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International Center for Numerical Methods in Engineering
Gran Capitán s/n, 08034 Barcelona, España

HYBRID CONTROL OF BUILDINGS WITH NONLINEAR BASE ISOLATION

A. H. Barbat[1], J. Rodellar[2], E. P. Ryan[3] and N. Molinares[4]

ABSTRACT

A hybrid seismic control system for building structures is considered, which combines a class of passive nonlinear base isolator with an active control system. The active control forces are applied to the structural base with the objetive of reducing its displacements. An adaptive control law is fornulated to compute the control forces in order to assure a form of stable behaviour of the system under seismic excitation and in the presence of uncertainties in the characteristics of the building and the base isolator. Numerical simulations are performed to assess the effectiveness of the hybrid control system. The global behaviour of the structure-base isolator system is such that the absolute base displacement is significantly reduced, the price paid being a slight increase of the response of the structure.

INTRODUCTION

The main objective in seismic design of a structure is to keep its response within limits defined by safety, service and human comfort conditions. This objective can be achieved by applying traditional seismic design principles, which assume that earthquakes act upon the structure across its fixed base, to assure partial dissipation of the induced energy. Plastic deformation of certain members can occur and, as a consequence, the structure is damaged to a certain degree.

This disadvantage can be avoided by using passive control systems such as base isolators or a variety of energy dissipation devices. Base isolation attempts to uncouple the structure from the seismic ground motion by means of replaceable devices, placed between the building and the foundation, capable of absorbing part of the energy induced by earthquakes. Both elastomeric bearings, which are hysteretic systems, and

[1] Department of Structural Mechanics, Technical University of Catalonia, Barcelona, Spain

[2] Department of Applied Mathematics III, Technical University of Catalonia, Barcelona, Spain

[3] School of Mathematical Sciences, University of Bath, England

[4] Graduate student, Technical University of Catalonia, Barcelona, Spain

sliding bearings, which are frictional systems (Kelly 1986, Skinner et al. 1993), are used as seismic isolators. Base isolation systems can provide in certain circumstances a level of performance beyond the normal design requirements. The most important disadvantage of such systems is the dependence of their effectiveness on the frequency of the excitation. Moreover, they cannot be applied in the case of tall or heavy structures, due to the size of the dynamic forces involved and to the risk of endangering the global stability of the structure.

The seismic behaviour of buildings can be improved by incorporating into the structure mechanisms equipped with external actuators capable of producing control forces with the aim of reducing their seismic response. These mechanisms are included in a closed loop governed by a computer and calculate in real time the control forces to be applied to the structure as a function of the measured structural response. Such active control systems are able to adapt themselves to the frequency characteristics of the seismic action (Soong 1987), but they may require large energy to govern the actuators.

Recently another class of structural control systems, referred to as hybrid systems, has been suggested. They combine different passive and/or active systems with the objective of reducing some of the limitations (Reinhorn 1987). One of the hybrid systems with most interesting perspectives for buildings consists of the combination of a base isolation with an active system applying control forces upon the base.

The base isolation component can reduce by itself both the inter-story drift and the absolute accelerations of the structure. Thus the structure tends to behave like a rigid body, the price paid being a significant displacement of the base. The objective of the active control component is to reduce this displacement by means of forces applied on the base. From a practical point of view, this hybrid scheme is appealing since it is possible to achieve the above mentioned objective by means of a single force which, moreover, does not exceed some acceptable limits due to the high flexibility of the base isolators. From a theoretical point of view, the development of a control law to calculate the active force involves difficulties associated with the nonlinear behaviour of the base isolators and to the uncertainties in the models describing the structure-base isolator system and in the seismic excitation. A robust control law for linear systems has been proposed in a previous work (Kelly et al., 1987). Also for linear systems, the application of predictive control has been considered (Inaudi et al., 1992), as well as a form of bang-bang control (Inaudi and Kelly, 1991). The nonlinearity of the isolators has been considered in the work by Yang et al. (1992) assuming no uncertainties in the structure-base model. Some experimental works with small-scale hybrid systems have been recently reported (Feng et al., 1993; Riley et al. 1992; Nagarajaiah et al., 1992).

In this paper we present a new active control law for base isolated structures. In subsequent sections, the control problem is stated, a model of the system is described, the control strategy is presented and some issues concerning its implementation are discussed. Also numerical examples are presented to illustrate the effectiveness of the control strategy.

ACTIVE CONTROL PROBLEM

Consider the building with a hybrid control system illustrated in Figure 1. The passive component consists of a base isolator, while the active component provides control forces applied upon the structural base. We address the problem of designing a feedback control law to generate the active forces. Since the effect of these forces is additional to that of the base isolation, it is important to understand the essential behaviour of the structure under purely passive conditions before defining the objective of the active control law.

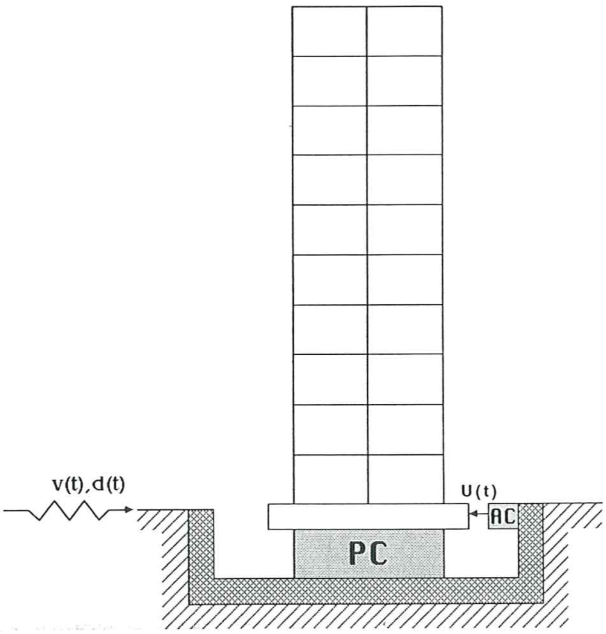


Figure 1 Building structure with a hybrid control system. **PC** – passive control system. **AC** – active control system.

When the parameters of the isolator are well tuned to the characteristics of the earthquake, good performance of the structure, with a reduced inter-story drift, may be expected. Nevertheless, this desirable inter-story behaviour can be associated with unacceptably high base displacements. Therefore, the main purpose of the active control forces is to reduce these base displacements. However, since the base and the stories are coupled, the application of forces at the base level can produce a negative effect, increasing the inter-story drift as compared to that of the structure with purely passive control. Thus the active control law has to limit this effect as well.

Some essential difficulties in the active control problem arise from the nonlinear behaviour of the isolation system, the uncertainties in the models describing the structure and the isolator as well as the uncertainties inherent to the seismic actions. These difficulties are magnified by the fact that the overall structure-base system is to be controlled by applying forces only to the base.

In previous work (Rodellar and Ryan 1993), a control method has been proposed for a class of uncertain nonlinear mechanical systems that can be decomposed into two coupled subsystems with feedback control on one of them. Under appropriate assumptions, this control assures a form of practical stability. The control law has an adaptive nature that does not require *a priori* knowledge of either the system parameters or the external excitations. The hybrid structural control problem we are dealing with herein can be cast within the framework of this class of systems, the base being the subsystem with control and the structure the other one. Following the guidelines detailed in (Rodellar and Ryan 1993), the objective of the active control law is essentially to drive the displacement and velocity of the base asymptotically to an arbitrarily small neighborhood of the equilibrium position, while keeping the inter-story drift within acceptable bounds.

