The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5 – 9 June 2022, Oslo, Norway

BAYESIAN SYSTEM IDENIFICATION AND DYNAMIC VIRTUALIZATION USING INCOMPLETE NOISY MEASUREMENTS

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Key words: Bayesian Inference, Filter-type Techniques, Structural Identification, Uncertainty Quantification, Virtual Sensing.

Abstract. This study presents the application of Bayesian Expectation-Maximization (BEM) methodology to coupled state-input-parameter estimation in both linear and nonlinear structures. The BEM is built upon a Bayesian foundation, which utilizes the EM algorithm to deliver accurate estimates for latent states, model parameters, and input forces while updating noise characteristics effectively. This feature allows for quantifying associated uncertainties using response-only measurements. The proposed methodology is equipped with a recursive backward-forward Bayesian estimator that provides smoothed estimates of the state, input, and parameters during the Expectation step. Next, these estimates help calculate the most probable values of the noise parameters based on the observed data. This adaptive approach to the coupled estimation problem allows for real-time quantification of estimation uncertainties, whereby displacement, velocity, acceleration, strain, and stress states can be reconstructed for all degrees-of-freedom through virtual sensing. Through numerical examples, it is demonstrated that the BEM accurately estimates the unknown quantities based on the measured quantities, not only when a fusion of displacement and acceleration measurements is available but also in the presence of acceleration-only response measurements.

1 INTRODUCTION

System identification and virtual sensing allow extracting maximum information from the available data, permitting a successful application of structural health monitoring, fatigue assessment, and damage diagnosis. These methods have evolved during the past few decades from deterministic approaches to probabilistic algorithms that consider the effect of noise and other sources of uncertainties. Amongst probabilistic approaches to system identification and response reconstruction, filtering techniques have received growing attention due to their simplicity and efficiency.

One of the most visited filters for reconstructing the states of a dynamical system is Kalman filter [1]. This recursive optimal filter calculates the posterior distribution of the latent states considering both the measurement and process noise, allowing for the sequential estimation of the unknown dynamical states over time. However, the accuracy and stability of the estimations obtained through the Kalman filter depend on the level of available information about dynamical behaviour and the applied loadings [2].

Additionally, the performance of the Kalman-type filters is highly dependent on the appropriate choice of noise characteristics, and regularization approaches such as the L-curve method have been suggested for fair-tunning of these filters. However, these tuning methods will encounter computational overheads as the number of unknown input forces increases. Moreover, they do not yield a viable realization of the associated uncertainties, which is crucial for evaluating the obtained results and decision-making.

As an alternative, a fully Bayesian framework, named Bayesian Expectation-Maximization (BEM), is developed for the simultaneous estimation of the unknown quantities of interest and identification of the noise parameters based on the incomplete noisy measurement. This framework has shown to be efficient and accurate when used for the joint input-state and joint input-state-parameter estimation of the linear dynamical systems [3,4]. This framework utilizes pseudo-input measurements to improve the observability of the low-frequency components in the absence of the measured displacement responses. The presence of the low-frequency drift components in the absence of displacement measurements, specifically when the applied loading is unknown is a common threat to the accuracy of the estimates made by Kalman-type filters. Thus, implementation of such stabilizing strategies immensely helps reduce the deteriorating effect of the drift in the estimation of the input and dynamical states.

In this study, the system identification and response reconstruction of dynamical systems are addressed through the BEM framework. The nonlinear nature of the problem at hand is tackled by deploying the Unscented transformation and smoothing in the Expectation step to drive the joint input-state-parameter estimation for the dynamic structure. The noise characteristics are then updated based on the available information and observed quantities during the Maximization step. The iterative BEM algorithm transfers the identified noise parameters into the Expectation step. Updating the noise characteristic based on the measurement data enhances the accuracy of the estimates and yields a reasonable realization of the involved uncertainties. The full-field response of the system can then be reconstructed based on the identified quantities and estimated latent states.

2 STATE-SPACE AND OBSERVATION MODELS

The continuous-time state-space equation describes the dynamical behavior of an N-DOF structure as a function of its dynamical state, comprising of the displacement and velocity responses ($\mathbf{x} = \begin{bmatrix} \mathbf{u}^T & \dot{\mathbf{u}}^T \end{bmatrix}^T \in \mathbb{R}^{2N}$), parameters ($\boldsymbol{\theta} \in \mathbb{R}^{N_\theta}$), external loading ($\mathbf{p} \in \mathbb{R}^{N_p}$), and the process noise vector ($\mathbf{v} \in \mathbb{R}^{2N}$), giving:

$$\dot{\mathbf{x}}(t) = f\left(\mathbf{x}(t), \boldsymbol{\theta}(t), \mathbf{p}(t)\right) + \mathbf{v}(t) \tag{1}$$

