

STRUCTURAL ASSESSMENT OF THE SEISMIC BEHAVIOR OF THE DOME OF THE TAJ MAHAL

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Abstract. *This paper presents the results of a study of the structural assessment of the seismic behavior of the main dome of the Taj Mahal, in Agra, India. Built by the Moghul emperor Shah Jahan between 1632 – 1648 AD, the structure is one of most famous buildings in the world and the finest and most sophisticated symbol of Moghul architecture in India. As it is located in a zone of moderate seismicity, there is an urgent need for ensuring the structural safety and preservation of this iconic cultural heritage for future generations. The objective of this study is to develop a simplified approach to enhance our conceptual understanding of the mechanism of load path/transfer in the dome of the Taj Mahal under both gravity and seismic loads, as well as to better understand the influence of the geometry of the dome on its structural behavior. Using the software SAP2000, a finite element model of the dome is created and analyzed under the action of gravity and seismic loads, with the seismic loading assumed to have a PGA of 0.25g based on the results of a study conducted by the Indian National Disaster Management Authority. The results obtained include plots of the distribution of nodal reactions at selected levels to represent the lateral load behavior of the dome of the Taj, as well as plots illustrating the paths of load transfer under the action of gravity and seismic loads. Free body diagrams of selected rings are also presented to illustrate the ‘hoop’ forces in the dome, while simplified internal stresses are computed and compared with known material properties. Observations are also presented about the effect of the dome geometry, especially the bulging part of the dome, on the distribution of reactions and the resulting mechanism of load transfer in the structure under both gravity and seismic loading.*

1 INTRODUCTION

World cultural heritage has found rich and creative expression in beautiful and expansive works of domed architecture such as the Pantheon in Rome, built by Emperor Hadrian between 120-124 AD, the Hagia Sophia in Istanbul, Turkey built in 6th century AD and the Taj Mahal in Agra, India built by the Emperor Shah Jahan between 1632-1648 AD. The survival of these structures to this day inspires the fundamental curiosity - how did the ancient builders build such structures with the materials (e.g. stone or brick, and mortar) and knowledge available to them at that time? A lot can be learned from their wisdom, specifically through their understanding of materials, geometry of structural forms, structural stability, and the techniques of construction.



Figure 1: (a) The Taj Mahal in Agra, (b) Humayun's tomb in Delhi and (c) Itmad-Ud-Daulah's tomb in Delhi (CC BY-SA 4.0)

While structures such as the Pantheon and the Hagia Sophia have been the subject of intense curiosity and have received considerable attention and study over the years, the Taj Mahal, on the other hand, has not received the adequate attention and study it deserves. Most studies conducted on the Taj to-date have focused predominantly on architectural, visual, and spatial aspects of the structure as well as on the Moghul gardens that surround it [1-4], while limited studies have been carried out from a structural engineering perspective [5,6,8,9]. In particular, the large, unique onion-shaped dome of the structure invites curiosity and merits further investigation.

This study aims to investigate the behavior of the main dome of the Taj Mahal under gravity and seismic loads in order to enhance our conceptual understanding of the load paths and mechanism of load transfer in the structure; and to better understand the influence of the geometry of the dome on its structural behavior under gravity as well as seismic loads.

In order to do this, the background and construction history of the Taj Mahal are first studied and available drawings, material information and results of previous structural investigations are examined.

Based on these studies, the dome geometry is established, which is subsequently used to generate a finite element (FE) model in SAP2000 [7]. In the first instance, a relatively simple FE model is generated by dividing the dome into a series of circular rings, which are then divided into sixteen radial segments (lunes). This FE model is then analyzed under the action of gravity and seismic loads, with the seismic loading corresponding to the maximum expected PGA on site. The results of this model provide insight into the distribution of forces/reactions in the dome under the action of gravity and seismic loads.

A second more refined FE model of the Taj Mahal dome was then developed in order to investigate the distribution of meridional and hoop stresses in the dome (both over the height and across the dome thickness), under the same loading conditions. The resulting stresses are compared to those reported by previous structural investigations of the Taj Mahal dome [5,6] as well to the material properties of the structure.

