Review of new flow friction equations: Constructing Colebrook's explicit correlations accurately

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List of Equations:

$$\frac{1}{\sqrt{f}} = -2 \cdot log_{10}\left(\frac{2.51}{Re} \cdot \frac{1}{\sqrt{f}} + \frac{\varepsilon}{3.71}\right) \tag{1}$$

$$\left.\begin{aligned} \frac{1}{\sqrt{f}} &= \frac{z}{2.51} \cdot (B + y) \\ z &= \frac{2 \cdot 2.51}{ln(10)} \\ A &= \frac{Re}{z} \cdot \frac{\varepsilon}{3.71} \\ B &= ln(Re) - ln(z) \\ x &= A + B \\ y &= W(e^x) - x = \omega(x) - x \end{aligned}\right\} \tag{2}$$

$$s_1(x) = ln(x) \cdot \left(\frac{1}{x} - 1\right) \tag{3}$$

$$s_2(x) = \frac{ln(x)}{2 \cdot x^2} \cdot (ln(x) - 2) \tag{4}$$

$$s_3(x) = \frac{ln(x)}{6 \cdot x^3} \cdot (2 \cdot ln^2(x) - 9 \cdot ln(x) + 6) \tag{5}$$

$$s_4(x) = \frac{ln(x)}{12 \cdot x^4} \cdot (3 \cdot ln^3(x) - 22 \cdot ln^2(x) + 36 \cdot ln(x) - 12) \tag{6}$$

$$s_5(x) = \frac{ln(x)}{60 \cdot x^5} \cdot (12 \cdot ln^4(x) - 125 \cdot ln^3(x) + 350 \cdot ln^2(x) - 300 \cdot ln(x) + 60) \tag{7}$$

$$y_1 \approx s_1(x) \tag{8}$$

$$y_2 \approx s_1(x) + s_2(x) \tag{9}$$

$$y_3 \approx s_1(x) + s_2(x) + s_3(x) \tag{10}$$

$$y_4 \approx s_1(x) + s_2(x) + s_3(x) + s_4(x) \tag{11}$$

$$y_5 \approx s_1(x) + s_2(x) + s_3(x) + s_4(x) + s_5(x) \tag{12}$$

$$y \approx s_1(x) + 0.00056 \tag{13}$$

$$y \approx s_1(x) + s_2(x) - 0.0014 \tag{14}$$

$$y \approx s_1(x) + s_2(x) + s_3(x) - 0.000093 \tag{15}$$

$$\left.\begin{aligned} y &\approx -ln(x) + \frac{ln(x)}{x} + \xi \\ \xi &= \frac{0.3896 \cdot ln(x) \cdot (ln(x) - 1) - 0.9873}{0.8421 \cdot x^2 + 0.01274 \cdot x \cdot (ln(x))^4 + x + 5.882} \end{aligned}\right\} \tag{16}$$

$$y_{sr} \approx -\ln(x) + \frac{1.038 \cdot \ln(x)}{x + 0.332} \tag{17}$$

$$y_{sr} \approx -\ln(x) + \frac{1.0119 \cdot \ln(x)}{x} + \frac{\ln(x) - 2.3849}{x^2} \tag{18}$$

$$y_{sr} \approx -ln(x) + \frac{ln(x)}{x - 0.5564 \cdot ln(x) + 1.207} = Y \tag{19}$$

$$y_{sr} \approx Y - \xi_1 = Y - \frac{x \cdot Y^2 + 3.0636 \cdot x \cdot Y + 18.58}{19.5 \cdot (Y^2 \cdot x^2 + x^3) + 169.9 \cdot Y^2 + 1260 \cdot x + 18178} \tag{20}$$

$$\left.\begin{aligned} \frac{1}{\sqrt{f}} &\approx 0.8686 \cdot \left(B - C + \frac{C}{x}\right) \\ A &\approx \frac{Re \cdot \varepsilon}{8.0878} \\ B &\approx ln(Re) - 0.7794 \\ C &= ln(x) \\ x &= A + B \end{aligned}\right\} \tag{21}$$

$$\frac{1}{\sqrt{f}} \approx 0.8686 \cdot \left[B - C + \frac{C}{x} + 0.00056\right] \tag{22}$$