For this system, let measurements ($\mathbf{y} \in \mathbb{R}^{N_m}$) be obtained in a discrete-time manner, which relate to the dynamical state, parameters, and input loading through the discrete-time observation model, given by

$$\mathbf{y}_k = H\left(\mathbf{x}_k, \boldsymbol{\theta}_k, \mathbf{p}_k\right) + \mathbf{w}_k \tag{2}$$

where $\mathbf{w}_k \in \mathbb{R}^{N_m}$ is measurement noise described as a zero-mean Gaussian process with $\mathbf{R} \in \mathbb{R}^{N_m \times N_m}$ covariance matrix. The subscript k in Eq. 2 denotes the current time step that corresponds to the time $t = k\Delta t$, where Δt indicates the length of each time step in seconds. Dynamic states are obtained at each time increment through the discrete-time state-space model below:

$$\mathbf{x}_k = F\left(\mathbf{x}_{k-1}, \boldsymbol{\theta}_{k-1}, \mathbf{p}_{k-1}\right) + \mathbf{v}_k \tag{3}$$

In this equation, the process error vector $\mathbf{v}_k \in \mathbb{R}^{2N}$ is a zero-mean Gaussian white process with the covariance matrix $\mathbf{Q} \in \mathbb{R}^{2N \times 2N}$, and the function F is obtained by integrating the continuous-time function f in Eq. (1) over each time step:

$$F\left(\mathbf{x}_{k-1}, \boldsymbol{\theta}_{k-1}, \mathbf{p}_{k-1}\right) = \mathbf{x}_{k-1} + \int_{(k-1)\Delta t}^{k\Delta t} f\left(\mathbf{x}(t), \boldsymbol{\theta}(t), \mathbf{p}(t)\right) dt \tag{4}$$

In this study, the transition of the input and model parameters are described by two separate random-walk models, where the fluctuations between two consecutive steps are governed by zero-mean error vectors $\mathbf{v}_k^p \in \mathbb{R}^{N_p}$ and $\mathbf{v}_k^\theta \in \mathbb{R}^{N_\theta}$ with covariance matrices $\mathbf{Q}^p \in \mathbb{R}^{N_p \times N_p}$ and $\mathbf{Q}^\theta \in \mathbb{R}^{N_\theta \times N_\theta}$, respectively. Then, one can write:

$$\mathbf{p}_k = \mathbf{p}_{k-1} + \mathbf{v}_k^p \tag{5}$$

$$\boldsymbol{\theta}_k = \boldsymbol{\theta}_{k-1} + \mathbf{v}_k^\theta \tag{6}$$

This model assumption allows expanding the original state vector by augmenting the unknown model parameters and external loading and forming the augmented state vector $\mathbf{z}_k = \begin{bmatrix} \mathbf{x}_k^T & \boldsymbol{\theta}_k^T & \mathbf{p}_k^T \end{bmatrix}^T \in \mathbb{R}^L$, where $L = 2N + N_p + N_\theta$. The augmented state-space equation is written as

$$\mathbf{z}_k = \bar{F}\left(\mathbf{z}_{k-1}\right) + \mathbf{v}_k^z \tag{7}$$

where the augmented process error ($\mathbf{v}_k^z \in \mathbb{R}^L$) is a zero-mean Gaussian white process with the covariance matrix $\mathbf{Q}^z = \text{block-diag}\left\lfloor \mathbf{Q}^x, \mathbf{Q}^\theta, \mathbf{Q}^p \right\rfloor$.

3 BAYESIAN EXPECTATION-MAXIMIZATION FRAMEWORK

The Bayesian Expectation-Maximization framework allows for the recursive estimation of the unknown quantities of interest for dynamical systems using incomplete noisy measurements while updating the noise characteristics and evaluating the estimation uncertainties. Assuming the prior distribution for the augmented state vector $\mathbf{z}_0 \sim N\left(\mathbf{z}_0 \mid \mathbf{z}_{0|0}, \mathbf{P}_{0|0}^z\right)$ to be Gaussian, the smoothed estimates for the expanded state vector are obtained by the Expectation (E) step, and the noise characteristics are updated by the Maximization (M) step based on the available observations. Once convergence is achieved, the complete response of the system is then reconstructed through the virtual sensing step, utilizing the identified information.

In this work, the Unscented Kalman Filter and Smoother are deployed to deliver simultaneous estimations of the states, model parameters, and unknown input forces, aiming to accommodate parametric and computational nonlinearities. In the filtering step, sequential estimations for the expanded state vector at each time step are obtained based on the past and current data. These estimations are then corrected during the smoothing process, incorporating the future state and measurements. Considering data points beyond the current time step for state estimations reduces the error and instabilities in the estimates and enables updating noise characteristics in the M-step. Additionally, including N_p pseudo-input measurements is suggested to further improve the estimation accuracy in the absence of displacement measurements. Each pseudo-input data is a zero-vector with a an unknown covariance matrix that is required to be updated based on the data. Incorporating this auxiliary information in the original measurement vector enforces a prior knowledge on the low-frequency components of the input force, which is usually not captured in the acceleration responses.

