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PARTITIONED MPM-FEM COUPLING APPROACH FOR ADVANCED NUMERICAL SIMULATION OF MASS-MOVEMENT HAZARDS IMPACTING FLEXIBLE PROTECTIVE STRUCTURES

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Abstract. The intensity and frequency of natural hazards such as landslides, debris flow, and mud flows have increased significantly over the last years due to climate change and global warming. These catastrophic events are responsible for numerous destructions of infrastructures and landscapes and often even claim human lives. Therefore, in addition to the prediction, the design and installation of protective structures are of tremendous importance.

In recent decades, highly flexible protective structures have been favored due to their enormous energy absorption capacity while adapting well to the environment. However, dimensioning such protective structures is a very complex task requiring advanced numerical simulation techniques. To capture the behavior of such natural hazards on the one hand and the highly flexible protection structures, including complex elements such as sliding cables or brakes on the other hand, a partitioned coupling approach is proposed in this work. This way, the most appropriate solvers, treated as black-box solvers, can be selected for each physics involved while the interaction is shifted to the shared interface.

The Material Point Method (MPM) is particularly well suited to capture flow processes with large strains and time-dependent material behavior due to its hybrid approach of an Eulerian background grid combined with Lagrangian moving integration points. At the shared

interface, non-conforming essential boundary conditions are introduced within the MPM model to reflect the behavior of the flexible protective structures. These, in turn, are modeled using the Finite Element Method (FEM) to capture the advanced structural model behavior, receiving the impact forces at the shared interface as external loads. Consequently, a Gauss-Seidel iteration scheme is applied, exchanging the interface conditions for the equilibrium at the shared interface.

1 INTRODUCTION

Mass-movement hazards involving rapid and large soil deformations have increased significantly during the past decades due to climate change and global warming. These catastrophic events like debris flow, avalanches, and mud flows are responsible for numerous destruction of infrastructures and landscapes and, on top of that, often even claim human lives. According to the Centre for Research on the Epidemiology of Disasters (CRED) and the UN Office for Disaster Risk Reduction (UNDRR) [1], climate-related disasters within the year 2020 have been responsible for $15,050$ deaths worldwide, with more than 98.4 million citizens affected, and about US171.3 billion economic losses. Therefore, installing and designing protective structures in affected regions is of tremendous importance such that the damage induced by the dynamic soil forces of mass-movement hazards can be minimized.

The dimensioning of such protective structures is a complex task requiring advanced numerical simulation techniques, especially in the case of highly flexible protective structures that are often preferred due to their high energy absorption ability. In this case, the Lagrangian finite element method (FEM) is the most appropriate numerical method to model complex structures commonly consisting of a protective net stretched between piers and braced by steel cables, which often contain braking elements to absorb the energy due to plastic deformation.

However, considering mass-movement hazards flowing down a mountainous region, classical FEM and other Lagrangian mesh-based techniques will likely suffer from mesh entanglement and distortion, requiring computationally expensive re-meshing schemes. Therefore, continuum-based particle methods are the natural choice for simulating those large strain events, including topological changes in the material. Among them, the Material Point Method is particularly suited, as it combines the advantages of both mesh-free and mesh-based methods. As initially proposed by Sulsky et al. [2], the governing equations are solved at an Eulerian background grid which brings along many similarities to the classical updated-Lagrangian finite element technique, while the physical domain is discretized by Lagrangian moving particles called material points, which carry the history-dependent variables. Thus, it can be interpreted as a modified updated-Lagrangian finite element technique with moving integration points while the background grid is systematically reset at the end of each time step circumventing the problem of mesh entanglement and the computational expense of re-meshing during the simulation of large strain problems.

Therefore to capture the behavior of the natural hazards flowing down a mountainous region on the one hand and to model the effects of such dynamic soil events on highly flexible protective structures, we are proposing a partitioned MPM-FEM coupling scheme. This allows solving each involved physic in its preferred reference frame while the interaction is shifted to the shared interface by exchanging respective boundary conditions.