2 BACKGROUND

2.1 Design and construction history

The Taj Mahal, considered as one of the most accomplished examples of Moghul architecture in India, was built between 1632 – 1648 AD by the Moghul emperor Shah Jahan in memory of his wife Mumtaz Mahal, who died while giving birth to their fourteenth child.

While designing the structure, Shah Jahan drew inspiration from both traditional Persian architecture as well as classic Mughal architecture as exemplified by structures such as Humayun's tomb and Itmad-Ud-Daulah's tomb (Figures 1b and c). In fact, according to [1], 'Taj Mahal was the logical conclusion and synthesis of several strands that already existed in Moghul architecture'. Various designs were submitted to Shah Jahan and modified according to his own suggestions, and while the identity of the architect of the final structure is heavily debated, it is widely accepted that the principal designer was Ustad Ahmad Lahauri of Lahore. Lahauri, who was a Persian, and said to be heavily involved in the construction of the structure, which was such a colossal venture that it required a whole team of architects, as well as masons, stonecutters, inlayers, carvers, painters, calligraphers, dome builders and other artisans requisitioned from the whole Moghul empire, central Asia and Iran. In fact, it took seventeen years for construction of the entire monumental complex to be completed.

2.2 Plan and configuration

The Taj Mahal has often been described as the embodiment of architectural perfection [4]. It is well recognized that geometry played an everlasting role in its architecture, and the plan layout and configuration of the Taj Mahal has been studied and analyzed extensively by many scholars over the years [1-5].

The tomb, which is the central focus of the entire Taj Mahal complex, is a large white marble-clad structure which stands on a square plinth and consists of a symmetrical building with an iwan (an arch shaped doorway) topped by a large dome and a finial. The base structure is a large multi-chambered cube with chamfered corners forming an unequal octagon that is approximately 54.9 m on each of the four sides. This irregular octagonal plan is a variation of hasht bihisht ('eight paradises') plan, which typically comprises a central chamber surrounded by four chambers at the diagonals and four open fore-halls on the perpendicular axes as illustrated by Figure 2a.

The base structure is surmounted by an inner dome and a cylindrical drum, which in turn is topped by the monumental bulbous dome. While the inner dome keeps the height of the interior space in proportion, the large outer dome helps the structure stand out visually. An elevation drawing illustrating this structural system can be found in Figure 2b [6].

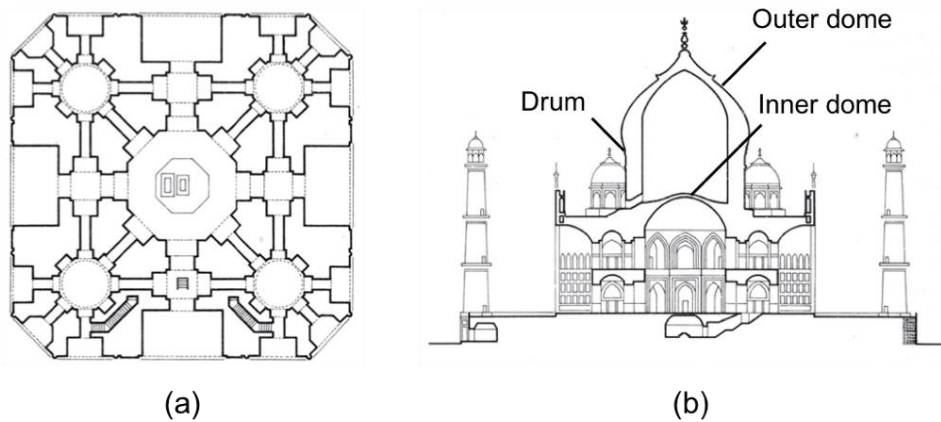


Figure 2: Taj Mahal (a) floor plan and (b) section (source ref. [2])

2.3 Materials used for construction

The materials used for construction of the Taj Mahal were sourced from all over India and Asia, and have been systematically investigated in detail and documented in [3,5,6]. The walls in the complex, many of which are several feet thick, were constructed of bricks with rubble inner cores and façades of either marble or sandstone fastened with iron dowels and clamps. The bricks typically measured 17.8-19.0 x 10.9-12.7 x 1.9-3.0 cm and were combined with lime mortar, while the façade slabs had a thickness of about 15.24 cm. The bricks were fired in local kilns, and the sandstone came from quarries 45 km away near Fatehpur Sikri. The white marble was brought all the way from quarries belonging to Raja Jai Singh in Makrana, Rajasthan, located 402 km away. The interior surface of the dome is finished with polished plaster, made with a combination of burnt lime, ground shells, marble dust, gum, sugar and egg whites, and applied in sequential step process.