SYSTEM DESCRIPTION

The dynamic behaviour of the structure with the hybrid control system can be described by means of a model composed of two coupled systems : Σ_r (the building) and Σ_c (the base). It is assumed that the structure behaves linearly due to the effect of the base isolation. The behaviour of the isolator is assumed nonlinear, including hysteretical and/or frictional characteristics. The motion of the structure is described by a vector $\boldsymbol{D}$ which represents the horizontal displacements of the n degrees of freedom respect to an inertial frame, while the displacement of the structural base is described by a single degree of freedom with horizontal displacement d_b relative to the above mentioned frame. The dynamic excitation is produced by a horizontal seismic ground motion, characterized by a displacement $d(t)$ and a velocity $v(t)$. A single horizontal control force $u(t)$ acts upon the structural base. Thus, the equations of motion are

$$\Sigma_r : \qquad \boldsymbol{M}\ddot{\boldsymbol{D}} + \boldsymbol{C}\dot{\boldsymbol{D}} + \boldsymbol{K}\boldsymbol{D} = \boldsymbol{C}\boldsymbol{J}\dot{d}_b + \boldsymbol{K}\boldsymbol{J}d_b \tag{1a}$$

$$\Sigma_c : \qquad \begin{aligned} & m_b\ddot{d}_b + [c_b + \boldsymbol{J}^T\boldsymbol{C}\boldsymbol{J}]\dot{d}_b + [k_b + \boldsymbol{J}^T\boldsymbol{K}\boldsymbol{J}]d_b \\ & - \boldsymbol{J}^T\boldsymbol{C}\dot{\boldsymbol{D}} - \boldsymbol{J}^T\boldsymbol{K}\boldsymbol{D} - c_b v - k_b d + f(d_b, \dot{d}_b, d, v) = u \end{aligned} \tag{1b}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices of the structure, respectively. The vector $\boldsymbol{J}$ expresses the rigid body motion according to the degrees of freedom of the model (in this case it is an unit vector). m_b, c_b and k_b are the mass, damping and stiffness of the base. The last two parameters correspond to the elastic and damping forces which appear on the base due to the linear effects of the isolator; f is an additional the horizontal force produced on the structural base by nonlinearities in the isolator.

Before formulating the control law, the description of the system is completed with the following assumptions:

1. The mass matrix $\boldsymbol{M}$ is invertible and the damping and stiffness matrices $\boldsymbol{C}$ and $\boldsymbol{K}$ are positive definite.
2. The displacement and the velocity of the seismic ground motion are bounded so that the following holds:

$$|c_b v(t) + k_b d(t)| \leq \epsilon \tag{2}$$

   for almost all t, ν being an unknown scalar.
3. The function f is such that, for some known continuous function γ', the following holds for some (unknown) scalar α':

$$|f(d_b, \dot{d}_b, d(t), v(t))| \leq \alpha' \gamma'(d_b, \dot{d}_b) \tag{3}$$

   for almost all t and all d_b and $\dot{d}_b$.

CONTROL STRATEGY

Fix $\lambda > 0$ and $\bar{k} > 0$ (design parameters). Define

$$p_b(t) = \eta d_b(t) + \dot{d}_b(t) \tag{4}$$

and

$$\gamma(d_b, \dot{d}_b, \boldsymbol{D}, \dot{\boldsymbol{D}}) = [1 + \gamma'(d_b, \dot{d}_b)^2 + d_b^2 + \dot{d}_b^2 + \sum_{i=1}^{n} (D_i^2 + \dot{D}_i^2)]^{1/2} \tag{5}$$

The control strategy is formulated as

$$u(t) = -k(t)[p_b(t) + \gamma(d_b, \dot{d}_b, \boldsymbol{D}, \dot{\boldsymbol{D}}) s_\lambda(p_b)] \tag{6a}$$

$$\dot{k}(t) = \bar{k}[|p_b(t)| + \gamma(d_b, \dot{d}_b, \boldsymbol{D}, \dot{\boldsymbol{D}})] d_\lambda(p_b) \tag{6b}$$

$$k(0) = k_0 \quad \text{(initial condition)} \tag{6c}$$

In equations (6), s_λ, d_λ are functions defined as

$$s_\lambda(p_b) = \begin{cases} \frac{p_b}{|p_b|}, & \text{if} \quad |p_b| \geq \lambda \\ \frac{p_b}{\lambda}, & \text{if} \quad |p_b| < \lambda \end{cases} \tag{7a}$$

$$d_\lambda(p_b) = \begin{cases} |p_b| - \lambda, & \text{if} \quad |p_b| \geq \lambda \\ 0, & \text{if} \quad |p_b| < \lambda \end{cases} \tag{7b}$$

The rationale for this control strategy lies in its capability of assuring a form of practical stability. We omit here a detailed stability analysis, which can be found in the reference (Rodellar et al. 1993). We only summarize the main result in terms of its physical interpretation.

By substituting the equations (6) into the equations (1), it can be considered that the global controlled system is characterized by the set of state variables, namely $(d_b, \dot{d}_b, \boldsymbol{D}, \dot{\boldsymbol{D}}, k)$. In this case, a stability analysis, as the one detailed in (Rodellar et al. 1993), can show that for $\lambda > 0$ and for any initial condition of the system, the following properties are satisfied:

(1) $\lim_{t\to\infty} k(t)$ exists and is finite.

(2) The state of the base, characterized by its coordinates $(d_b, \dot{d}_b)$, tends asymptotically to a ball of radius λ centered in zero.

(3) The state of the structure, characterized by the vectors $\boldsymbol{D}$ and $\dot{\boldsymbol{D}}$, tends asymptotically to a ball (centered at zero) with radius proportional to λ, however the proportionality constant depends on unknown bounds on the system uncertainties and so cannot be calculated *a priori.*

It is important to emphasize the paucity of knowledge about the system that is required *a priori*. From the control law, it is apparent that parameters of the system, such as masses, damping and stiffness are not present, so they need not to be nown to the designer. Also the external seismic excitation is unknown, assuming only that it is bounded by unknown constant as in (2). Regarding the nonlinear force f produced by the isolators, the control strategy allows it to be unknown but bounded, modulo arbitrary scaling, by a known continuous function as in (3). This function enters into the control law in the definition of the function γ in (5). The adaptive nature of the control law, associated with the time-varying gain $k(t)$, guarantees the above stability properties be assured for any realization of the unknown parameters satisfying assumptions 1-3.

For implementation of the control law, the absolute displacement and velocity responses of the base and the structure are required as feedback information. With this information, equations (6) and (7) are used to calculate the value of the control $u(t)$. The parameters (positive-valued) η, λ, k_0 and $\bar{k}$ are open to choice by the designer. λ is the most significant of these parameters. It defines the guaranteed stability ball and has a primary influence in achieving the control objective.

NUMERICAL EXAMPLES

For the purpose of assessing the effectiveness of the above control law, numerical simulations are presented for two types of base isolated structures. In the first example, the structure is modelled as a single degree of freedom system. The second example considers a 10-story shear building.

For the force f, we need to specify the model describing the nonlinear behaviour of the isolating devices. Two types of behaviours are considered, one frictional and the other one hysteretical. In Figure 2, three isolators are defined for different combinations of the above behaviours. These are the New Zealand (NZ) isolator (Figure 2(a)) (Su et al. 1989), characterized by a purely hysteretical performance, the NZ with a purely frictional system connected in series (NZ+F_SER) (Figure 2(b)) and the NZ with a purely frictional system connected in parallel (NZ+F_PAR) (Figure 2(c)).

