ODDLS: An overlapping domain decomposition level set method for simulation of free surface problems

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Introduction

(Naval) Free Surface Flows

Smoothed Particle Hydrodynamics

Mesh type	FS algorithm	FS BC	FS accuracy	Forces accuracy	Turbulence	CPU req.
None	Tracking	Simplified	High	Low	¿?	High

Volume of Fluid

Mesh type	FS algorithm	FS BC	FS accuracy	Forces accuracy	Turbulence	CPU req.
Structured	Capturing	Simplified	Medium	High	Modelled	Medium

Others such us Particle FEM, Level set methods, …

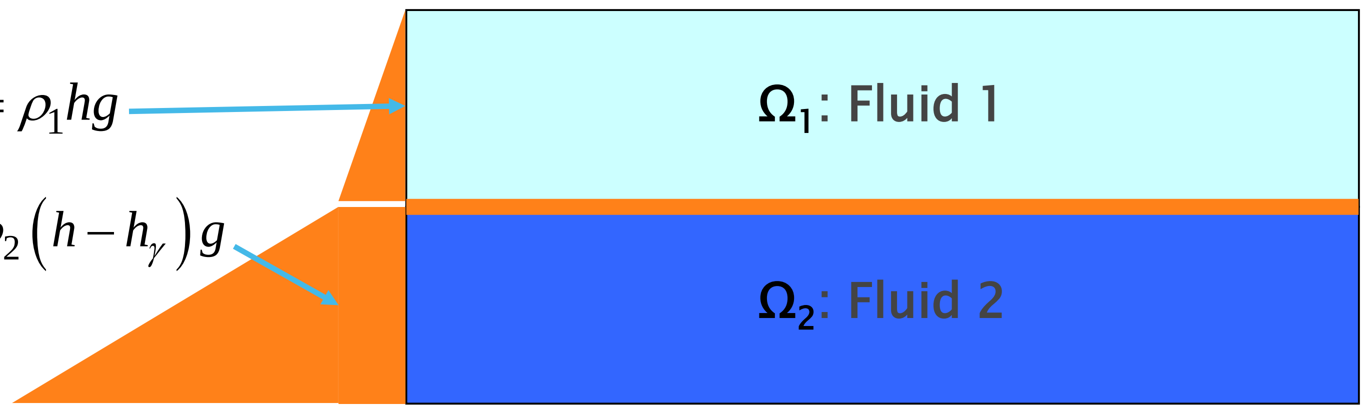
Introduction

- The main problem of the free surface capturing algorithms is the treatment of the pressure (gradient) discontinuity at the interface due to the variation of the properties there.

$$\mathbf{u}, p, \rho, \mu = \begin{cases} \mathbf{u}_1, p_1, \rho_1, \mu_1 & x \in \Omega_1 \\ \mathbf{u}_2, p_2, \rho_2, \mu_2 & x \in \Omega_2 \end{cases}$$

$$p = \rho_1 h g$$

$$p = \rho_1 h_\gamma g + \rho_2 \left(h - h_\gamma\right) g$$



- In most of these algoritms the sharp transition of the properties at the interface is smoothed (extended to a band of several elements width) and therefore the accuracy in capturing the free surface is reduced.

Introduction

- A new free surface capturing approach able to overcome/improve most of the difficulties/ shortcomings of the existing algorithms is presented.
- This approach is based on:
  - Stabilisation of governing equations (incompressible two-fluids Navier-Stokes equations) by means of the FIC method.
  - Application of ALE techniques.
  - Free surface movement is solved using a level set approach.
  - Monolithic Fractional-Step type Navier-Stokes integration scheme.
  - Application of domain decomposition techniques to improve the accuracy in the solution of governing equations in the interface between both fluids.

Problem Statement: Two fluids Navier Stokes equations

- Two (incompressible) fluids (non honogeneous) Navier Stokes equations:

$$\partial_t \rho + \nabla(\rho \mathbf{u}) = 0$$

$$\partial_t(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) - \nabla \cdot \boldsymbol{\sigma} = \rho \mathbf{f}$$

$$\nabla \cdot \mathbf{u} = 0$$

- With:

$$\Omega = \operatorname{int}\left(\overline{\Omega_1(t) \cap \Omega_2(t)}\right) \quad \forall t \in (0, T]$$

$$\rho(\mathbf{x}, t), \mu(\mathbf{x}, t) = \begin{cases} \rho_1, \mu_1 & \mathbf{x} \in \Omega_1(t) \\ \rho_2, \mu_2 & \mathbf{x} \in \Omega_2(t) \end{cases} \quad \forall (\mathbf{x}, t) \in \Omega \times (0, T]$$

- And the necessary initial and boundary conditions

Problem statement: Level set equation

- Let be Ψ a function (level set), defined as follows:

$$\psi(\mathbf{x},t) = \begin{cases} d(\mathbf{x},\Gamma(t)) & \mathbf{x} \in \Omega_1(t) \\ 0 & \mathbf{x} \in \Gamma(t) \\ -d(\mathbf{x},\Gamma(t)) & \mathbf{x} \in \Omega_2(t) \end{cases}$$

- Therefore we can re-write the density field as:

$$\rho(\psi(t,\mathbf{x})) = \begin{cases} \rho_1 & \psi(t,\mathbf{x}) < 0 \\ \rho_2 & \psi(t,\mathbf{x}) \geq 0 \end{cases}$$

- And finally, obtain an equivalent equation for the density in terms of Ψ (level set equation):

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0 \longrightarrow \partial_t \psi + (\mathbf{u} \cdot \nabla) \cdot \psi = 0$$

Problem Statement: Level set – Navier Stokes equations

- Two (incompressible) fluids level-set type Navier Stokes equations:

$$\partial_t(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) - \nabla\cdot\boldsymbol{\sigma} = \rho\mathbf{f}$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\partial_t\psi + (\mathbf{u}\cdot\nabla)\psi = 0$$

- With:

$$\Omega = \operatorname{int}\left(\overline{\Omega_1(t)\cap\Omega_2(t)}\right) \quad \forall t\in(0,T]$$

$$\rho(\mathbf{x},t),\mu(\mathbf{x},t) = \begin{cases}\rho_1,\mu_1 & \mathbf{x}\in\Omega_1(t)\\ \rho_2,\mu_2 & \mathbf{x}\in\Omega_2(t)\end{cases} \quad \forall(\mathbf{x},t)\in\Omega\times(0,T]$$

- And the necessary initial and boundary conditions

Problem Statement: ALE formulation

- We may easily re-write the previous equations in an Arbitrary Lagrangian-Eulerian frame:

$$\partial_t(\rho\mathbf{u}) + \left[\left(\mathbf{u} - \mathbf{u}^m\right)\cdot\nabla\right](\rho\mathbf{u}) - \nabla\cdot\boldsymbol{\sigma} = \rho\mathbf{f}$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\partial_t\psi + \left[\left(\mathbf{u} - \mathbf{u}^m\right)\cdot\nabla\right]\psi = 0$$

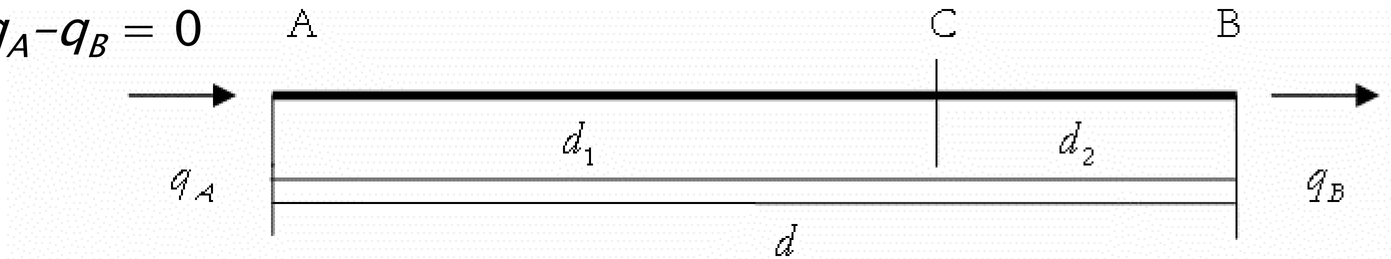
- Where $\mathbf{u}^m$ is the (mesh) deformation velocity of the moving domain:

$$\Omega(t) = \mathrm{int}\left(\overline{\Omega_1(t)\cap\Omega_2(t)}\right)\ \forall t\in(0,T]$$

Finite Calculus (FIC) method: The bases

Consider a convection-diffusion problem in a 1D domain of length L. The equation of balance of fluxes in a sub-domain of size d is:

$q_A - q_B = 0$



where q_A and q_B are the incoming fluxes at points A and B. The flux q includes both convective and diffusive terms.