2.4 Previous structural investigations

The first known structural investigation of the Taj Mahal, including a thorough examination of the sub-soil conditions and the foundation system, was carried out by Dayalan [5]. The dimensions of the structure were established using precise measurements and detailed drawings made available by the Archaeological Survey of India. Based on these dimensions, the weight of the dome was estimated to be 119,600 kN and the weight of the supporting drum to be 59,800 kN. Dayalan's structural analysis consisted of dividing the dome (up to a height of 22.9 m) into 30 equal sections, constructing a diagram of forces acting at the center of gravity of each of the sections, based on which the stresses were determined. From this analysis, the maximum compressive stress at the base of the dome was determined to be 383.07 kPa, while a maximum compressive stress of 574.54 kPa was recorded at the base of the supporting drum. According to [5] the walls of the cenotaph chamber that carry the weight of the drum and dome are very heavily loaded resulting in a stress level in the cenotaph walls of 766.01 kPa which is almost twice the safe stress limit level for present day bricks with lime mortar. In fact, the signs of crushing detected in the cenotaph chamber are most likely due to the squeezing of mortar (which had likely deteriorated over time) under the action of these superimposed loads.

A more detailed structural analysis of the dome of the Taj under gravity loading was

conducted using finite element modelling in Abaqus FEA [6]. From this analysis it was found that the maximum stresses experienced by the dome in both the hoop and meridional directions were less than the strengths of the brick and mortar, with the over-dimensioning of the structure effectively resulting in a safety factor of 5 for the dome [6]. This compares reasonably well with analyses conducted on the complete Taj Mahal structure using 3D finite element modelling in SESAM, which yielded a safety factor between 4 and 8 for the monument under vertical loading [9].

Studies concerning the seismic analysis and safety of the Taj Mahal have also been conducted at the University of Roorkee, however these make use of relatively simplified models such as the lumped mass and spring model [8].

3 TAJ MAHAL DOME – DIMENSIONS AND GEOMETRY

In order to determine the dimensions of the dome of the Taj Mahal needed to build the computational model for structural analysis, Mehrotra and Glisic [6] established one known reference dimension namely the external dimension of the drum as documented by Dayalan [5]. Using this as a reference dimension, all other defining dimensions were determined relative to this reference dimension. Figure 3a presents the major dimensions of the dome as presented by Mehrotra and Glisic [6]. The dome has a constant thickness of 4 meters, from its base to a height of 15.5 meters above it. Dome thickness varies in the conical portion of the structure (i.e. the top 8 meters of the dome), starting with a thickness of 4.3 meters at the lower end, which increases to 4.8 meters near the top as shown in Figure 3a.