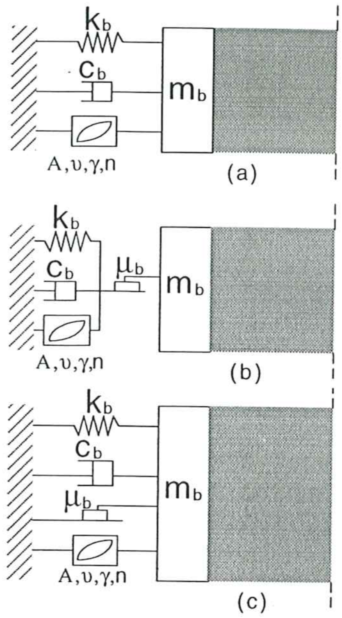


Figure 2 Base isolation systems: (a) New Zealand (NZ); (b) NZ with frictional system connected in series (NZ+F_SER); (c) NZ with frictional system connected in parallel (NZ+F_PAR).

For frictional isolation devices, the formulation of a purely frictional force f_1 is proposed in the references by Nagarajaiah et al. (1989 and 1991) by means of the equation

$$f_1 = -\text{sgn}(x)[\mu_{max} - \Delta\mu \, e^{-\beta|\dot{x}|}]Q \tag{8}$$

where Q is the force normal to the friction surface and μ is the friction coefficient. β is a constant, μ_{max} is the coefficient for high sliding velocity and $\Delta\mu$ is the difference between μ_{max} and the friction coefficient for low sliding velocity, which is considered to be constant. x is the displacement of the base relative to the ground, i.e., $x = d_b - d$.

One way of formulating a hysteretical isolation device is to use constitutive models defined by means of differential equations. In this paper, with the purpose of illustrating the application of the proposed control law, Wen's uniaxial model is adopted (Wen 1976). The hysteretic force f_2 is expressed by means of the following equation

$$f_2 = f^y z \tag{9}$$

where f^y is the yielding force and z is an auxiliary variable defined by the differential equation

$$\frac{dz}{dx} = A \pm (\nu_1 \pm \nu_2) z^m \tag{10}$$

The parameters A, ν_1, ν_2 and m allow the description of the hysteretical cycles for a wide class of materials, ranging from elastic to elasto-plastic ones (Wen 1976).

For the control law to be implemented, the force f is required to be bounded in the form expressed in (3). In the cases we consider here, f is a combination (as illustrated in Figure 2) of forces of the types defined by equations (8)-(10). Therefore, we consider that the force f is bounded by a constant. This implies that the function γ' in (3), also appearing in the control law in (5), is just equal to 1. It simplifies the implementation of the control law and, as ultimately verified through the numerical simulations, gives satisfactory results.

Example 1

The purpose of this example is to analyze the effectiveness of the three hybrid control systems with the proposed control law for a frequency range covering a wide class of building structures. This effectiveness is compared with that of the corresponding purely passive systems. In order to do this, a single degree of freedom model with a mass of 6×10^5 kg is considered. Its stiffness is varied in such a way that the natural period lies in the range between 0.1 s and 1.0 s. The mass of the base is also 6×10^5 kg, while its stiffness is 1.184×10^7 N/m. The characteristics of the frictional bearings are: $\mu = 0.1$, $\mu_{max} = 0.185$, $\Delta\mu = 0.09$ and $\beta = 2.0$. For the hysteretic bearings the following parameters are used: $f^y = 1.5 \times 10^3$ N, $A = 1.0$, $\nu_1 = \nu_2 = 0.5$ and $m = 1$.

The model is subjected to the El Centro (1940) earthquake. For each natural period considered in the analysis, the structural response is simulated solving the equations of motion (1) by a Newmark integration method and the equation (10) by a Runge-Kutta scheme of 4th order. This simulation is performed for the three isolators under purely passive conditions (active control is disconnected) as well as for the hybrid case (active control is operating). The parameters associated to the control law have chosen: $\nu = 1$, $\lambda = 0.42$, $\bar{k} = 10^7$ and $k_0 = 0$.

The maximum response values are plotted in Figures 3-5 against the natural period: the absolute base displacement in Figure 3, the displacement of the structure relative to the base in Figure 4 and the absolute acceleration of the structure in Figure 5. From these figures, it can be noted that the maximum absolute displacement is significantly reduced by the hybrid system. This was the main objective of the proposed control law. The price paid is the increase of the maximum relative displacement and the maximum absolute acceleration of the structure, which still remain within adequate bounds. In Figure 6 the maximum value of the control force is plotted against the period for the three isolators, showing an acceptable magnitude.

A more detailed frequency analysis can be seen in figures 7-13. In these figures, a comparison is performed between the effect on the structural behaviour of a pure

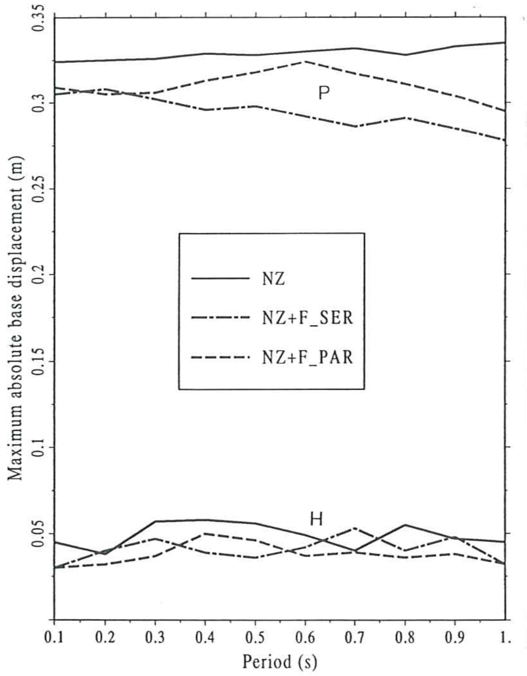


Figure 3 Plot of the maximum absolute base displacement against the structural period for the model with passive (P) and hybrid (H) control systems.

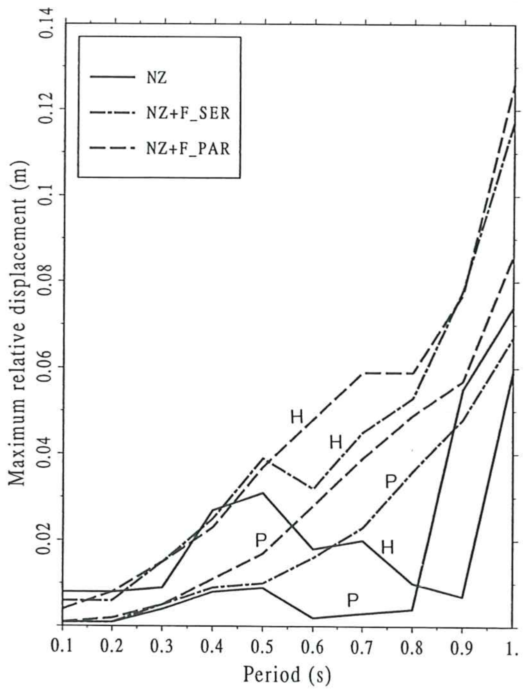


Figure 4 Plot of the maximum relative displacement against the structural period for the model with passive (P) and hybrid (H) control systems.

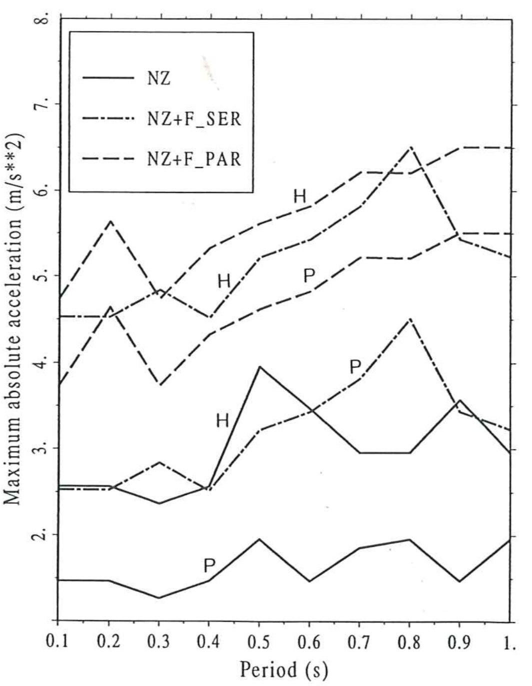


Figure 5 Plot of the maximum absolute acceleration against the structural period for the model with passive (P) and hybrid (H) control systems.

