Enforcing boundary conditions for finite elements in problems requiring continuity higher than C^0

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ABSTRACT

In [1] two families of two-dimensional finite element interpolation functions of general degree with general continuity are presented. These nodal interpolation functions can be used to obtain "standard" finite elements, which by nodal assembly lead to approximations with a prescribed order of continuity.

The simplest and most classical application of these elements is the problem of modelling bending plates using Kirchhoff's Theory, which assumes that the rotations of the fibres are related with the (vectorial) derivatives of the transverse displacement, so that the generalised strains of the problem, the curvatures, are the (tensorial) second derivative of the transverse displacement. In this case a C^1 approximation is a requirement for compatibility, and either the aforementioned functions or the classical Hsieh, Clough and Tocher's triangle (HCT) [2], Fraeijs de Veubeke's quadrilateral (FVQ) [3] and Argyris TUBA triangular elements [4] can be used.

Combining the problem of Kirchhoff's bending with strain-gradient elasticity leads to a requirement of C^2 continuity, which can be directly addressed by the interpolation functions in [1].

Applying such finite elements to simple domains, where each support (Dirichlet boundary condition) is located along a reference axis is a "reasonably simple" task. However, when the supports are located along arbitrary directions and non-homogenous conditions are considered, the situation is considerably more delicate.

If boundary conditions are imposed by explicitly removing the prescribed generalised nodal displacements it is quite easy to introduce errors in the model, wherein incomplete (allowing movements that should be blocked) or excessive (blocking movements which are supposed to be free) boundary conditions are imposed.

As an alternative, it is possible to work with hybrid conditions along the sides. In this case the difficulty is associated with the possibility of imposing dependent constraints, which imply a system of equations that is singular. Furthermore, in the case of non-homogeneous boundary conditions, such dependent constraints can be inconsistent.

In this presentation these questions are discussed and illustrated. A general approach (valid for arbitrary orders of continuity) to formulate the hybrid conditions is discussed. In it the dependent conditions are identified and set aside during the assembly process. This allows to check whether they are consistent after the solution is obtained.

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