The BEM algorithm is an iterative algorithm that requires its convergence criteria to be carefully chosen for fast and satisfactory convergence. In this regard, a combination of the iteration number and the improvement rate is recommended for terminating the algorithm.

3.1 Unscented Kalman filter and smoother

The UKF is employed to approximate the posterior distribution of the augmented state vector at each time increment with a Gaussian distribution through the unscented transformation of a set of carefully chosen points known as sigma points. The mean and covariance of the augmented state at time step k-1 are used to calculate 2L+1 points ($\chi_k^i \in \mathbb{R}^L,\ i=0,...,2L$) and their associated weights ($\omega_i^m \in \mathbb{R}^L,\ i=0,...,2L$). These sigma points are perturbated through the state-space and observation functions to find the statistics of the predictive prior and posterior distribution of the state at the next time step.

The sigma points at each time increment are chosen such that they maintain the expected mean and covariance of the augmented state at the current time step. The predictive set of sigma points for time step k, given the data acquired till the time stamp k-1 are obtained as:

$$\begin{cases} \chi_{k|k-1}^0 = \mathbf{z}_{k-1|k-1},\ \omega_0^m = \dfrac{\lambda}{\lambda+L},\ \omega_0^c = \omega_0^m + \left(1-\alpha^2+\beta\right) & i=0 \\ \chi_{k|k-1}^i = \mathbf{z}_{k-1|k-1} + \left(\sqrt{\dfrac{L}{1-\omega_0^m}\mathbf{P}_{k-1|k-1}^z}\right)_i,\ \omega_i^m = \omega_i^c = \dfrac{1-\omega_0^m}{2L} & 1 \le i < L \\ \chi_{k|k-1}^i = \mathbf{z}_{k-1|k-1} - \left(\sqrt{\dfrac{L}{1-\omega_0^m}\mathbf{P}_{k-1|k-1}^z}\right)_i,\ \omega_i^m = \omega_i^c = \dfrac{1-\omega_0^m}{2L} & L+1 < i \le 2L \end{cases} \tag{8}$$

where, the sub-script i in $\left(\sqrt{\frac{L}{1-\omega_0^m}\mathbf{P}_{k-1|k-1}^z}\right)_i \in \mathbb{R}^L$ represents the i^{th} column of the term inside the parentheses. The position of the sigma points with respect to the mean is controlled by their associated weights. A smaller weight projects points to close vicinity of the state mean vector. In this study, the corresponding weights are calculated based on the parameters $\lambda = \alpha^2 (L+\kappa) - L$, α , and β . The literature suggests the constant parameters α , β , and κ be 1, 2, and 0, respectively [5]. The statistics of the predictive prior are then calculated based on the sigma points:

$$\mathbf{z}_{k|k-1} = \sum_{i=0}^{2L} \omega_i^m \chi_{k|k-1}^i \tag{9}$$

$$\mathbf{P}_{k|k-1}^z = \sum_{i=0}^{2L} \omega_i^c \left(\mathbf{z}_{k|k-1} - \chi_{k|k-1}^i \right) \left(\mathbf{z}_{k|k-1} - \chi_{k|k-1}^i \right)^T + \mathbf{Q}^z \tag{10}$$

A prediction of the measured quantities is calculated by perturbation of the predictive sigma points through the observation equation, which leads to

$$\overline{\mathbf{y}}_{k|k-1} = \sum_{i=0}^{2L} \omega_i^m H\left(\chi_{k|k-1}^i \right) \tag{11}$$

The predictive measurement vector calculated through the above equation allows updating the previously obtained predictive prior distribution of the augmented state based on the newly observed data to find the posterior distribution for the expanded state vector As follows:

$$\mathbf{z}_{k|k} = \mathbf{z}_{k|k-1} + \mathbf{K}_G \left(\mathbf{y}_k - \overline{\mathbf{y}}_{k|k-1} \right) \tag{12}$$

$$\mathbf{P}_{k|k}^z = \mathbf{P}_{k|k-1}^z - \mathbf{K}_G \mathbf{P}_k^y \mathbf{K}_G^T \tag{13}$$

$$\mathbf{K}_G = \mathbf{P}_k^{zy} \left(\mathbf{P}_k^y - \mathbf{R} \right) \tag{14}$$