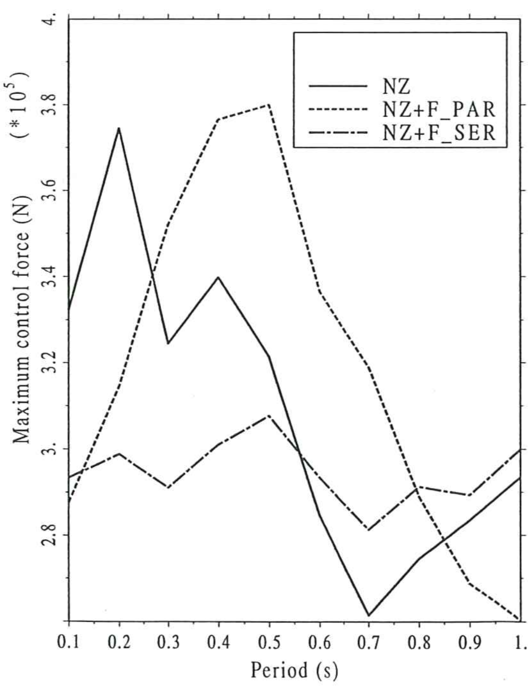
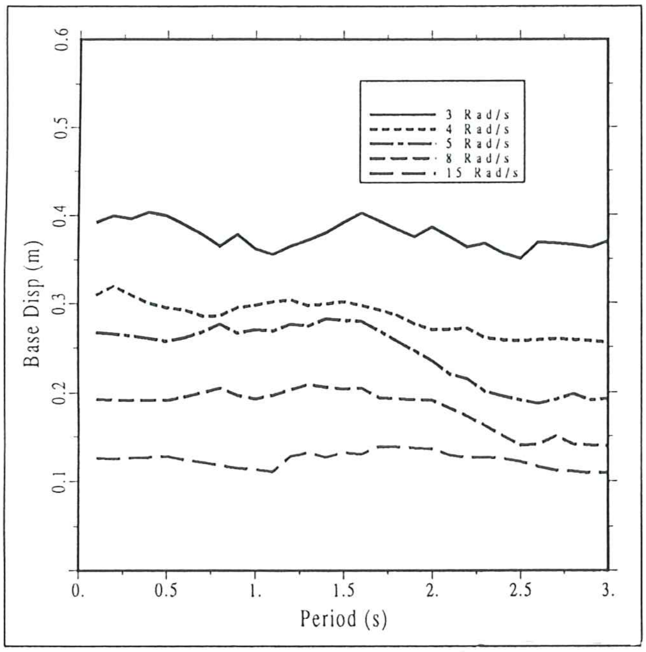


Figure 6 Plot of the maximum control force in function of the structural period for the model with hybrid control system.



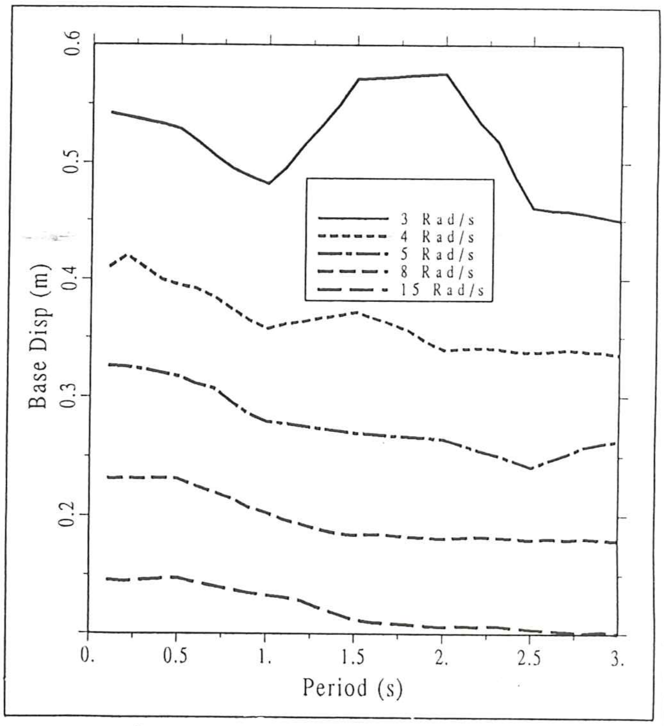
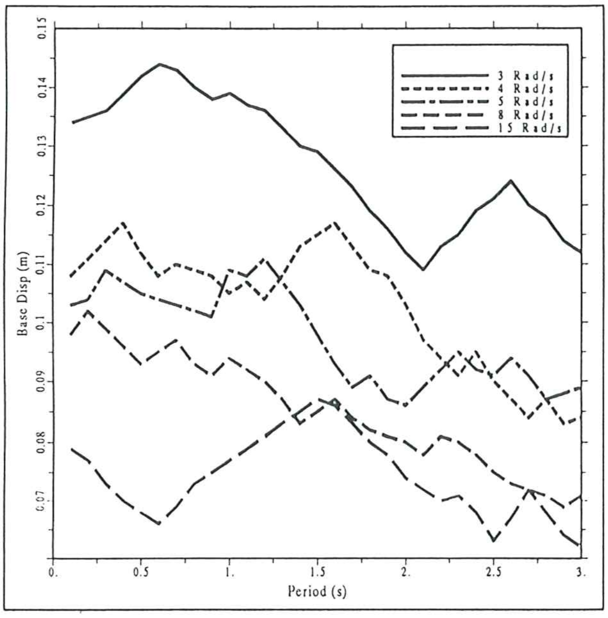


Figure 7 Plot of the maximum absolute base displacement against the structural period for the model with passive base isolation. (a) pure friction case. (b) hysteretic case.



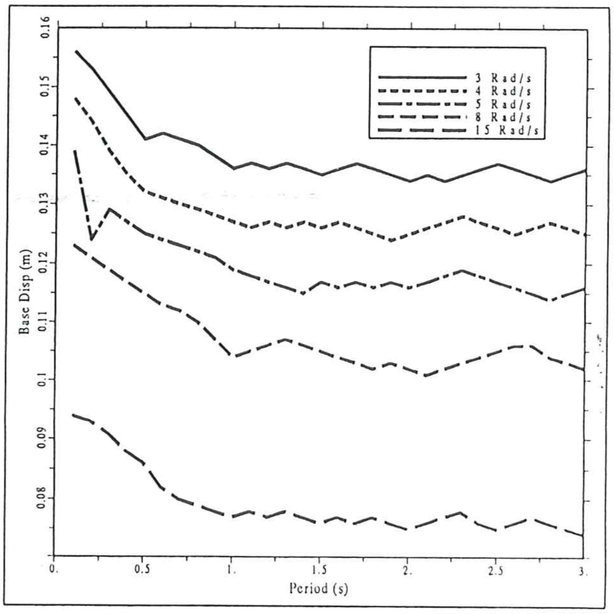
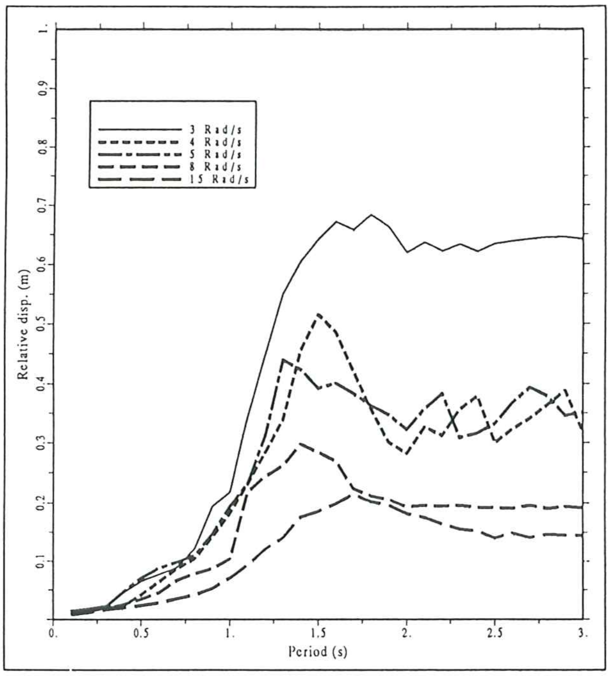