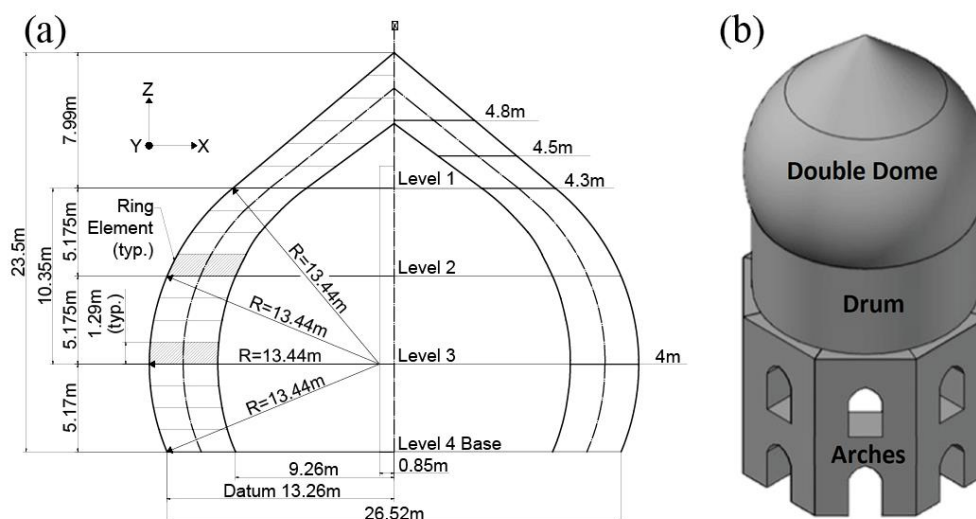


Figure 3: (a) Dome section showing ring dimensions and geometry and (b) Structural System - Taj Mahal

4 STRUCTURAL SYSTEM

The Moghul designers and engineers had developed an innovative structural system for the Taj Mahal consisting of the outer dome supported on a drum, seated on eight massive columns with arches spanning between these columns, as illustrated by Figure 3b (reproduced from [6]). As reported in [5], it is likely that the Taj's outer dome with its bulbous geometrical form must experience a certain amount of tension in the lower part of its structure. In the absence of

reinforcing techniques like outer metal bands during that period, the architects and master builders of the Moghul era managed to creatively achieve stability of the dome by using good quality bricks and mortar, and by gradually increasing the dome thickness [5].

As reported by Dayalan [5] and Viladkar et. al [9], this entire structure rests on a raft/mat 33.5 m thick, which rests on a thick sandy layer 9.1m in depth, overlaying a 63.6 m thick clayey strata, founded on rock.

5 STRUCTURAL ANALYSIS

Dayalan's simplified structural analysis of the Taj Mahal [5] as well as the more detailed structural analysis as carried out by Mehrotra and Glisic [6] only considered gravity loads in the analyses. In this paper, a detailed structural analysis of the Taj Mahal dome has been carried out under the action of both gravity and seismic loads. Ideally it would be desirable to carry out a comprehensive structural analysis of the entire structure, incorporating the entire structural system e.g. the outer dome, inner shallow dome, the drum, and the arched cenotaph and its raft foundation. However, this proved to be infeasible due to modelling complexities and computational constraints, resulting in only the main dome of the structure being considered in this phase of the study.

5.1 Finite Element Analysis – Part I

The objective of the first part of the detailed structural analysis was to investigate the transfer of forces/reactions in the Taj Mahal dome structure. Based on the geometry of the dome presented in Figure 3a, a simple 3D finite element (FE) model of the Taj Mahal dome was first developed using shell elements in the SAP2000. The dome was divided into seventeen rings over its height, which in turn are divided into sixteen radial segments (lunes), resulting in a total of 272 shell elements in the model (17 vertical x 16 radial = 272). Boundary conditions were assumed to be pinned at the base. A 3D view of this model is presented in Figure 4.

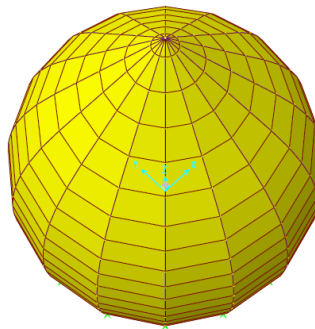


Figure 4: 3D FEA model of the Taj Mahal dome

The material properties adopted for this model are listed in Table 1. Note that the analyses conducted here assume an isotropic material model, which is not the case in reality. However the limited information regarding exactly how the bricks are arranged within the dome (i.e. concentrically/radially/comboination of the two), made it challenging to define anisotropic material properties for the model. Thus these effects were neglected in the adopted modelling approach.

Table 1: Material properties assumed for the FE model

Parameter	Value
Density	20.59 kN/m ³
Modulus of elasticity	3447.4 MPa
Poisson's ratio	0.2
Unit compressive strength [11]	2430 kPa
Unit tensile strength [11]	300 kPa
Bond tensile strength [12]	170 kPa

The following three load cases are considered for analysis:

1. Dead load (self-weight only) = 101,705.83 kN
2. Seismic load = 0.25g, as obtained from the results of a study conducted by the Indian National Disaster Management Authority [10]
3. Combined load = Dead + Seismic

and the forces/reactions are studied at four different levels of interest as listed in Table 2 and indicated in Figure 3a.

