimality in space, we employ the elliptic reconstruction operator [2003, SIAM J. Numer. Anal., 41, pp. 1585-1594]. The results could be seen as a generalization of the work presented in [2006, Math. Comput., 75, pp. 1627-1658] for the deterministic parabolic PDEs to the parabolic PDE with small uncertainties. Numerical investigations confirm the theoretical findings.

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