Figure 8 Plot of the maximum absolute base displacement against the structural period for the model with active base isolation. (a) pure friction case. (b) hysteretic case.



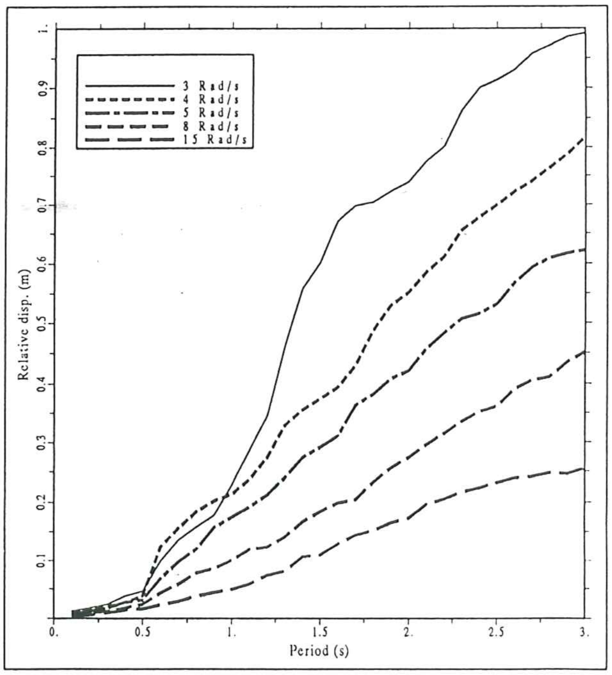
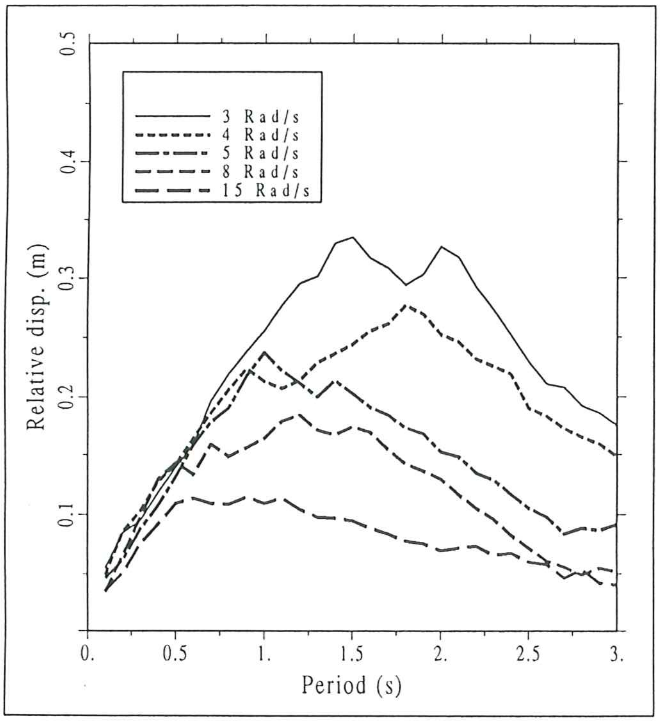


Figure 9 Plot of the maximum relative displacement against the structural period for the model with passive base isolation. (a) pure friction case. (b) hysteretic case.



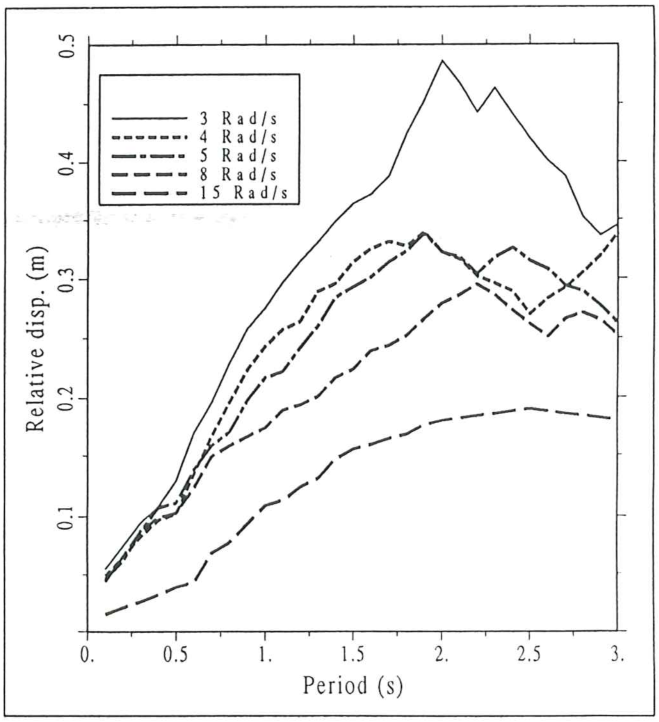
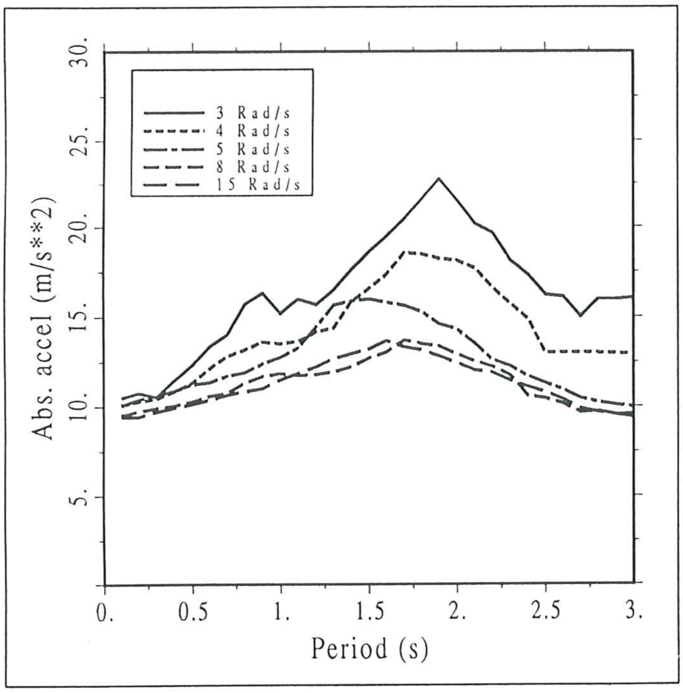


Figure 10 Plot of the maximum relative displacement against the structural period for the model with active base isolation. (a) pure friction case. (b) hysteretic case.