Table 2: Dome dimensions at the specified levels of interest

Level	Height above Base (m)	Diameter (m)	Dome Thickness (m)
4	0.00	12.90	4.0
3	5.04	13.91	4.0
2	10.08	12.90	4.0
1	15.13	9.19	4.3

Figure 5 presents the distribution of reactions under each of these different load cases. A comparison of the distribution of reactions under the dead load only in the XY plane (Figures 5b and c) illustrates that the forces appear to be pushing outward at the bulging part of the dome (Level 3, Figure 5c) and inward at the base (Level 4, Figure 5b).

A study of dome reactions under seismic loading at the base indicate that the reactions in the direction of the earthquake are maximum at the edges of the dome, and are minimum at the center of the dome in the line of the earthquake. Reactions in a direction normal to the earthquake show bulging in the left half of the dome; and contraction in the right half (Figure 5e). A study of the dome reactions under seismic loading at the bulge (Level 3) indicate that the reactions in the direction of the earthquake are more uniform on both the right and left side of the dome; and that the reactions in the direction normal to the earthquake show lesser bulging and contracting at this level (Figure 5d).

5.2 Finite Element Analysis – Part II

A more refined FEA model was also developed in SAP 2000 using eight node solid elements for investigating the stresses at different levels, and across the thickness of the dome.