Let us express now the fluxes q_A and q_B in terms of the flux at an arbitrary point C within the balance domain. Expanding q_A and q_B in Taylor series about point C up to second order terms gives:

$$q_A = q_C - d_1 \left.\frac{dq}{dx}\right|_C + \frac{d_1^2}{2} \left.\frac{d^2q}{dx^2}\right|_C - 0(d_1^3) \qquad q_B = q_C + d_2 \left.\frac{dq}{dx}\right|_C + \frac{d_2^2}{2} \left.\frac{d^2q}{dx^2}\right|_C + 0(d_2^3)$$

Substituting above eqs. into balance equation gives

$$\frac{dq}{dx} - \frac{h}{2}\frac{d^2q}{dx^2} = 0 \quad \text{with} \quad h = d_1 - d_2$$

FIC stabilised equations

- Applying the FIC concept to the Navier Stokes equations, the stabilized Finite Calculus form of the governing differential equations is obtained:

$$\mathbf{r}_m - \frac{1}{2}\mathbf{h}_{\mathrm{m}}\nabla\mathbf{r}_m = 0 \qquad \mathbf{r}_m = \rho\partial_t(\mathbf{u}) + \rho\left[\left(\mathbf{u}-\mathbf{u}^m\right)\cdot\nabla\right]\mathbf{u} - \nabla\cdot\boldsymbol{\sigma} - \rho\mathbf{f}$$

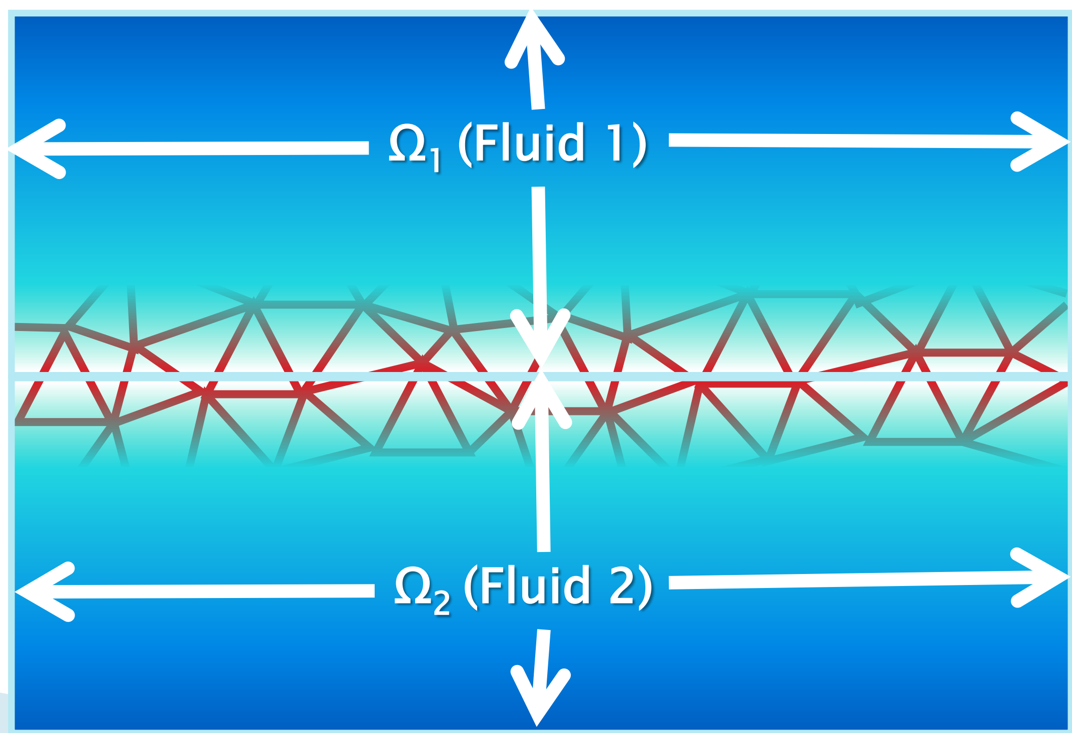
$$r_d - \frac{1}{2}\mathbf{h}_{\mathrm{d}}\nabla r_d = 0 \qquad r_d = \nabla\cdot\mathbf{u}$$

$$r_\psi - \frac{1}{2}\mathbf{h}_\psi\nabla r_\psi = 0 \qquad r_\psi = \partial_t\psi + \left[\left(\mathbf{u}-\mathbf{u}^m\right)\cdot\nabla\right]\psi$$

- It can be proved that a number of stabilized methods allowing equal order interpolation for velocity and pressure fields and stable and accurate advection terms integration can be derived from this formulation.

Overlapping Domain Decomposition Tecnique

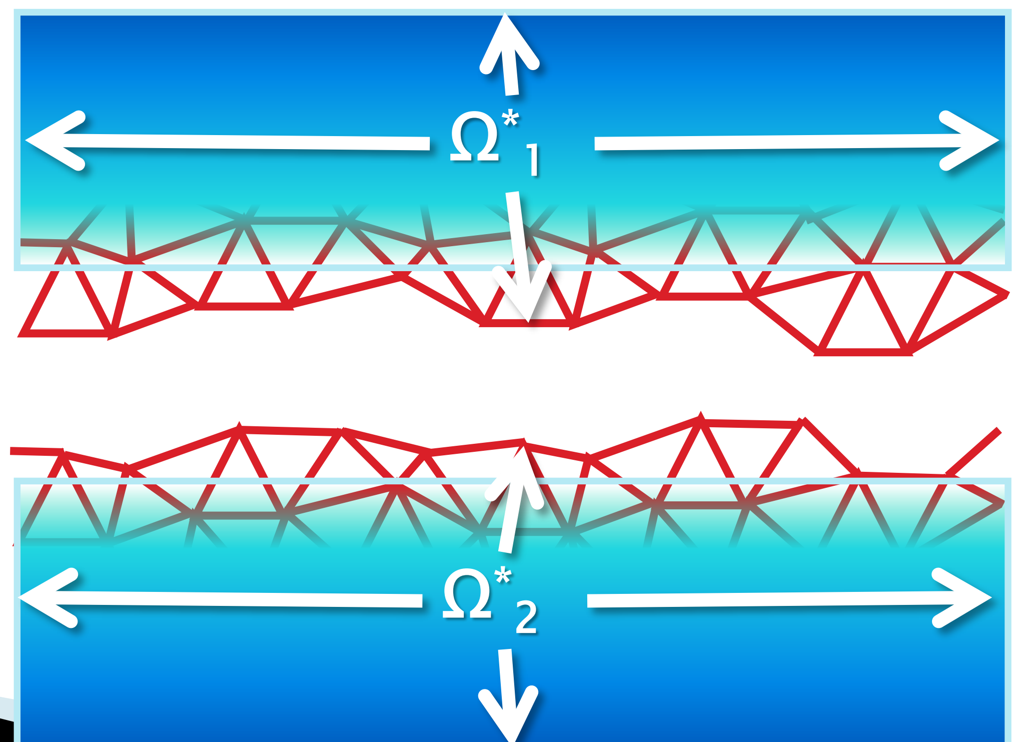
- Let K be a finite element partition of domain Ω, and consider a domain decomposition of Ω into three disjoint sub domains $\Omega_3(t)$, $\Omega_3(t)$ and $\Omega_5(t)$:
- $$\Omega_3(t) = \bigcup_e K_3^e,\ \forall \mathbf{x} \in K_3^e \,|\, \psi(\mathbf{x},t) > 0,\ \Omega_5(t) = \bigcup_e K_5^e,\ \forall \mathbf{x} \in K_5^e \,|\, \psi(\mathbf{x},t) < 0$$
  $$\Omega_4(t) = \Omega(t) \setminus \left(\Omega_3(t) \cup \Omega_5(t)\right)$$



Overlapping Domain Decomposition Tecnique

- From this partition let us define two overlapping domains Ω^*_1, Ω^*_2 in such a way that

$$\Omega_1^* := \operatorname{int}\left(\overline{\Omega_3(t) \cup \Omega_4(t)}\right),\ \Omega_2^* := \operatorname{int}\left(\overline{\Omega_4(t) \cup \Omega_5(t)}\right)$$



Overlapping Domain Decomposition Tecnique

- We can write an equivalent problem, using a standard Dirichlet-Neumann domain decomposition technique (using Ω^*_1, Ω^*_2 decomposition). The resulting variational problem is:

$$\left(\rho_1\partial_t\mathbf{u}_1,\mathbf{v}\right)_{\tilde\Omega_1}+\left\langle\rho_1(\mathbf{u}_1\cdot\nabla)\mathbf{u}_1,\mathbf{v}\right\rangle_{\tilde\Omega_1}+\left(\boldsymbol\sigma_1,\nabla\mathbf{v}\right)_{\tilde\Omega_1}+\frac{1}{2}\left(\mathbf{r}_{1m},(\mathbf{h}_{1m}\cdot\nabla)\mathbf{v}\right)_{\tilde\Omega_1}+$$

$$+\left(\bar{\mathbf{t}},\mathbf{v}\right)_{\Gamma_{1N}}+\left(\bar t_1\mathbf{g}+\bar t_2\mathbf{s},\mathbf{v}\right)_{\Gamma_{1M}}=\left\langle\rho_1\mathbf{f},\mathbf{v}\right\rangle_{\tilde\Omega_1}$$

$$\left(q,\nabla\cdot\mathbf{u}_1\right)_{\tilde\Omega_1}+\frac{1}{2}\left(r_{1d},(\mathbf{h}_{1d}\cdot\nabla)q\right)_{\tilde\Omega_1}=0$$

$$\mathbf{u}_1=\mathbf{u}_2\quad on\ \tilde\Gamma_1$$

Ω^*_1

$$\left(\rho_2\partial_t\mathbf{u}_2,\mathbf{v}\right)_{\tilde\Omega_2}+\left\langle\rho_2(\mathbf{u}_2\cdot\nabla)\mathbf{u}_2,\mathbf{v}\right\rangle_{\tilde\Omega_2}+\left(\boldsymbol\sigma_2,\nabla\mathbf{v}\right)_{\tilde\Omega_2}+\frac{1}{2}\left(\mathbf{r}_{2m},(\mathbf{h}_{2m}\cdot\nabla)\mathbf{v}\right)_{\tilde\Omega_2}+$$

$$+\left(\bar{\mathbf{t}},\mathbf{v}\right)_{\Gamma_{2N}}+\left(\bar t_1\mathbf{g}+\bar t_2\mathbf{s},\mathbf{v}\right)_{\Gamma_{2M}}+\left(\tilde{\mathbf{t}},\mathbf{v}\right)_{\tilde\Gamma_2}=\left\langle\rho_2\mathbf{f},\mathbf{v}\right\rangle_{\tilde\Omega_2}$$

$$\left(q,\nabla\cdot\mathbf{u}_2\right)_{\tilde\Omega_2}+\frac{1}{2}\left(r_{2d},(\mathbf{h}_{2d}\cdot\nabla)q\right)_{\tilde\Omega_2}=0$$

Ω^*_2

Monolithic Fractional-Step type scheme

- Integration of Navier-Stokes equations in every domain is done by means of a Monolithic Fractional-Step type scheme.
- This scheme is based on the iterative solution of the momentum equation, where the pressure is updated by using the solution of a velocity divergence free correction (m iteration counter):

$$\rho\frac{\mathbf{u}^{n+1,m+1}-\mathbf{u}^{n}}{\delta t}+\rho\left[\left(\mathbf{u}^{n+\theta,m}-\mathbf{u}^{m}\right)\cdot\nabla\right]\mathbf{u}^{n+\theta,m+1}+\nabla\cdot\boldsymbol{\sigma}^{n+\theta,m+1}-\frac{1}{2}\mathbf{h}_{m}\nabla\mathbf{r}_{m}^{n+\theta,m}=0$$

$$\delta t\Delta\left(p^{n+\theta,m+1}-p^{n+\theta,m}\right)=\nabla\cdot\mathbf{u}^{n+\theta,m+1}+\frac{1}{2}\mathbf{h}_{d}\nabla r_{d}^{n+\theta,m}$$

$$\frac{\psi^{n+1,m+1}-\psi^{n}}{\delta t}+\left[\left(\mathbf{u}^{n+\theta,m}-\mathbf{u}^{m}\right)\cdot\right]\psi^{n+\theta,m+1}-\frac{1}{2}\mathbf{h}_{\psi}\nabla r_{\psi}^{n+\theta,m}=0$$

- This scheme only requires to solve scalar problems, with the subsequent savings on CPU time and memory.