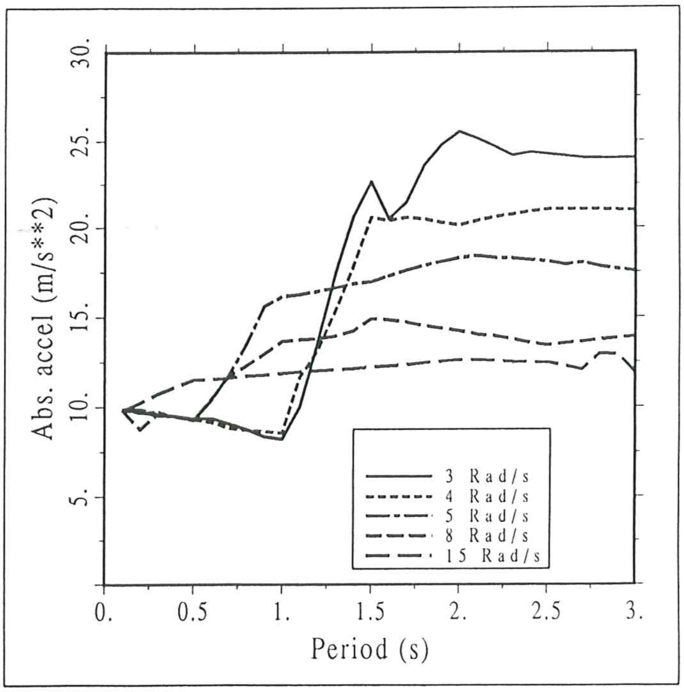
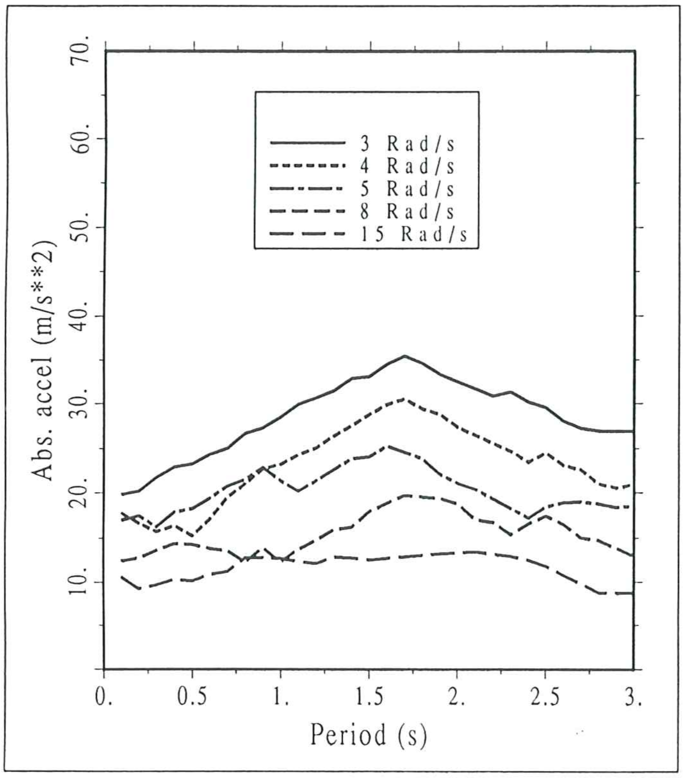


Figure 11 Plot of the maximum absolute acceleration against the structural period for the model with passive base isolation. (a) pure friction case. (b) hysteretic case.



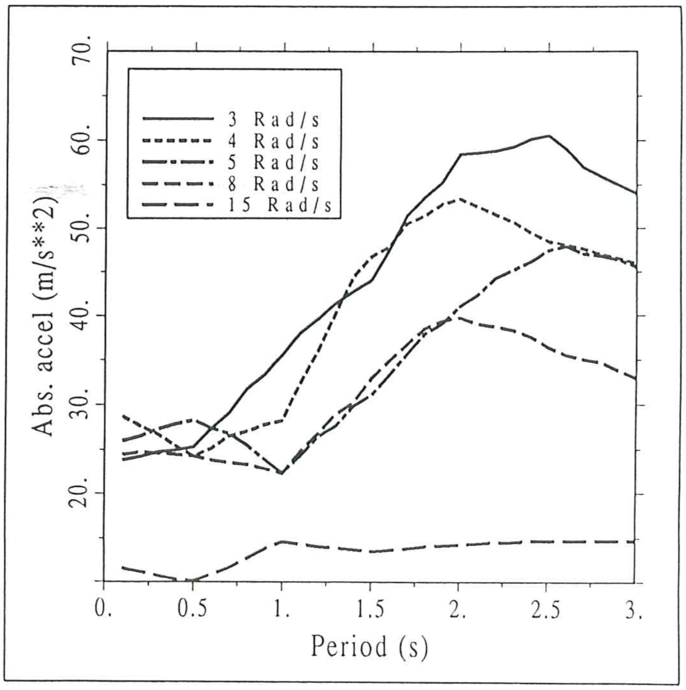
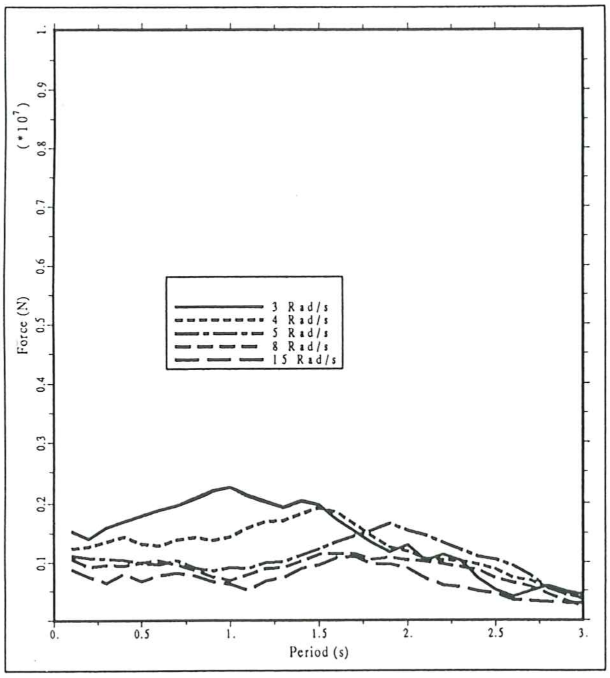


Figure 12 Plot of the maximum absolute acceleration against the structural period for the model with active base isolation. (a) pure friction case. (b) hysteretic case.



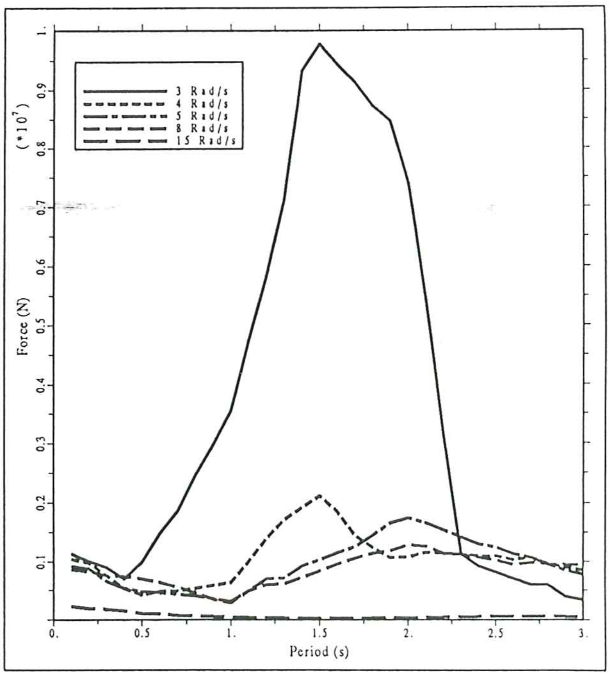


Figure 13 Plot of the maximum control force in function of the structural period for the model with hybrid control system. (a) pure friction case. (b) hysteretic case.

friction base isolation system and a hysteretic one. Results are included for both the pure passive case and the hybrid case. The single degree of freedom model has been subjected to 5 harmonic ground motions, having the same amplitude, but different frequencies.