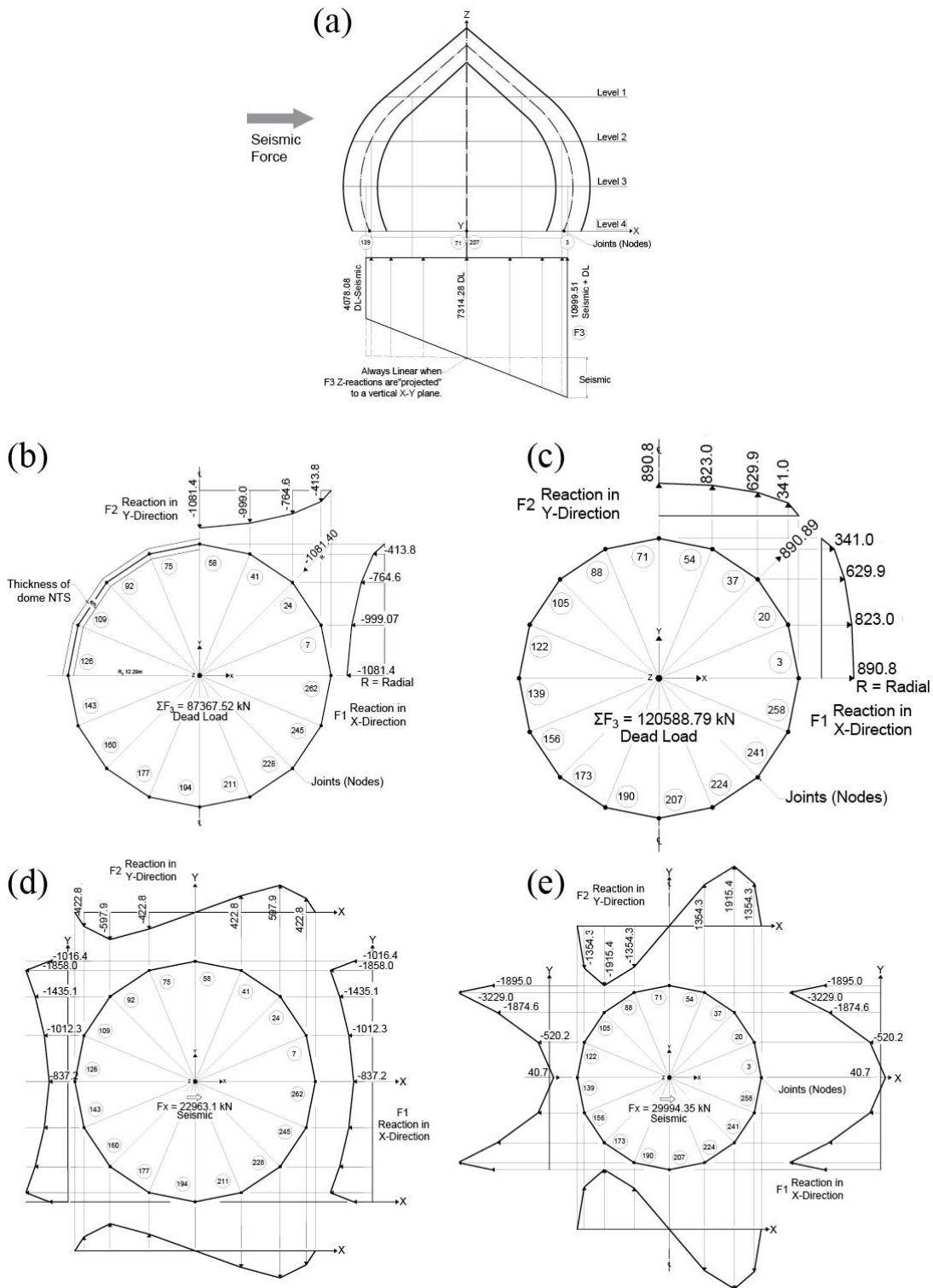


Figure 5: (a) distribution of vertical reactions – dead + seismic; (b) Level 3 (bulge) – XY reactions - dead load - kN; (c) Level 4 (base) – XY reactions - dead load - kN; (d) Level 3 (bulge) – XY reactions - seismic load - kN (e) Level 4 (base) – XY reactions - seismic load - kN

As in the case of the FE shell model, the boundary conditions were modelled as pinned ($UX=UY=UZ=0$), while the dome was divided into a series of rings over its height, which are then divided into one hundred and twenty radial segments (lunes). The dome was further discretized using solid elements, by dividing the dome thickness into four segments throughout the dome geometry up to 19.5 m above the base of the dome. Above the 19.5 m level, 4 m high solid cone shape elements were generated. The resulting FE model comprises 17644 solid elements, with 23050 joints (Table 3). Note that the material properties and load cases adopted for this model are the same as for the simplified model with the shell elements.

Table 3: Dome dimensions as they relate to SAP 2000 software

Level	No. of segments through dome thickness	No. of vertical divisions	No. of radial divisions	No. of 3D solid elements
4	4	8	120	3840
3	4	8	120	3840
2	4	8	120	3840
1	-	-	120	6124
			Total	17644

Plots of the maximum and minimum principal stress distributions obtained using this model can be found in Figure 6. Under dead load only, the maximum principal stress can be assumed to correspond to the hoop stress, while the minimum principal stress can be assumed to correspond to the meridional stress. It will thus be referred to as such in order to facilitate comparison with the results of other studies.

In general, the study of the hoop stress distributions in the dome of the Taj Mahal under dead load only (Figures 6a) indicates that:

- At level 3 (bulging part of the dome) there is hoop tension which is higher on the outside and lower on the inside of the dome
- At level 4 (at the base of the dome) there is hoop compression on the outer surface of the dome and lower compression on the inside surface of the dome indicating that there is rotation and bulging out even at the base of the dome

A comparison of the maximum hoop and meridional stresses obtained from this structural analysis, with those obtained by Dayalan [5] and Mehrotra & Glisic [6] are presented in Table 4. As Table 4 indicates, under the influence of the dead load only, maximum hoop tensile stress occurs at the outer surface of Level 3 (bulge), with a value of 70.8 kPa. This lies between the maximum hoop tensile stress values reported by Dayalan [5] and Mehrotra & Glisic [6], and is found to be within the safe limits of the 300 kPa tensile strength of brick [11] as well as the 170 kPa tensile bond strength of the brick-mortar interface (perpendicular to the joints, as this is the limiting case) [12]. At the base (Level 4) the hoop stress reverses to compression and varies from -202.0 kPa at the outer dome surface to -39.3 kPa at the inner dome surface, with an average value of -120.6 kPa which is larger than the maximum hoop stresses values reported by Dayalan [5] and Mehrotra & Glisic [6], but still within the safe limits of brick's compressive strength of 2430 kPa [11].