Problem statement: Boundary condition at the interface

- Dirichlet conditions are applied on Γ^*_1 (compatibility of velocities at the interface)

$$\mathbf{u}_1 = \mathbf{u}_2 \ \text{ on } \ \Gamma_1^*$$

- And Neumann condition are applied on Γ^*_2 (jump condition)

$$\mathbf{n}_2 \cdot \boldsymbol{\sigma}_2 = \mathbf{t}^* \ \text{ on } \ \Gamma_2^*$$

- $\mathbf{t}^*$ is evaluated from the resulting velocity and pressure field on Ω^*_1. Pressure evaluation must take into account the jump condition given by:

$$p_1 \cdot \mathbf{n} = p_2 \cdot \mathbf{n} + \gamma\kappa\mathbf{n} \ \text{ on } \ \Gamma$$

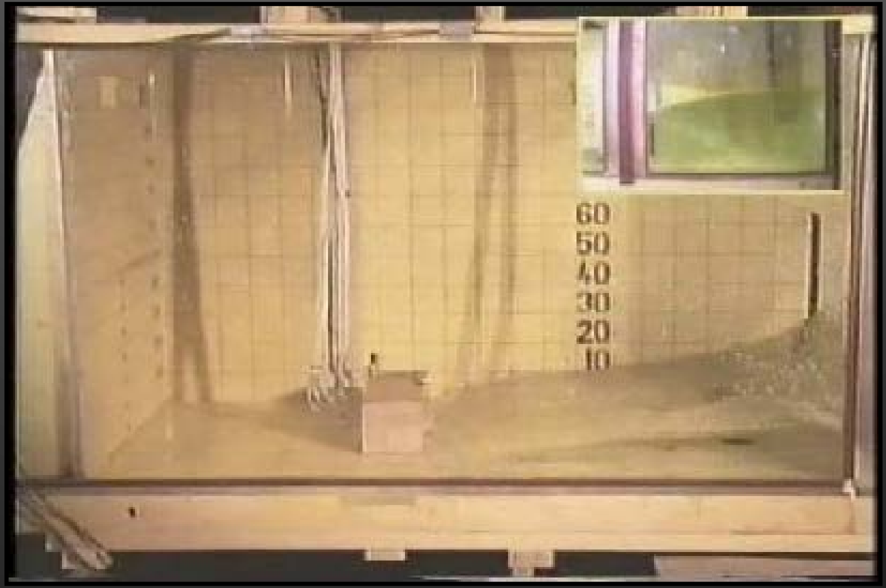
- Where γ is the coefficient of surface tension, and p_1, p_2 are the pressure values evaluated on the real free surface interface Γ.

Adaptation for solving Monophase Flow

- It is usual in many practical applications to have only one fluid of interest. These applications involve most of the flows of interest in naval/marine applications, where density and viscosity ratio are about 1000.
- It is important for these cases to adapt the ODDLS technique to solve monophase problems, reducing the computational cost and capturing the free surface with the necessary accuracy and maintaining the advantages of the proposed method.
- In these cases, the computational domain is reduced to the nodes in the fluid of interest plus those in the other fluid being connected to the interface (Ω^*_1). The later nodes are used to impose the pressure and velocity boundary conditions on the interface.

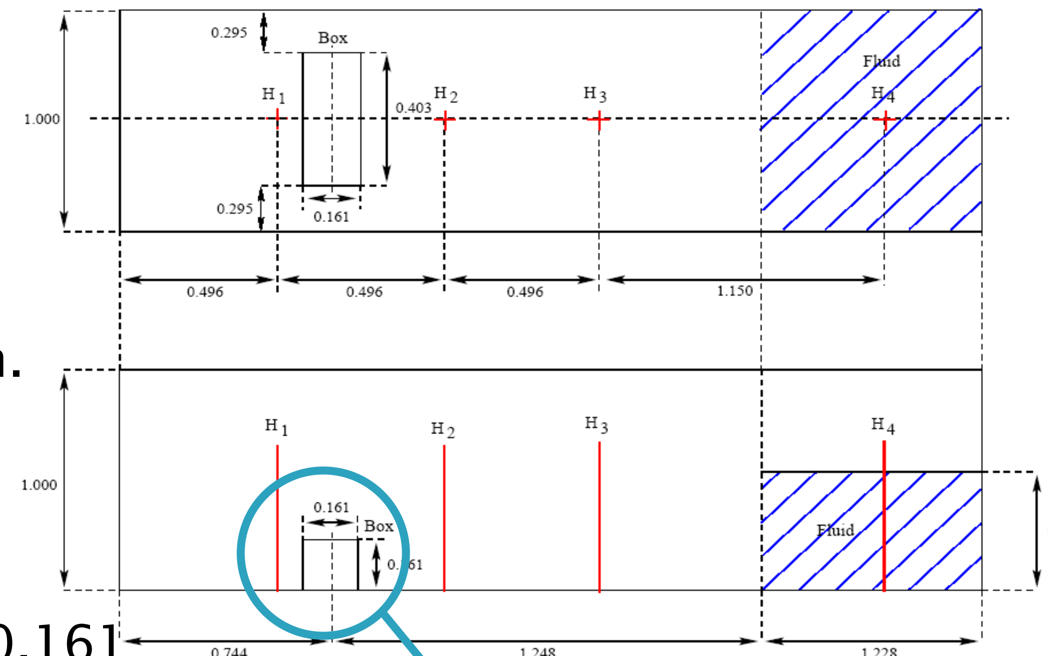
Application example

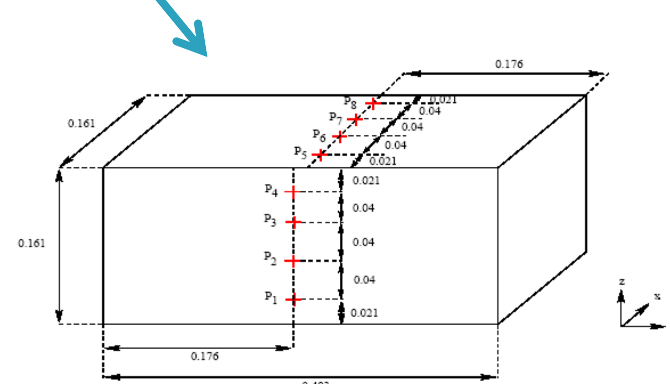
- Green waters experiment



Green waters experiment

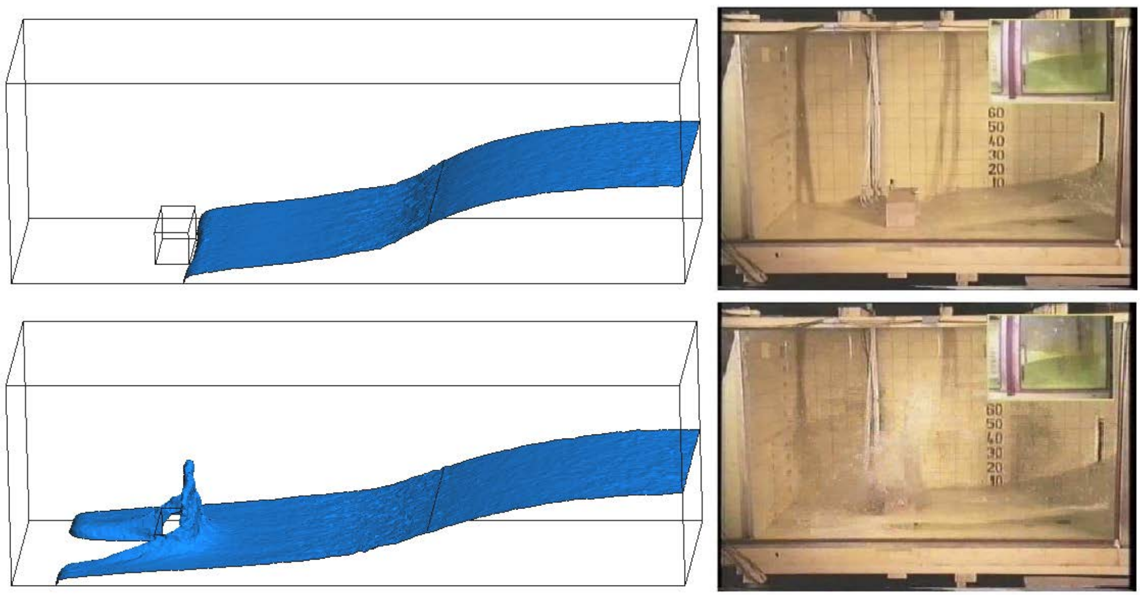
- This example shows a simple model of green water flow.
- The model consists of a tank with open roof of dimensions 3.22x1x1 m. A water column of 0.55 m. height is closed behind a door. The door is opened instantaneously by releasing a weight.
- A block of 0.161x0.403x0.161 meters is set in the middle of the tank, with 8 pressure gauges.
- The experiments in the model have been performed Maritime Research Institute Netherlands (MARIN).