where $\mathbf{z}_{k|k} \in \mathbb{R}^L$ and $\mathbf{P}_{k|k}^z \in \mathbb{R}^{L \times L}$ are the mean and covariance of the state posterior distribution, characterized based on the unscented transformation of the sigma point as an approximate Gaussian distribution. The Kalman Gain $\mathbf{K}_G \in \mathbb{R}^{L \times N_m}$ indicates the information gain from the measured responses. The covariance matrices $\mathbf{P}_k^y \in \mathbb{R}^{N_m \times N_m}$ and $\mathbf{P}_k^{zy} \in \mathbb{R}^{L \times N_m}$ are also calculated by the unscented transformation given as

$$\mathbf{P}_k^y = \sum_{i=0}^{2L} \omega_i^c \left(H\left(\chi_{k|k-1}^i \right) - \overline{\mathbf{y}}_{k|k-1} \right) \left(H\left(\chi_{k|k-1}^i \right) - \overline{\mathbf{y}}_{k|k-1} \right)^T + \mathbf{R} \tag{15}$$

$$\mathbf{P}_k^{zy} = \sum_{i=0}^{2L} \omega_i^c \left(\chi_{k|k-1}^i - \mathbf{z}_{k|k-1} \right) \left(H\left(\chi_{k|k-1}^i \right) - \overline{\mathbf{y}}_{k|k-1} \right)^T \tag{16}$$

The state estimates in the previous time instances are then smoothed through the Unscented Kalman Smoother to incorporate both past and future data to reduce possible errors and stabilize the estimations. The Unscented Kalman Smoother delivers the smoothed estimates of the state at time step *k*-1 as:

$$\mathbf{z}_{k-1|N}^s = \mathbf{z}_{k-1|k-1} + \mathbf{L}_G \left(\mathbf{z}_{k|N}^s - \mathbf{z}_{k|k-1} \right) \tag{17}$$

$$\mathbf{P}_{k-1|N}^{zs} = \mathbf{P}_{k-1|k-1}^{z} + \mathbf{L}_G \left(\mathbf{P}_{k|k}^{zs} - \mathbf{P}_{k|k-1}^{z}\right)\mathbf{L}_G^T \quad (18)$$

$$\mathbf{L}_G = \left(\omega_i^m \chi_{k|k-1}^i\right)\left(\omega_i^m F\left(\chi_{k|k-1}^i\right)\right)^T \mathbf{P}_k^{zy}\left(\mathbf{P}_k^y - \mathbf{R}\right) \quad (19)$$

where, $\mathbf{L}_G \in \mathbb{R}^{L\times N_m}$ is the smoother gain; $\mathbf{z}_{k-1|N}^s \in \mathbb{R}^L$ and $\mathbf{P}_{k-1|N}^{zs} \in \mathbb{R}^{L\times L}$ are the mean and covariance of the smoothed estimate of the augmented state.

3.2 Maximization-step (M-step)

The optimal values for the noise characteristics are updated in the M-step by maximizing the surrogate objective function $\mathbf{L}\left(\{\mathbf{z}_k\}_{k=0}^N, \mathbf{D}_N \left|\mathbf{Q}^z, \mathbf{R}\right.\right)$ with respect to the augmented process and measurement noise covariance matrices $\mathbf{Q}^z$ and $\mathbf{R}$. This process ultimately leads to the following explicit formulations:

$$\mathbf{R} = \frac{1}{N}\sum_{k=1}^{N}\mathbb{E}\left[\left(\mathbf{y}_k - H\left(\mathbf{z}_{k|N}^s\right)\right)\left(\mathbf{y}_k - H\left(\mathbf{z}_{k|N}^s\right)\right)^T\right] \quad (20)$$

$$\mathbf{Q}^z = \frac{1}{N}\sum_{k=1}^{N}\mathbb{E}\left[\left(\mathbf{z}_{k|N}^s - F\left(\mathbf{z}_{k-1|N}\right)\right)\left(\mathbf{z}_{k|N}^s - F\left(\mathbf{z}_{k-1|N}\right)\right)^T\right] \quad (21)$$

These identified covariance matrices are used in the next iteration of the EM algorithm to find smoothed estimates for the unknown quantities of interest. Once the convergence criteria for the EM algorithm are met, the algorithm is terminated, and the identified values are used to reconstruct the unknown quantities.

3.3 Virtual sensing

The full-field dynamical response of the system ($\mathbf{y}_k^e$) includes the strain ($\boldsymbol{\varepsilon}_k$), displacement ($\mathbf{x}_k$), velocity ($\dot{\mathbf{x}}_k$), and acceleration ($\ddot{\mathbf{x}}_k$) responses at each time increment. The relation of this complete response vector to the identified quantities is established through the function $\bar{H}$, which provides

$$\mathbf{y}_k^e = \bar{H}(\mathbf{z}_k) \quad (22)$$

Following this relation, the complete response vector could be described by a Gaussian process and its statistics are found based on the posterior distribution of the estimated dynamical state.