Similarly, a comparison of the meridional stresses under the dead load (Figure 6c) indicates that as in [5,6] the maximum meridional stress is recorded at the outer surface of Level 4 (base)

with a maximum value of -833.4 kPa (compression), which reduces to -168.6 kPa at the inner surface of the dome, resulting in average value of -501.0 kPa which is within the range of the values reported by Dayalan [5] and Mehrotra & Glisic [6] and well within the safe limit of brick's compressive strength.

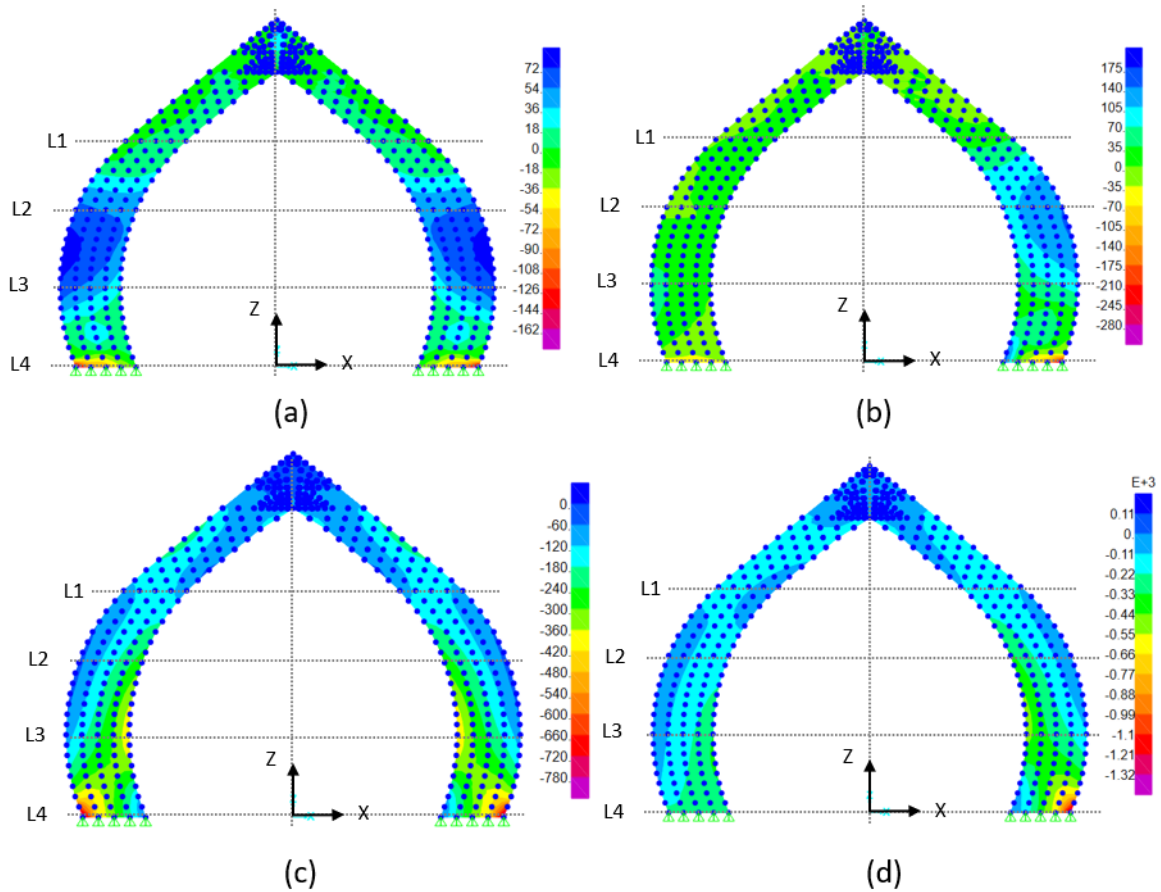


Figure 6: (a) Maximum principal stress - dead load (kPa) (b) maximum principal stress - combined dead + seismic load (kPa) (c) minimum principal stress – dead load (kPa) (d) minimum principal stress – combined dead + seismic load (kPa)