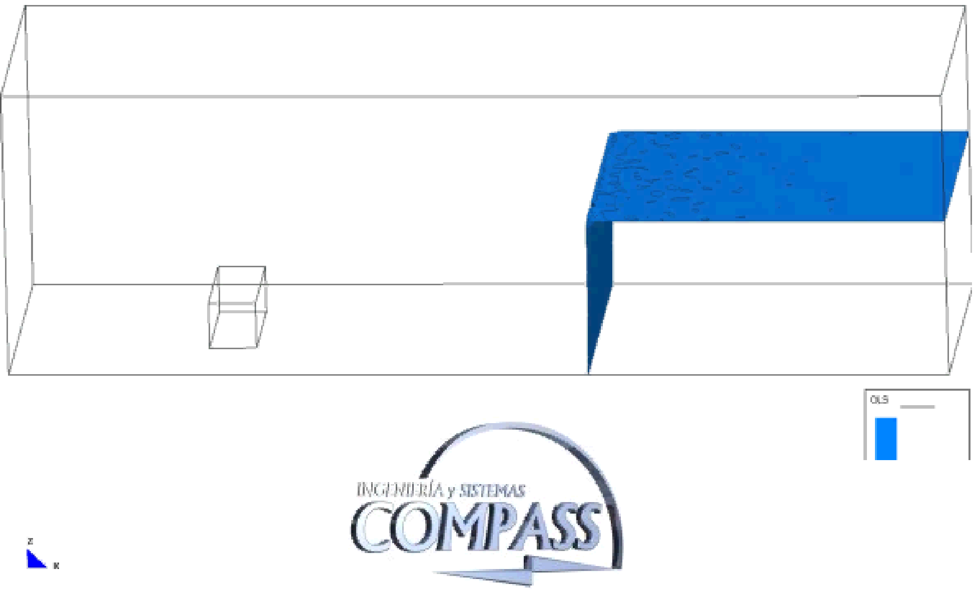
Green waters experiment

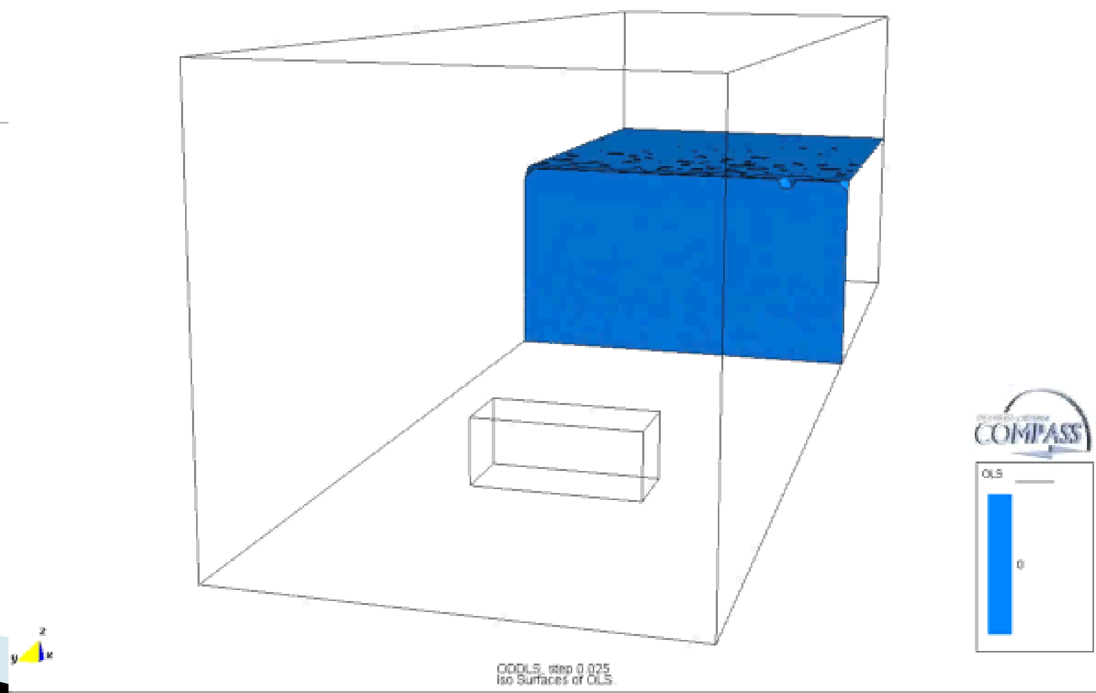
- On the walls of the tank the slipping boundary condition is imposed. The model consists of 1.16 million linear tetrahedra in an unstructured mesh.



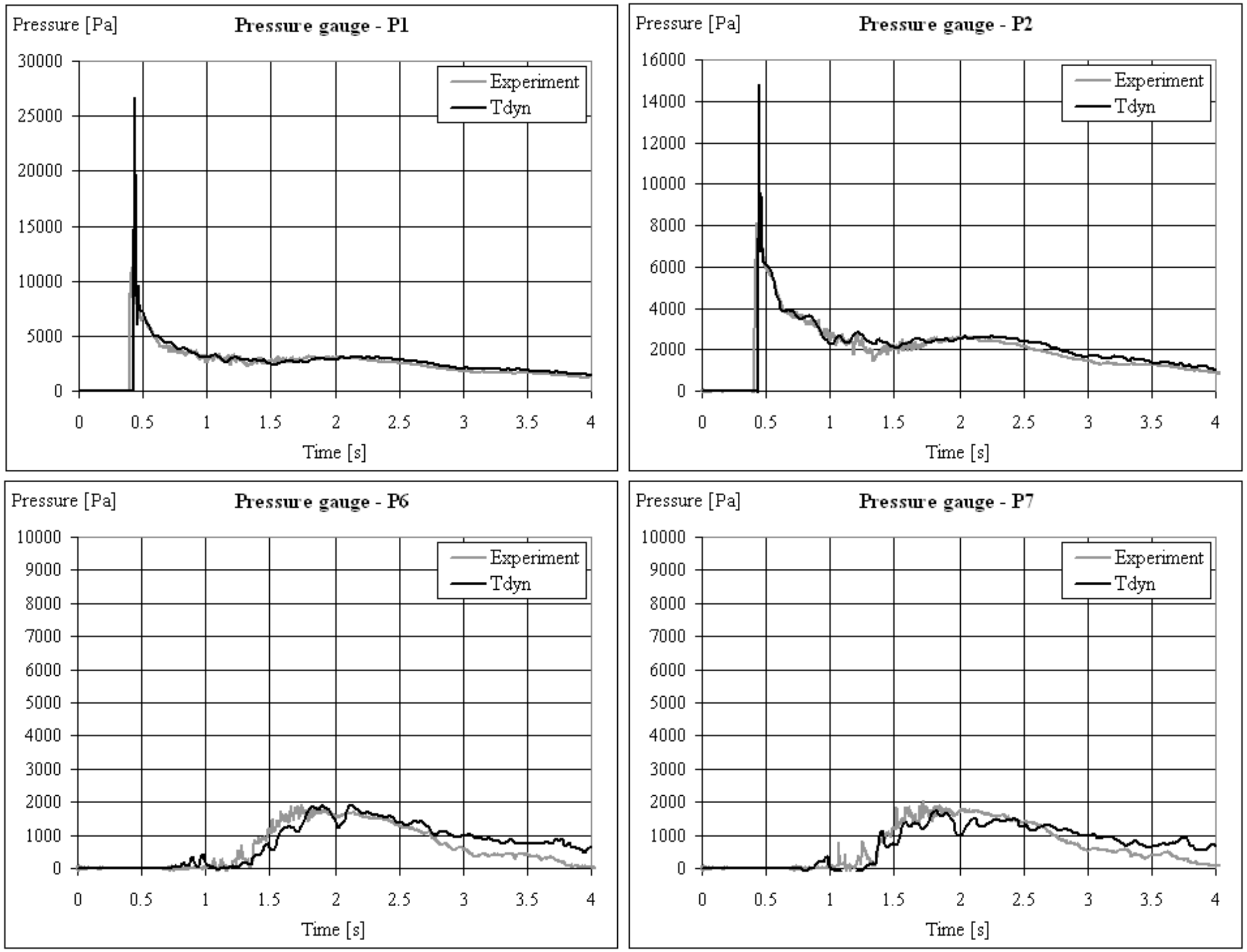
- Above figure shows a comparison between the zero level set function, the free surface, and the experimental water front.