4 ILLUSTRATIVE EXAMPLE

4.1 3-DOF system with Bouc-Wen parameter

The 3-DOF damped system shown in Fig. 1 is used to demonstrate the performance of the proposed method in delivering simultaneous estimations of the state, model parameters, and applied loading in the presence of mechanical nonlinearities when subjected to non-stationary excitation.

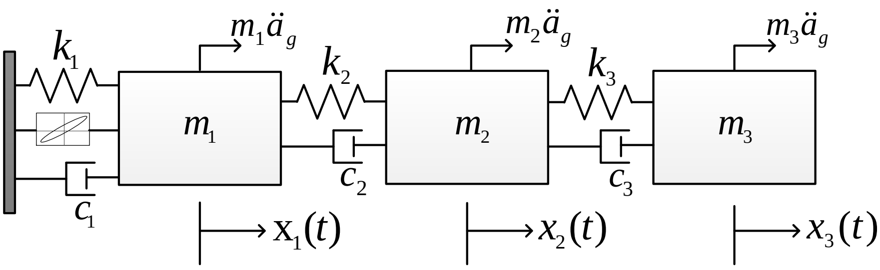

Figure 1: 3-DOF mass-spring-damper system with a Bouc-wen element.

Three masses, stiffnesses, and damping coefficients of the above system are assumed to be identical and equal to 1 kg, 12 kN/m, and 0.3 Ns/m, respectively. Additionally, a Bouc-Wen parameter is considered on the first DOF. The hysteretic behavior of this element is formulated as:

$$\dot{r}_1(t) = A\dot{x}_1(t) - \beta|\dot{x}_1(t)||r_1(t)|^{n-1} r_1(t) - \gamma \dot{x}_1(t)|r_1(t)|^n \quad (1)$$

The constant parameters A, β, λ, and n in Eq. 7 are the Bouc-wen parameters. The simulated responses are created assuming parameter A to be known and equal to 1 N.s/m. Parameters β, λ, and n are set to 2, 0.5, and 2, respectively. The response from the system is simulated by applying the base excitation $\ddot{a}_g$, described by a non-stationary process with a sampling frequency of 500 Hz. The data from three accelerometers installed on each element is used to identify the stiffness, damping, unknown Bouc-wen parameters, applied force, and latent states.

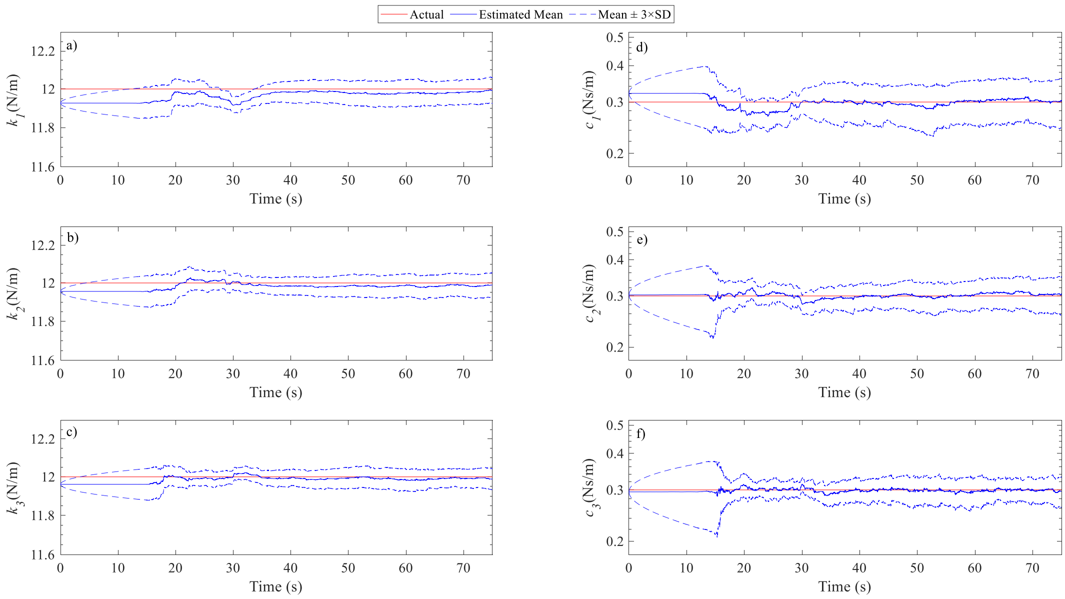

Figure 2: Estimations of the (a-c) stiffness and (d-f) damping parameters.