Table 4: Comparison of maximum hoop and meridional stresses obtained under dead load only with those obtained by Dayalan [5] and Mehrotra & Glisic [6]

Level	Max Hoop Stress (kPa)				Max Meridional Stress (kPa)			
	Dayalan	Mehrotra & Glisic	Outer	Inner	Dayalan	Mehrotra & Glisic	Outer	Inner
4	-65.0	-110.0	-202.0	-39.3	-565.0	-478.7	-833.4	-168.6
3	266.0	40.0	70.8	44.8	-339.0	-75.0	-152.7	-410.3
2	-123.0	-65.0	60.4	51.9	-158.0	-107.6	-94.5	-244.0
1			-66.9	-55.5			-151.1	-105.5

Under the influence of the combined dead and seismic loads, the maximum principal stresses are generally found to be higher on the east side of the dome, and lower on the west side, with the outer surface once again recording larger stresses than the inner surface (Figure 6b). The maximum principal tensile stress is found to migrate from Level 3 (bulge) to Level 2, with a maximum value of 139.8 kPa recorded on the outer surface of the east side of the dome (Table 5). Note that this tensile stress is once again within the limiting tension capacity of both the brick material and the brick-mortar interface.

Similarly, the minimum principal stresses under combined loading (Figure 6d) are generally found to be higher on the east side of the dome and lower on the west side. However, in this case the inner surface tends to record larger stresses than the outer surface (Table 6), with the exception of the base (Level 4) where a compressive stress of -1402 kPa is recorded on the outer surface of the dome. Note that this compressive stress is once again within the 2430 kPa compressive strength of brick. Furthermore, a tensile stress of 170.4 kPa is also recorded on the inner surface of the dome. While this is still less than the tensile strength of the brick unit, it is equal to the 170 kPa tensile strength of the brick-mortar interface, which could result in some cracking of the structure at its base.

Table 5: Maximum principal stress (kPa) under combined dead and seismic load

Level	West Side		East Side	
	Outer	Inner	Outer	Inner
4	-65.3	-61.2	-338.9	28.2
3	43.0	26.2	101.7	67.0
2	-31.0	-17.1	139.8	108.1
1	-133.3	-102.8	35.8	22.1

Table 6: Minimum principal stress (kPa) under combined dead and seismic load

Level	West Side		East Side	
	Outer	Inner	Outer	Inner
4	-266.1	-268.9	-1402.0	170.4
3	-106.7	-321.0	-199.6	-500.6
2	-94.5	-149.4	-95.2	-338.8
1	-162.5	-75.6	-151.9	-136.2

6 CONCLUSIONS

In this paper the results of a seismic analysis of the dome of the Taj Mahal are presented. Results include the mechanism of load transfer, and stress distributions in the dome under gravity and seismic loads. Under the influence of gravity alone (dead load), both the maximum hoop and meridional stresses recorded in the dome are found to be well within the tensile and compressive strengths of brick (safety factors of 4.2 and 2.9 respectively) as well as the tensile bond strength of the masonry (safety factor of 2.4), and compare reasonably well with the results of similar analyses conducted in the literature.

Similarly, under the influence of the combined gravity and seismic loads, the maximum principal tensile stress (139.8 kPa at level 2 at outer surface on the east side) was again found

to be within the limiting tensile capacity of both the brick material and brick-mortar interface, but with the safety factors now reducing to 2.1 and 1.2 respectively. Additionally, the maximum principal tensile stress was found to migrate from level 3 (bulge) to level 2 due to the dome geometry.

Furthermore, under the combined load case, the minimum (maximum magnitude) principal compressive stress (-1402 kPa at level 4 at outer surface on east side) was also found to be less than the compressive strength of the material, but with the safety factor now reducing to 1.7. Additionally, a tensile stress of 170.4 kPa was also recorded on the inner surface of the dome. While this is still less than the limiting tensile capacity of the brick unit, it is however equal to the strength of the brick-mortar interface, which could result in some cracking of the structure at its base. It is proposed to further investigate this potential cracking behavior in the next phase, while also examining the effect of the supporting substructure on the dynamic (seismic) response of the dome.

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