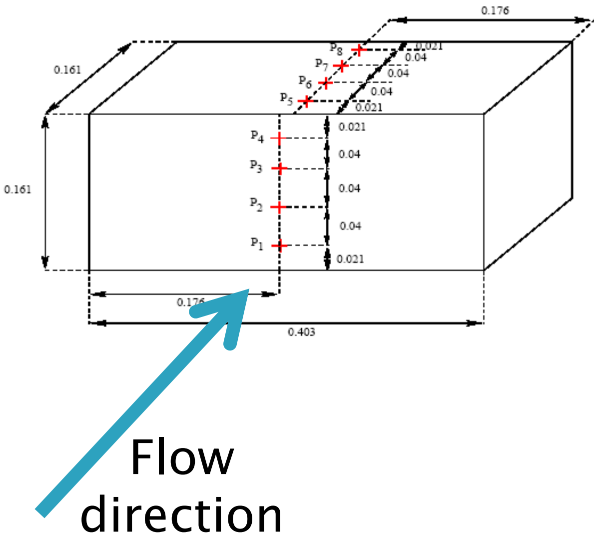
Green waters experiment





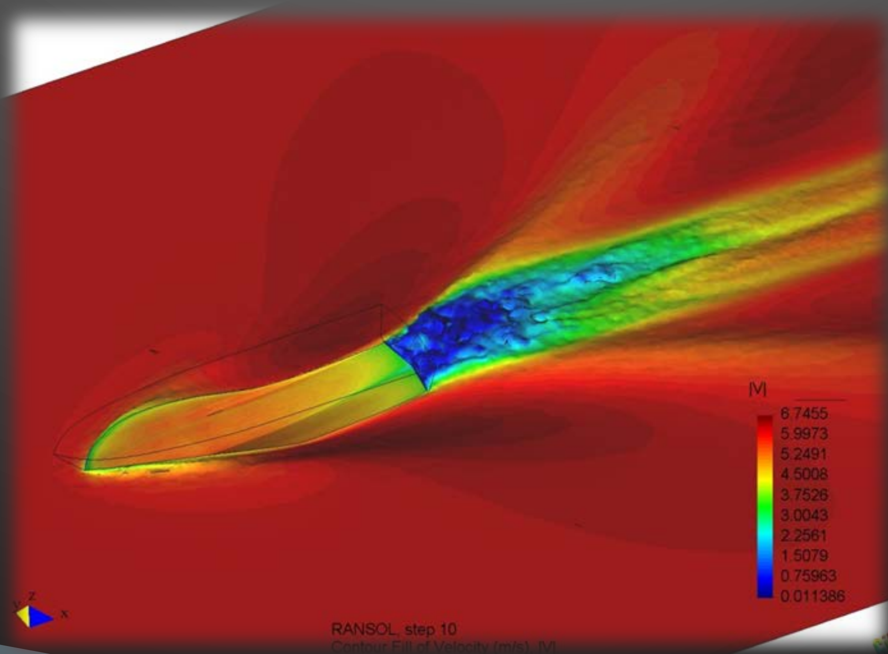
Green waters experiment





Application example

- Navtec NT-130

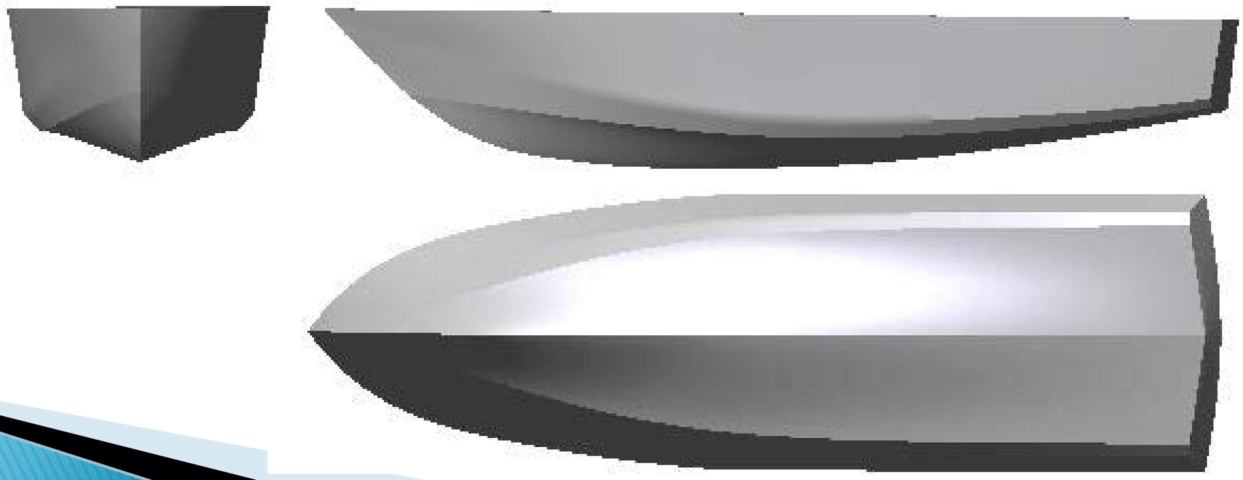


Navtec NT-130

- This example shows the application of the presented technique to the analysis of a semi-planning hull designed by Navtec. The general characteristics of this boat are shown in the following table:

Main Characteristics	
LOA	14.0 m
Moulded Draft	1.05 m
Moulded Beam	3.54 m
Design Speed	14 Kn (Fn = 0.65)

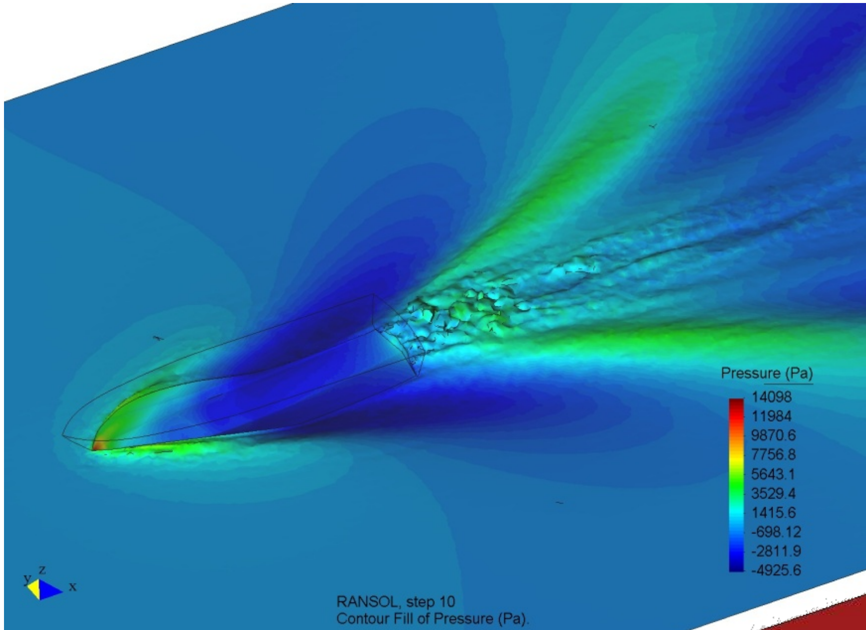
- The geometry of the boat has been defined by means of NURBS entities.



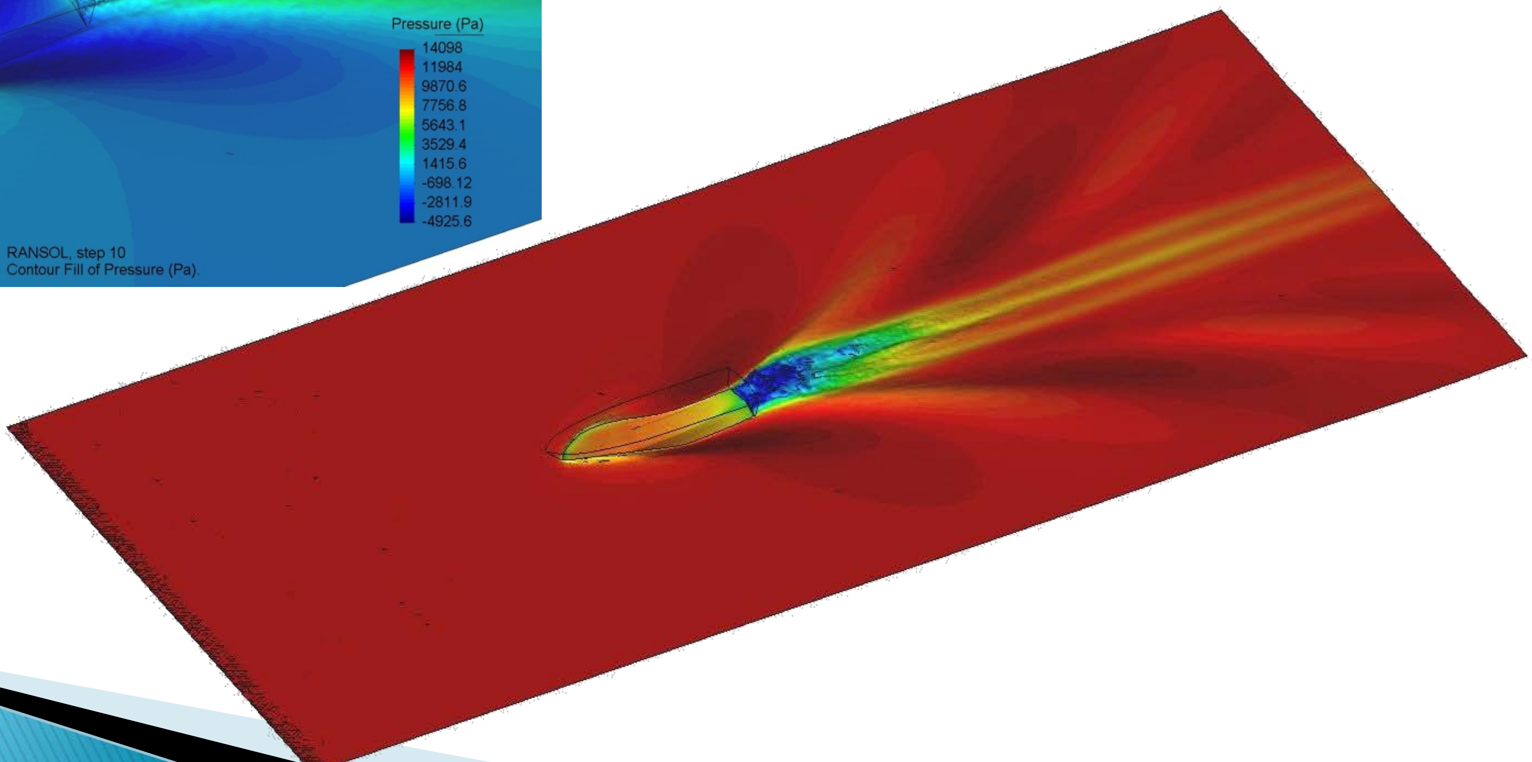
Navtec NT-130 Still Water Analyses

- The first analysis consist of the towing of the hull at different speeds (still water).
- Characteristics of the analyses:
  - 2 sets of analysis were done: fixed ship and free to sink and trim.
  - 4 different speeds were run for every set of analysis with an unstructured 3.2 million linear tetrahedra mesh.
  - Two more meshes of 1.8 and 15.8 million linear tetrahedra were used to study the influence of the mesh density in the results.
  - All the cases were run using an ILES-type turbulence model.