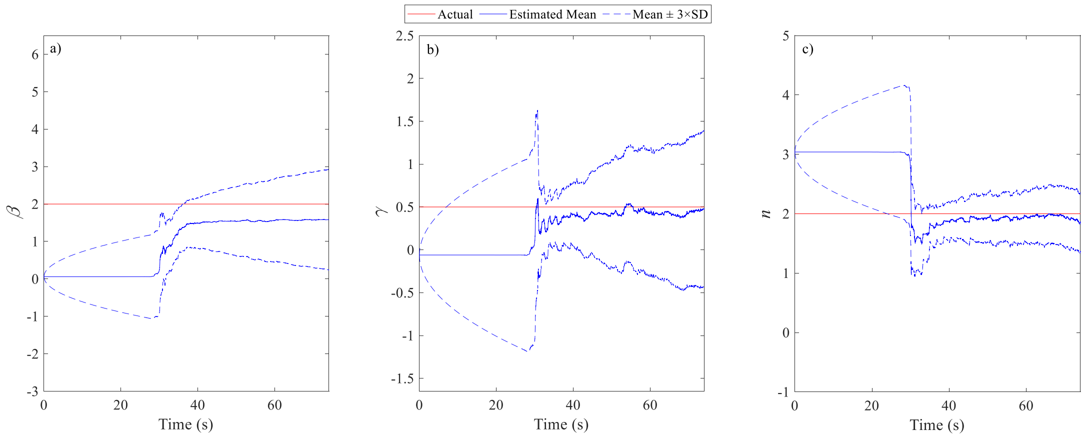

Figure 3: Estimations of the Bouc-wen parameters.

The initial values for all three stiffness parameters are set to be 6 kN/m and the damping parameters are initiated at 0.5 Ns/m. The initial values for the Bouc-wen parameters are also set to be $\beta_0 = 1$, $\lambda_0 = 2$, and $n_0 = 3$, while the noise covariance matrices are set to be $\mathbf{Q}^x = 10^{-12}\mathbf{I}_6$, $\mathbf{Q}^\theta = 10^{-7}\mathbf{I}_9$, $\mathbf{Q}^p = 10^{-2}$, $\mathbf{R} = 10^{-4}\mathbf{I}_3$, and $\mathbf{R}^{pd} = 10^3$. The proposed BEM algorithm iterated three times before converging. The results of the final iteration are presented herein.

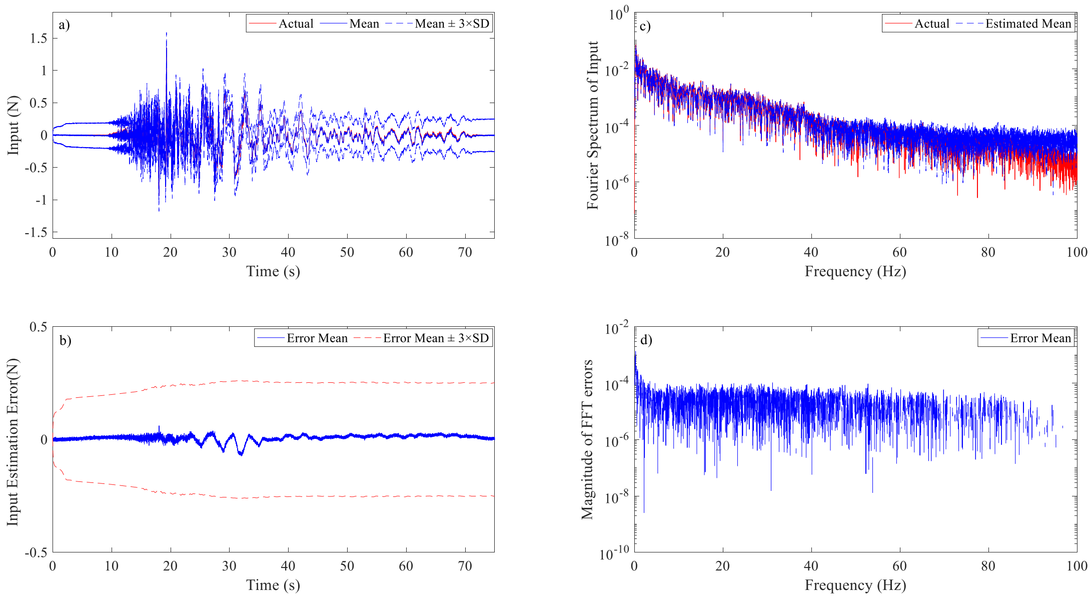

Figure 4: (a) Estimations of the applied excitation in time domain, (b) associated errors and uncertainty bounds in time domain, (c) estimation of the input in frequency domain, (d) estimation errors in frequency domain.

The estimations for the stiffness and damping parameters are shown in Fig. 2, indicating the convergence of the estimations to the actual values after experiencing a non-stationary input. Soon after the excitation dies out, the algorithm places the mean of the parameters in a close vicinity of the nominal values while the discrepancies are covered by the uncertainty bounds. A similar pattern is also observed in the estimation of the Bouc-wen parameters. Fig. 3 shows the estimation trajectory of the hysteresis parameters and the efficiency of the algorithm in providing a realistic uncertainty bound for the identified quantities.