$$\frac{1}{\sqrt{f}} \approx 0.8686 \cdot \left[B - C + \frac{C}{x} + \frac{C}{2 \cdot x^2} \cdot (C - 2) - 0.0014\right] \tag{23}$$

$$\frac{1}{\sqrt{f}} \approx 0.8686 \cdot \left[B - C + \frac{C}{x} + \frac{C}{2 \cdot x^2} \cdot (C - 2) + \frac{C}{6 \cdot x^3} \cdot (2 \cdot C^2 - 9 \cdot C + 6) - 0.000093\right] \tag{24}$$

$$\left.\begin{array}{c}\frac{1}{\sqrt{f}} \approx 0.86858896 \cdot \left[B - C + \frac{C}{x} + \varsigma\right] \\ \xi = \frac{0.3896 \cdot C \cdot (C-1) - 0.9873}{0.8421 \cdot x^2 + 0.01274 \cdot x \cdot C^4 + x + 5.882} \\ A \approx \frac{Re \cdot \varepsilon}{8.0884} \\ B \approx ln(Re) - 0.779397 \\ C = ln(x) \\ x = A + B\end{array}\right\} \quad (25)$$

$$\frac{1}{\sqrt{f}} \approx 0.8686 \cdot \left[B - C + \frac{1.038 \cdot C}{0.332 + x}\right] \quad (26)$$

$$\frac{1}{\sqrt{f}} \approx 0.8686 \cdot \left[B - C + \frac{1.0119 \cdot C}{x} + \frac{C - 2.3849}{x^2}\right] \quad (27)$$

$$\frac{1}{\sqrt{f}} \approx 0.8686 \cdot \left[B - C + \frac{C}{x - 0.5564 \cdot C + 1.207}\right] \quad (28)$$

$$\frac{1}{\sqrt{f}} \approx 0.8685972 \cdot \left[B - C + \frac{C}{x - 0.5588 \cdot C + 1.2079}\right] \quad (29)$$

$$\left.\begin{array}{c}\frac{1}{\sqrt{f}} \approx 0.868589 \cdot [B + Y - \xi_1] \\ \xi_1 = \frac{x \cdot Y^2 + 3.0636 \cdot x \cdot Y + 18.58}{19.5 \cdot (Y^2 \cdot x^2 + x^3) + 169.9 \cdot Y^2 + 1260 \cdot x + 18178} \\ Y = -C + \frac{C}{x - 0.5564 \cdot C + 1.207} \\ A \approx \frac{Re \cdot \varepsilon}{8.088387} \\ B \approx ln(Re) - 0.7793975 \\ C = ln(x) \\ x = A + B\end{array}\right\} \quad (30)$$

Notations

The following symbols are used in this paper:

A	variable that depends on Re and ε (dimensionless); $A = \frac{Re}{z} \cdot \frac{\varepsilon}{3.71}$
B	variable that depends on Re and z (dimensionless); $B = ln(Re) - ln(z) \approx ln(Re) - 0.7794$
C	variable that depends on x (dimensionless); $C = ln(x)$
f	Darcy (Moody) flow friction factor (dimensionless) – main output parameter
s	approximation of y by a truncated series about infinity; $y_i = \sum_{i=1}^{+\infty} s_i$
Re	Reynolds number (dimensionless) – main input parameter
x	variable that depends on A and B (dimensionless); $x = A + B$
y	approximation of $\omega(x) - x$, i.e. of $W(e^x) - x$
y_{sr}	approximation of $\omega(x) - x$, i.e. of $W(e^x) - x$ found by symbolic regression
Y	approximation of $\omega(x) - x$ from Eq. (19) and used in Eq. (20)
z	constant (dimensionless); $z = \frac{2 \cdot 2.51}{ln(10)}$
α	constant (dimensionless); half of the smallest term in the asymptotic series
ε	Relative roughness of inner pipe surface (dimensionless) – main input parameter
$\delta_{\%}$	Relative error (%); $\delta_{\%} = \frac{f - f_i}{f} \cdot 100\%$
ξ and ξ_1	error-functions developed using symbolic regression
e	exponential function
log_{10}	logarithm with base 10
ln	natural logarithm
W	Lambert W-function
ω	Wright ω-function
i	Counter
wrightOmega	Matlab's built-in function
Globalsearch	Matlab's built-in solver