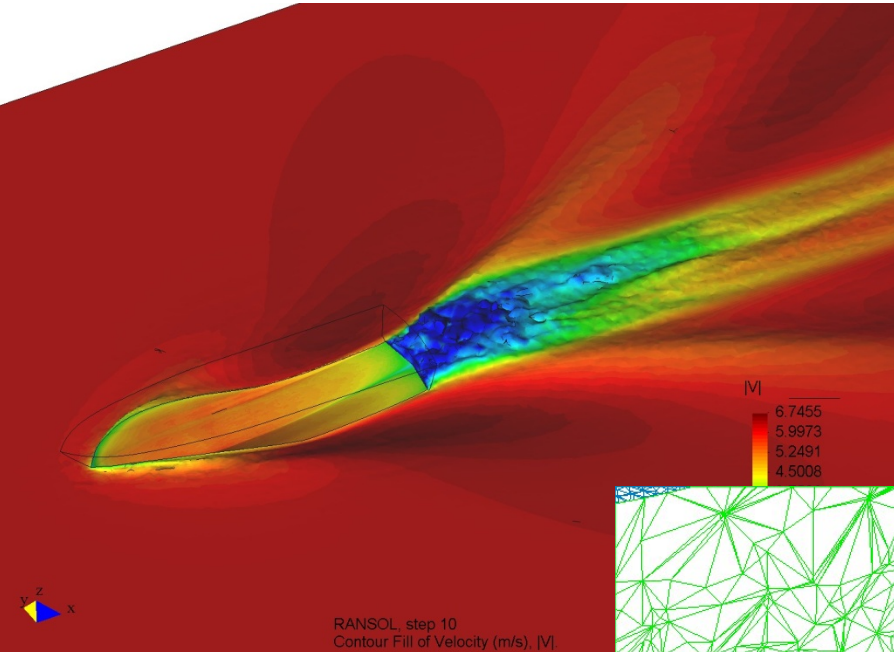
Navtec NT-130
Still Water Analyses



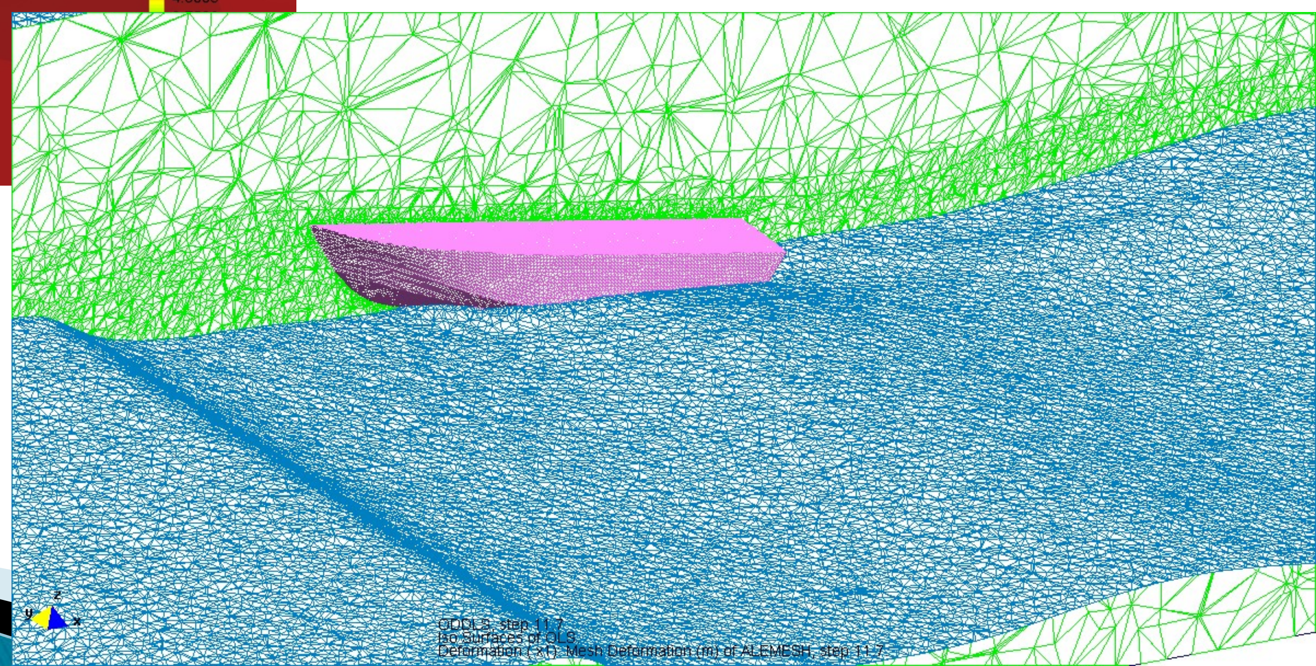
Snapshot of the results (V = 12kn):
- Mesh of 15.8 million tetrahedra
- Left: (dynamic) pressure
- Down: velocity modulus