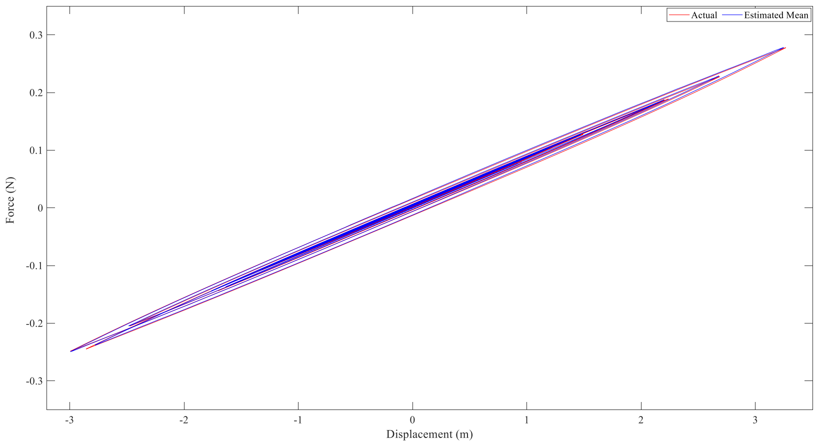

Figure 5: Estimations of hysteretic loop through the BEM algorithm.

The estimated base excitation applied to the system is compared with its actual counterpart in time domain in Fig. 4 (a). As seen in Fig. 4 (b), the estimation error is zero-mean and well-contained within the uncertainty bounds. Comparing the frequency content of the estimations for this non-stationary process with the actual values (Fig. 4 (c-d)) shows the good match between the two curves, and the efficiency of the pseudo-input measurements in mitigating the estimation error and in particular the discrepancies in the identified low-frequency components.

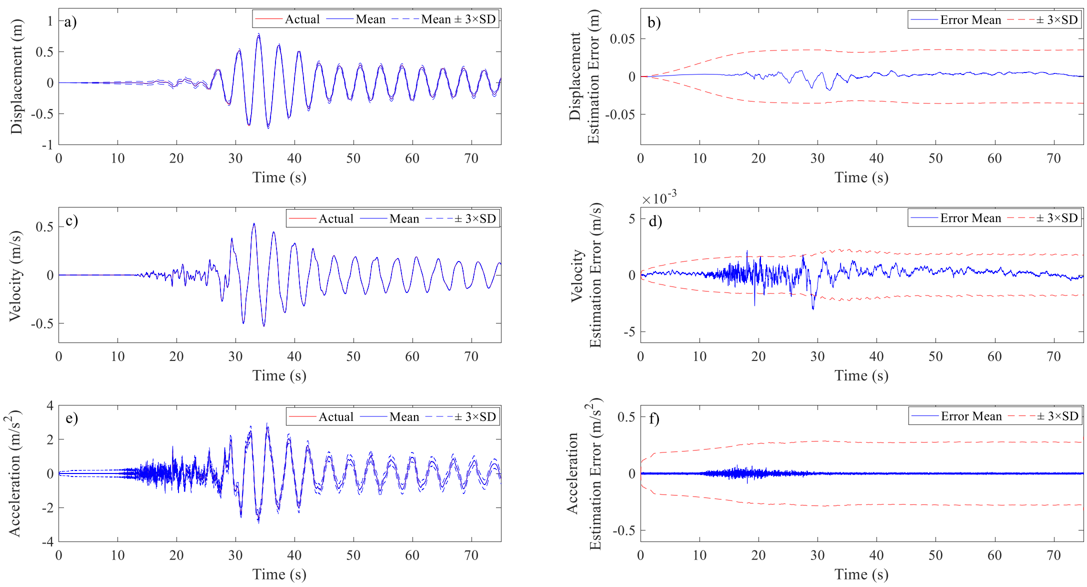

Figure 6: Estimations of the (a-b) displacement and the associated errors, (c-d) velocity and associated errors, and (e-f) acceleration and associated errors for the second DOF.

Fig. 5 compares the hysteretic loops generated during the identification with those representing the true hysteresis behavior, indicating the close match between the two. The latent states of the system and the associated uncertainties identified through the BEM further indicates the efficacy of this approach. The estimations for the displacement, velocity, and acceleration of the second element are compared with the actual hysteresis loops shown in Fig. 6. The associated errors remain zero-mean and well-captured by the uncertainty bounds.