Example 2

To assess the behaviour of the hybrid system in a more demanding situation, consider now a 10 story building structure as illustrated in Figure 1. The mass of each level, including that of the base, is 6×10^5 kg. The stiffness varies in 5×10^7 N/m between levels, from 9×10^8 N/m the first one to 4.5×10^8 N/m the top one. The damping ratio is 0.05. The characteristics of the base isolation devices are the the same as in the single degree of freedom case.

For the case of the New Zealand base isolator and considering El Centro (1940) earthquake as excitation, the control law has been applied. Figures 14 and 15 show the time histories of the absolute base displacement and the displacement of the top floor relative to the base for the passive and hybrid case in which the parameter $\lambda = 0.9$. In Figure 14, it can be observed that the absolute base displacement in the hybrid case rapidly enters into a neighborhood around zero with a response level much more lower than the corresponding to the passive case. From Figure 15 it can be seen that there is an initial period in which the hybrid relative displacement of the top floor is higher than the passive one, but it reduces more rapidly than in the passive case to a neighborhood centered at zero. The behaviour observed in Figures 14 and 15 is according with the theoretical objective of the control law. In Figure 16 the time variation of the absolute response acceletration of the top floor has been represented, while Figure 17 shows that the corresponding control force is reasonable.

As we mentioned before, λ is the most important parameter in the control law, since it defines the "size" of the neighborhood to which the controlled base response has to approach. In order to evaluate its influence, Figures 18 and 19 show the maximum values of the relative top floor displacement and the control force for different values of λ. An increase of λ produces higher maximum values of the relative displacement and lower maximum control forces. This is as expected, since an increase of λ implies a less demanding control.

In order to have more detailed insight on the structural behaviour, it can be useful to examine the inter-story drift ratio, which is defined as the difference between the displacements of two adjacent floors divided by the floor height. Figure 20 describes the variation of the maximum inter-story drift ratio with the floor level for the three base isolators in the cases purely passive and hybrid respectively. For comparison purpose, the same is shown for the structure with fixed base. An increase of the maximum values for the hybrid cases are observed as compared with the passive cases. Similar pattern can be seen in Figure 21, in which the maximum floor accelerations are plotted for the same cases. Nevertheless, when considering together both the structure and the base, an improvement of the global behaviour can be recognized.

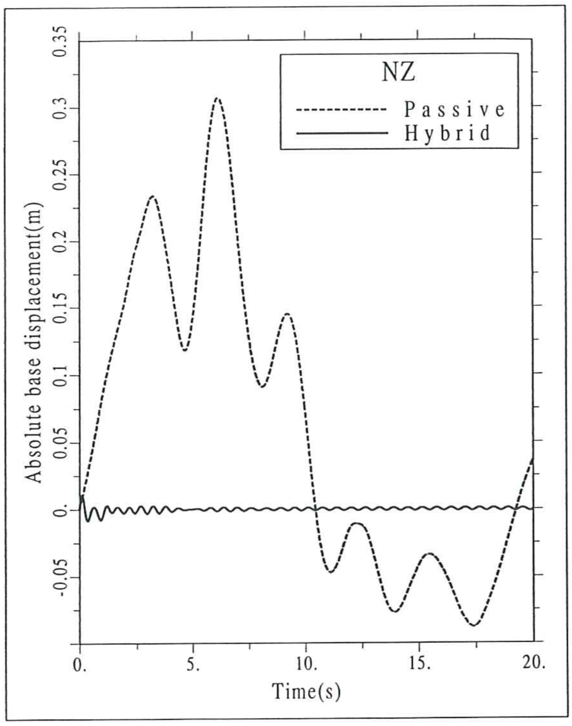


Figure 14 Absolute base displacement of the 10 degrees of freedom model.

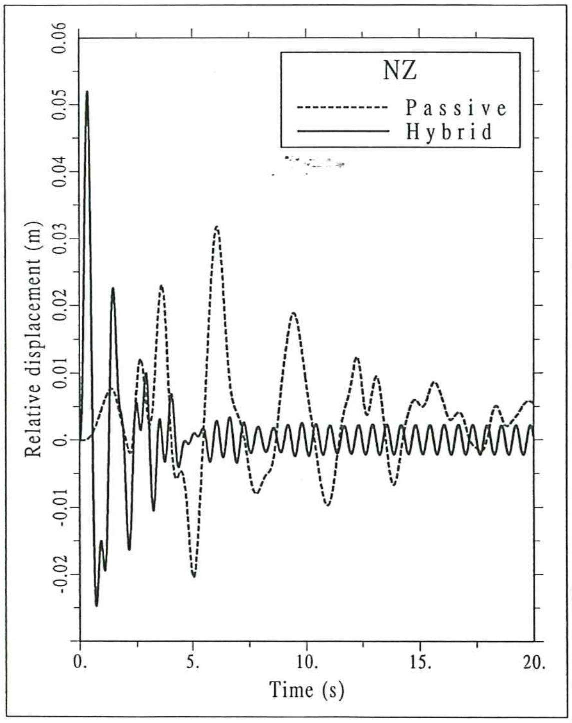


Figure 15 Top floor displacement relative to the base.

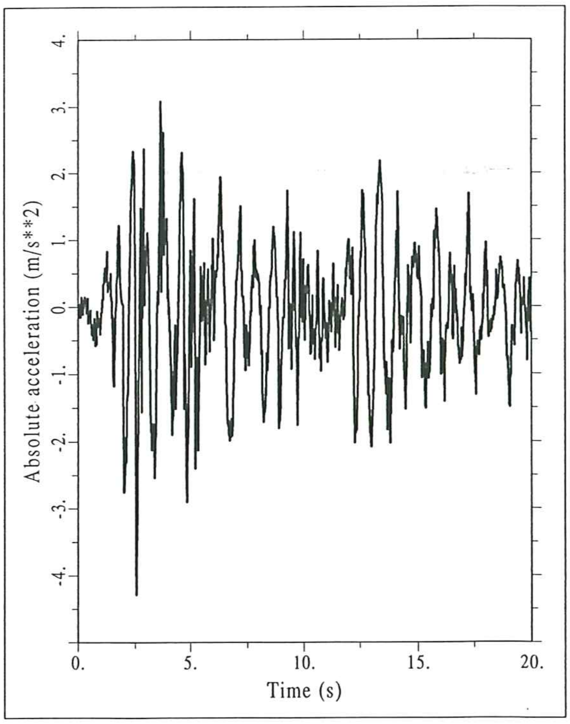


Figure 16 Absolute acceleration of the top floor

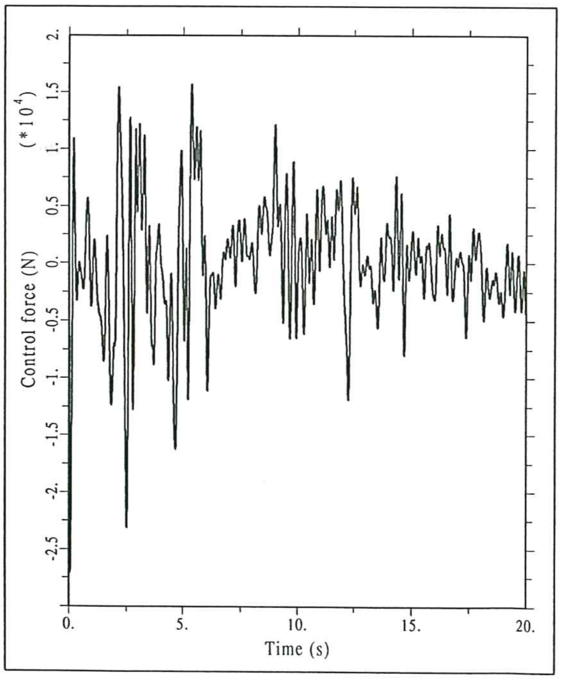


Figure 17 Active control force.