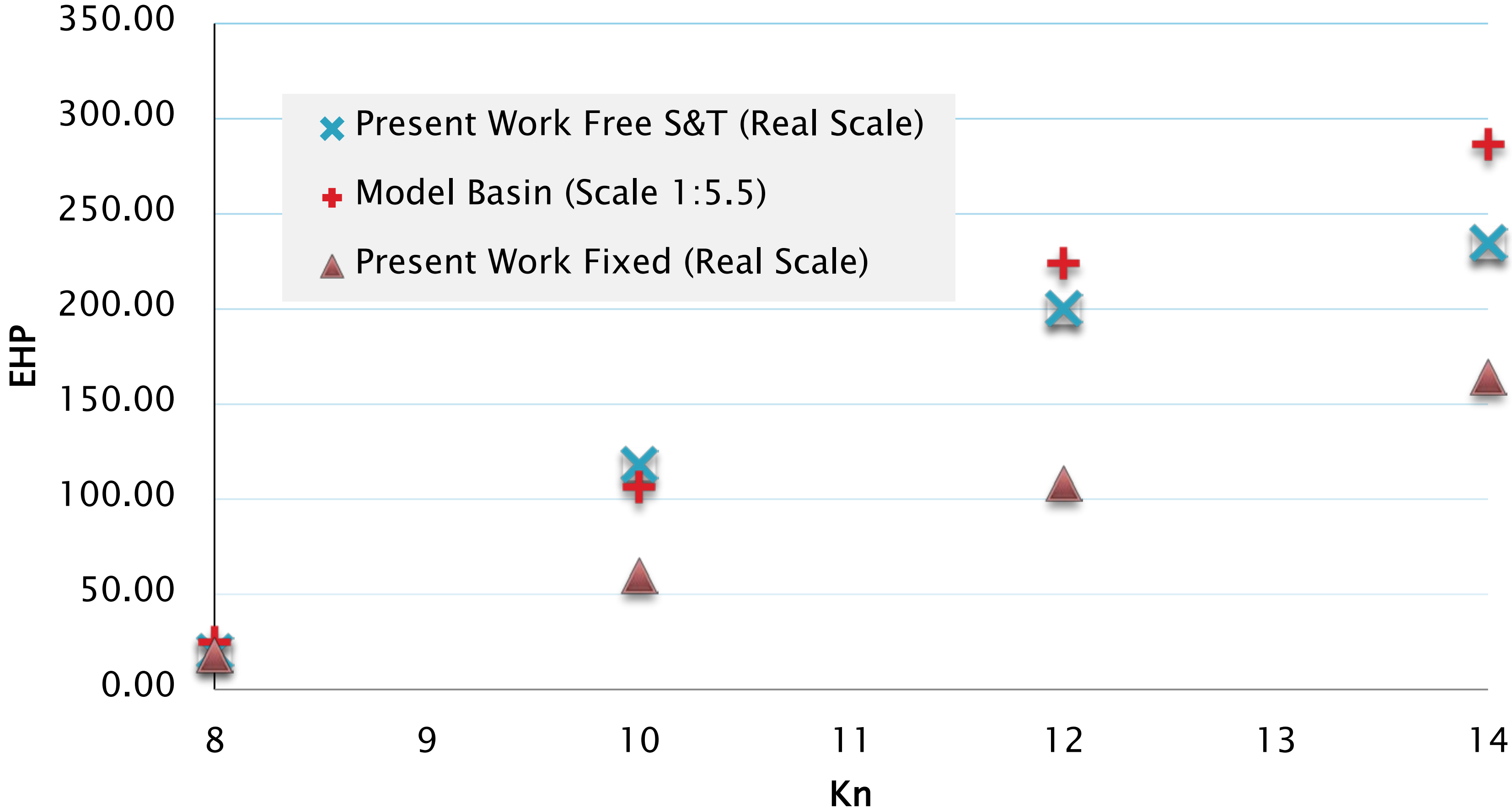
Navtec NT-130
Still Water Analyses



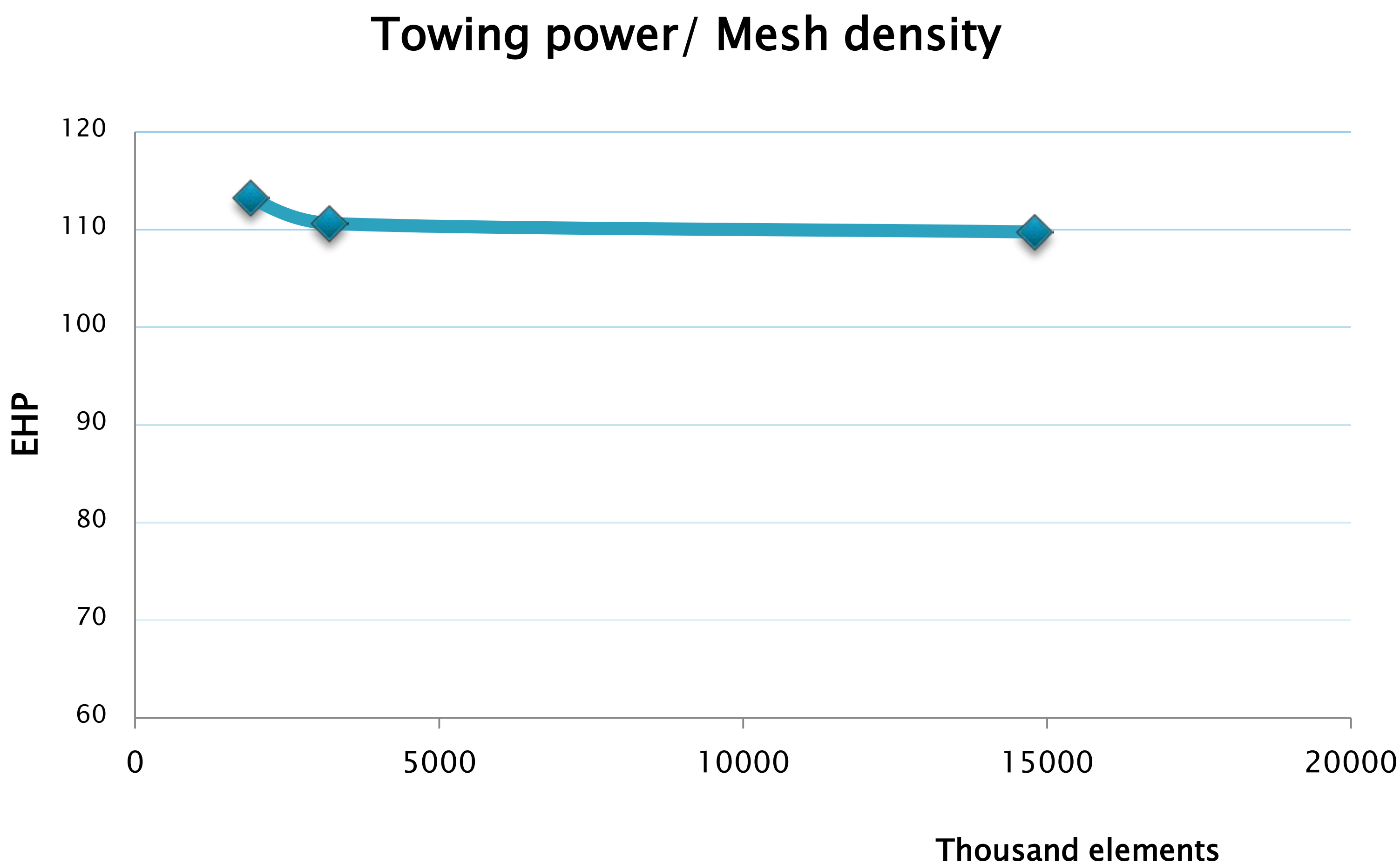
Snapshot of the results (V = 12kn):
- Left: velocity modulus (mesh of 15.8 million tet.)
- Down: View of the mesh of 1.8 million tets.



Navtec NT-130



Navtec NT-130



Navtec NT-130 Head Waves Analyses

- The second analysis consist of the towing of the hull at different speeds with head waves.
- Characteristics of the analyses:
  - The analyses were carried out with the ship free to sink and trim.
  - 4 different speeds were run for every set of analyses with an unstructured 2.5 million linear tetrahedra mesh.
  - The selected wave lenght for the analyses was 1.5xLOA (about critical values for slamming).
  - All the cases were run using an ILES-type turbulence model.

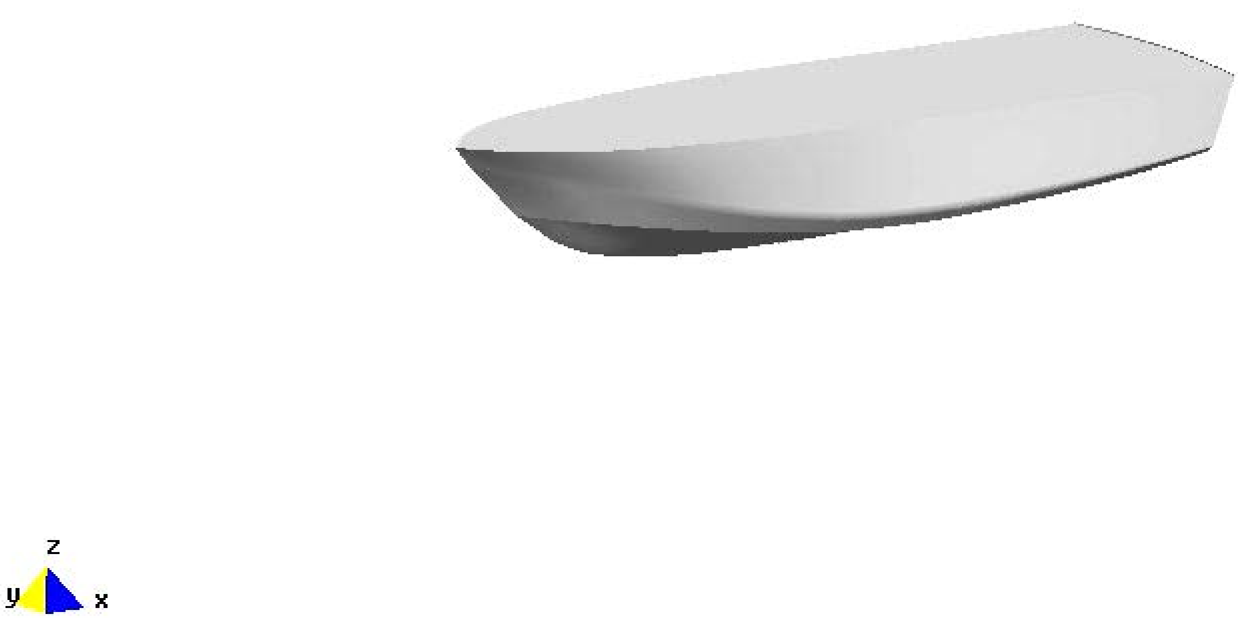
Navtec NT-130 Wave Generator

- The waves are generated by defining an oscillating velocity boundary condition at the inlet of the basin. The velocity is obtained from the equivalent movement of a wall given by $x = d \cdot \sin(\omega t)$, where ω is the wave generator frequency and d is the amplitude of the movement (stroke). The resulting boundary condition is as follows (k is the wave number and V_o is the ship speed):

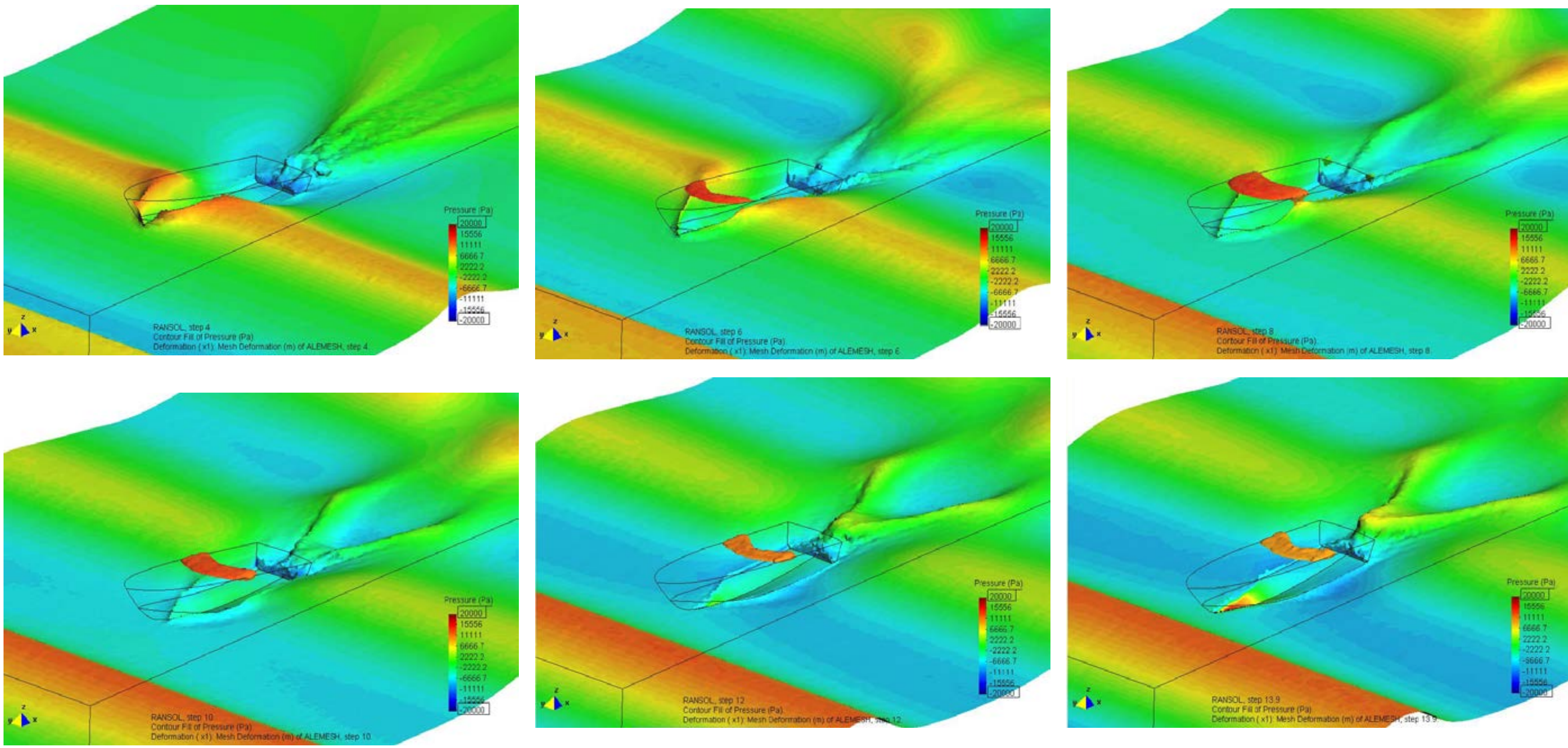
$$V_x = V_0 + d \cdot \omega \cdot \cos\left[\left(\omega + k \cdot V_0\right)t\right]$$

- For this problem linear wave theory states that the relation between wave number, channel depth and wave-maker frequency is $\omega^2 = g \cdot k \cdot \tanh(kh)$. Where k is the wave number, h is the water depth and g the gravity acceleration
- The waves are dumped at the right hand side of the tank by disposing a beach.