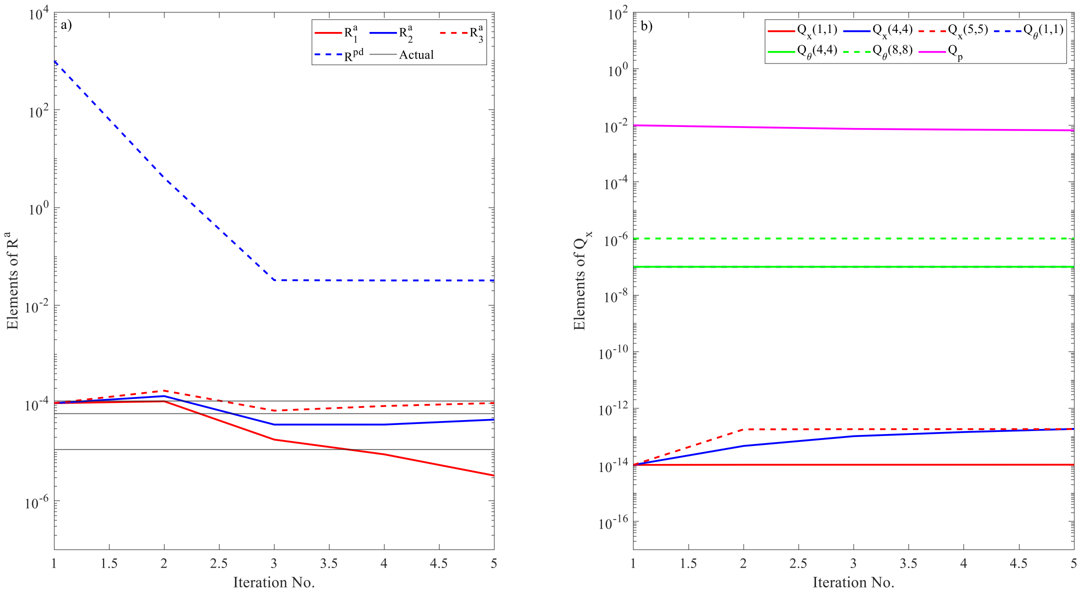

Figure 7: Estimations of the (a) measurement noise variances and (b) variance of some elements of the process noise covariance matrix.

The identification of the noise parameters through different iterations of the BEM algorithm are presented in Fig. 7, where a close match for the components of the measurement noise is observed. Although there are no reference values to evaluate the accuracy of the identified values for the process noise components, the stability of the identification is noteworthy. The covariance of the pseudo measurement vector is also updated based on the data and stabilized at its optimum value, which improves the accuracy of the estimates by successfully mitigating the low-frequency drift components.

5 CONCLUSIONS

In this study, the simultaneous estimation of the input-state-parameter for nonlinear dynamical systems is addressed through a Bayesian Expectation-Maximization (BEM) framework, where the parametric and computational nonlinearities are tackled by using the Unscented filtering and smoothing to derive the smoothed estimations for the augmented state vector in a recursive manner in the E-step. The noise characteristics are updated based on the available information through the M-step, allowing a robust approach for quantifying the estimation uncertainties. The efficacy of the proposed algorithm in delivering simultaneous estimations for the latent states, unknown model parameters, and input loading in the presence of non-stationary excitations is verified through a 3-DOF system. The estimations obtained through this method using acceleration-only responses are stable and free from low-frequency drift, attributed to the successful identification of the noise parameters and the inclusion of the pseudo-input measurements that help mitigate the instabilities due to the lack of displacement measurements. Overall, the BEM framework is a suitable tool for reliable reconstruction of the complete state of the system and the concerning uncertainties, allowing for a smooth application of fatigue monitoring, health evaluation, and operational maintenance approaches.

6 AKNOWLEDGEMENTS

Financial support from the Hong Kong Research Grants Council under project No. 16212918 and 16211019 is gratefully acknowledged.

REFERENCES

[1] Kalman, R.E. A New Approach to Linear Filtering and Prediction Problems. *J. Basic Eng. 82* (1960) 35.

[2] Simon, D. *Optimal state estimation: Kalman, H infinity, and nonlinear approaches*. John Wiley & Sons (2006).

[3] Teymouri, D. and Sedehi, O. and Katafygiotis, L.S. and Papadimitriou, C. Input-State-Parameter-Noise Identification and Virtual Sensing in Dynamical Systems: A Bayesian Expectation-Maximization (BEM) Perspective. (2022). http://arxiv.org/abs/2207.01828.

[4] Teymouri, D. and Sedehi, O. and Katafygiotis, L.S. and Papadimitriou, C. A Bayesian Expectation-Maximization (BEM) methodology for joint input-state estimation and virtual sensing of structures. *Mech. Syst. Signal Process*. 169 (2022) 1–31.

[5] Särkkä, S. *Bayesian filtering and smoothing*. Cambridge University Press., (2013).