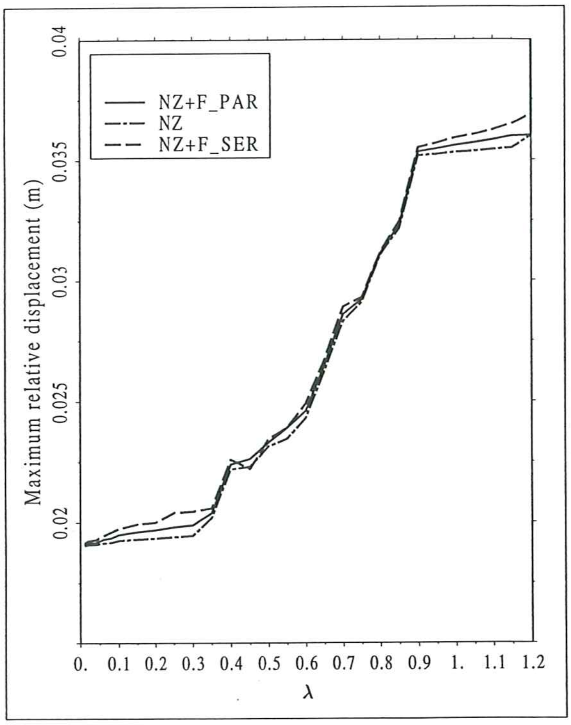


Figure 18 Maximum top floor relative displacement model for different values of λ.

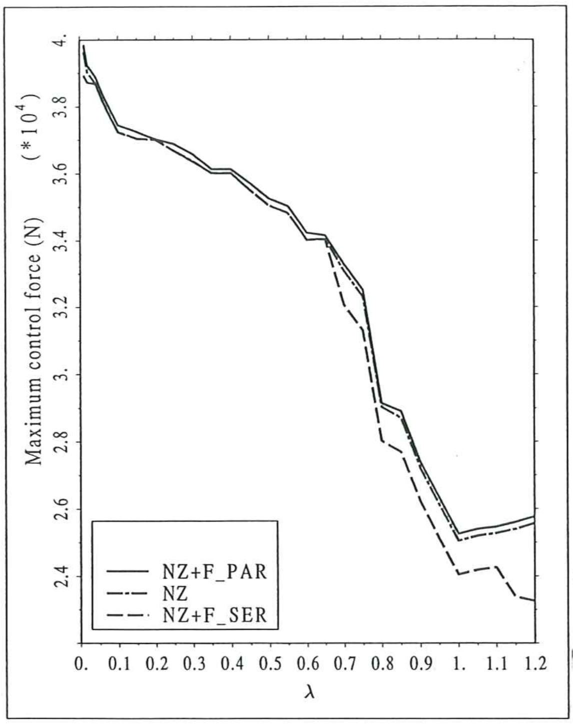


Figure 19 Maximum control force for different values of λ.

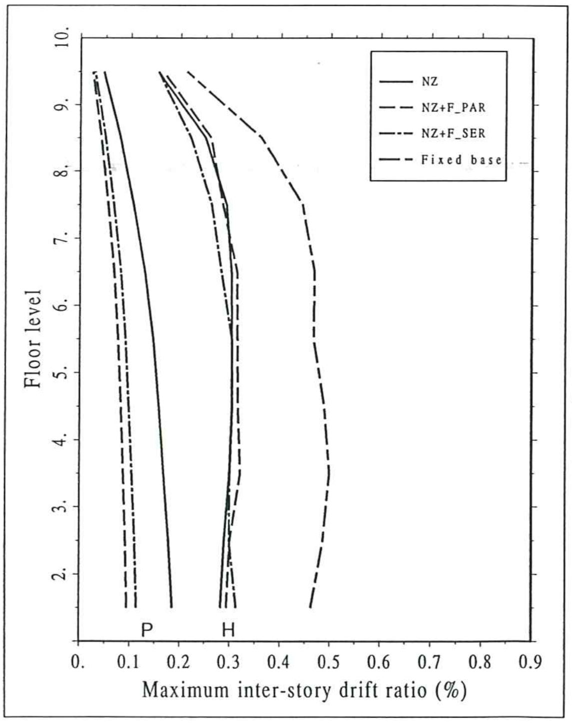


Figure 20 Variation of the maximum inter-story drift ratio with the floor level.

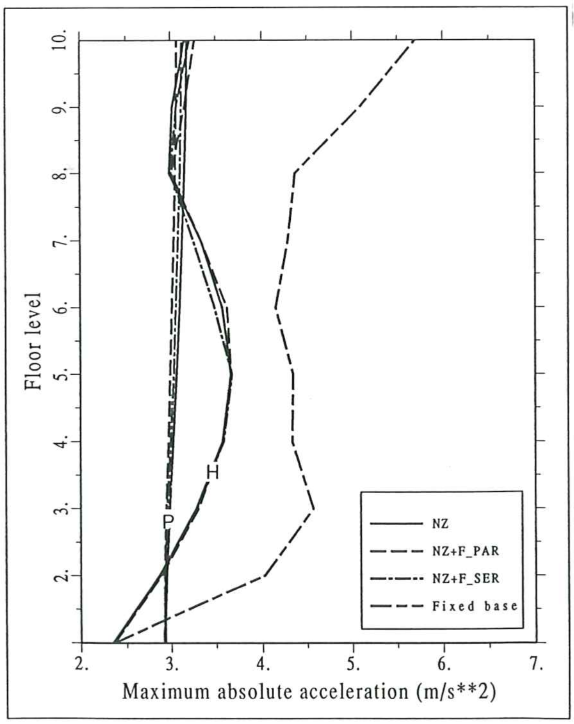


Figure 21 Variation of the maximum absolute acceleration with the floor level.

CONCLUSIONS

The hybrid control system discussed in this paper combines a nonlinear base isolation component with an active one applying forces at the base level. The main purpose has been to propose a new feedback control law for the active component and to explore its potential through simulation examples.

The objective for the control law has been posed in terms of stability and two very important features present in hybrid systems have been taken into account: (i) uncertainties in knowledge of the structure, the isolators and the excitation and (ii) nonlinear behaviour in the isolator devices. Regarding the uncertainties, the main remark is that no knowledge of the parameters of the base and tructure model are required in the control law because of its adaptive nature. Concerning the nonlinearities in the isolator, for illustrative purposes, a variety of hysteretic and frictional characteristics have been considered in the examples.

In the evaluation of the structural behaviour, the following variables have been checked: the absolute base displacement, the inter-story drift and the absolute acceleration of the structure. The numerical results show that the active control force reduces significantly the base displacement of the structure, producing a slight increase of the structural response with respect to the pure passive case. However, the global structural behaviour improves when the hybrid system is used. This can be particularly significant for excitations having predominant frequencies in the range in which the purely passive base isolated structure has its maximum response.

The emphasis of the paper has been made in the proposal of the adaptive control law, the examples having the motivation of illustrating its features. The results presented show a clear potential for this control law. Its capabilities need to be more extensively assessed by means of a systematic numerical application in a variety of operating conditions: different structural models, isolator nonlinearities, seismic excitations, etc., completed with experimental verification.

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