Navtec NT-130: Head Waves

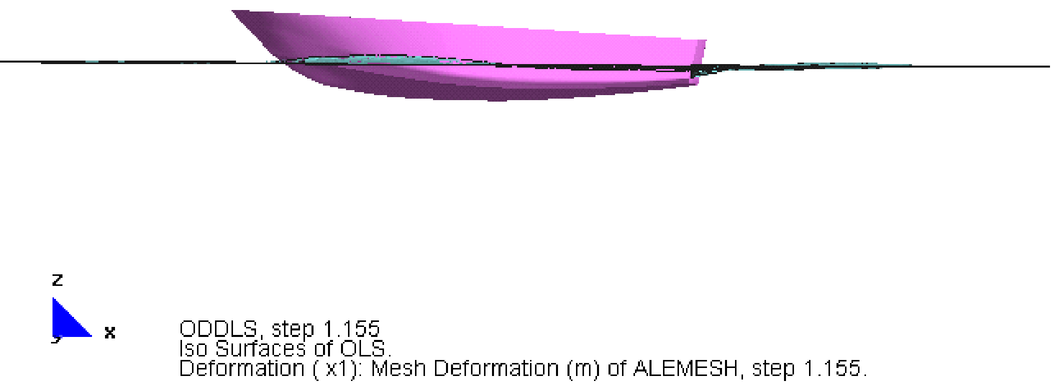


Navtec NT-130: Head Waves



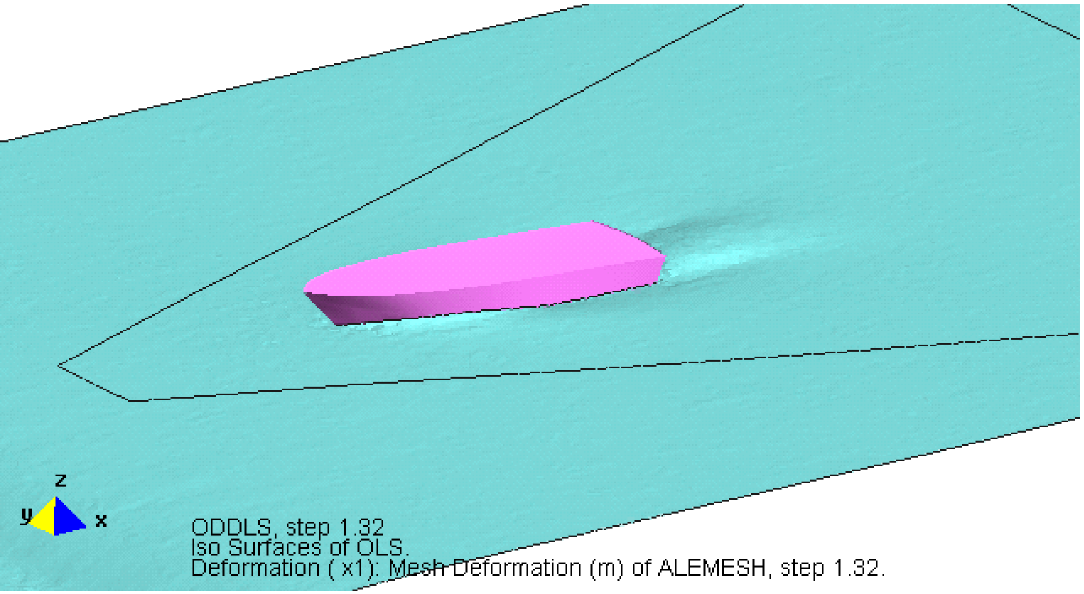
Dynamic pressure field (sequence). Fn = 0.65.

Navtec NT-130: Head Waves

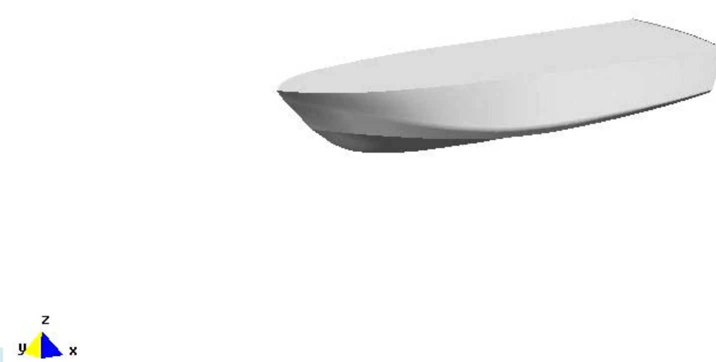


Head waves: side view
Left: Fn = 0.56
Down: Fn = 0.65

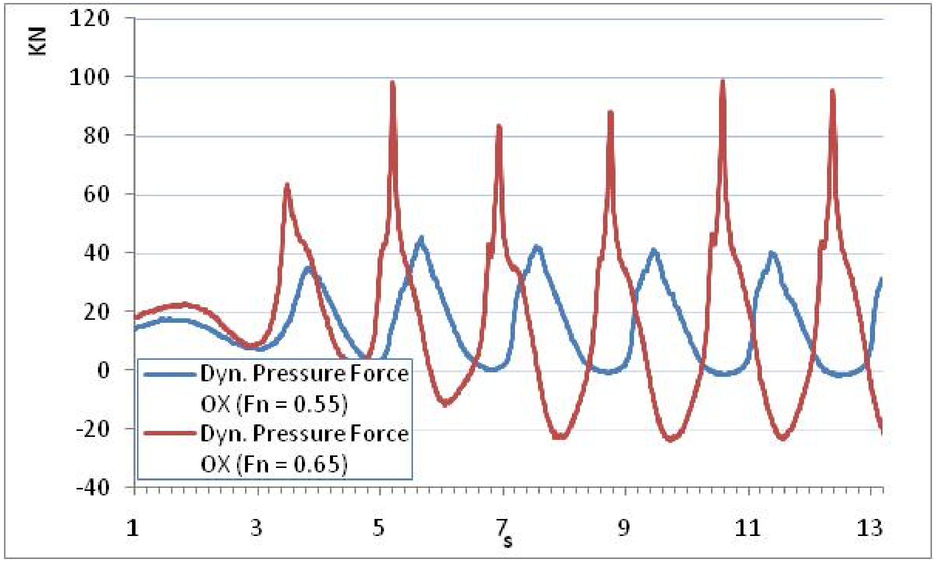
Navtec NT-130: Head Waves



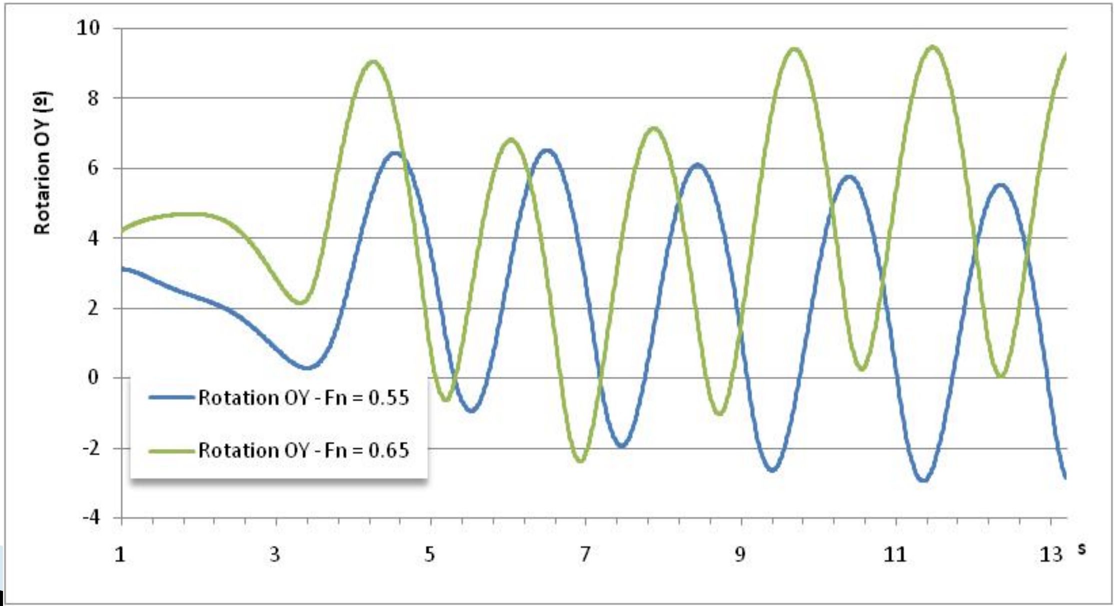
Head waves: front view
Left: Fn = 0.56
Down: Fn = 0.65



Navtec NT-130: Head Waves



Left: Pressure forces (OX)
Down: Trim angle



Conclusions

- The present work describes a new methodology for the analysis of free surface flows so-called ODDLS.
- ODDLS method is based on the domain decomposition technique combined with the Level Set technique and a FIC stabilized FEM. The ODDLS approximation increases the accuracy of the free surface capturing (level set equation) as well as the solution of the governing equations in the interface between two fluids. The greater accuracy in the solution of the interface between the fluids allows the use of non-structured meshes, as well as larger elements in the free surface.
- The method can be simplified by solving only one of the two fluids, which increases the efficiency in most of the naval/marine applications where the effect of one of the fluids can be neglected.
- The proposed ODDLS methodology has also been integrated with an ALE algorithm for the treatment of moving meshes.
- The ODDLS technique has been applied in the analysis of different free surface flows problems. The good qualitative and practical results obtained in comparison with experimental data show the capability of the ODDLS methodology for solving free surface flows problems of practical interest.

Further Information

- Contact email address:
  info@compassis.com
- A demo version of the software can be downloaded from:
  http://www.compassis.com
- The validation cases can be downloaded from:
  ftp://ftp2.compassis.com/papermodels

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