2 GOVERNING EQUATIONS

2.1 Strong Form

The displacement $\mathbf{u}$ of each point within a continuum body $\mathcal{B}$ occupying a domain Ω with regular boundary Γ in the three-dimensional Euclidean space $\mathcal{E}$ is defined by

$$\mathbf{u} = \mathbf{x} - \mathbf{X}. \tag{1}$$

It relates each position $\mathbf{X}$ in the undeformed reference configuration to its position $\mathbf{x}$ in the deformed configuration at time t (see Figure 1).

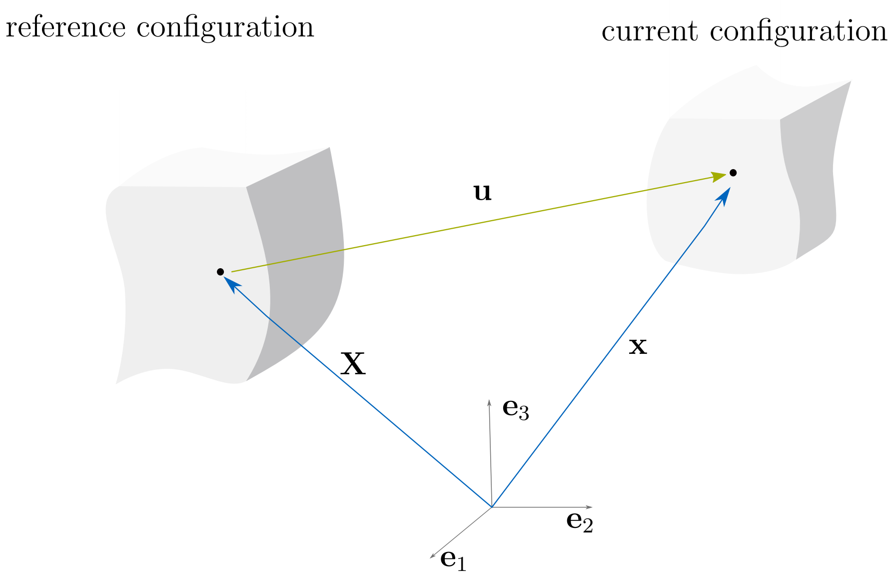

Figure 1: Kinematic description of the body $\mathcal{B}$.

Cauchy's first equation of motion, which is the governing equation to solve the given problem, is defined by

$$\rho \ddot{\mathbf{u}} = \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{b} \qquad \text{in} \qquad \Omega. \tag{2}$$

Within this equation, $\mathbf{b}$ denotes the volume acceleration, $\boldsymbol{\sigma}$ is the symmetric Cauchy stress tensor, and ρ is the spatial mass density, while the first and second material time derivatives of the displacement field $\mathbf{u}$ are the velocity and the acceleration, respectively. This balance equation implies the conservation of mass and holds for every point $\mathbf{x} \in \Omega$ for all times t assuming an isothermal setting.

This balance equation is to be solved numerically, considering the following Dirichlet and Neumann boundary conditions:

$$\mathbf{u} = \overline{\mathbf{u}} \qquad \text{on} \quad \Gamma_D \tag{3}$$

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \overline{\mathbf{p}} \qquad \text{on} \quad \Gamma_N \; . \tag{4}$$

Herein, $\overline{\mathbf{u}}$ is a prescribed displacement field on the Dirichlet boundary Γ_D and $\overline{\mathbf{p}}$ is a traction vector on the Neumann boundary Γ_N with the outward normal $\mathbf{n}$.

2.2 Weak Form

Since in general, a closed-form solution for the given problem cannot be found, the momentum balance equation is formulated through the *Principle of Virtual Work* [3]

$$\delta W = -\int_{\Omega} \boldsymbol{\sigma} : \delta \mathbf{e} d\Omega - \int_{\Omega} \rho \ddot{\mathbf{u}} \cdot \delta \mathbf{u} d\Omega + \int_{\Omega} \rho \mathbf{b} \cdot \delta \mathbf{u} d\Omega + \int_{\Gamma_N} \overline{\mathbf{p}} \cdot \delta \mathbf{u} d\Gamma_N = 0 \tag{5}$$

equalizing zero for systems in equilibrium. In this equation $\delta\mathbf{u}$ are the virtual displacements while $\delta\boldsymbol{\epsilon}$ is the virtual strain arising from the gradient of the virtual displacement field.

The stress boundary defined by equation (4) is part of the weak form and, therefore, often referred to as the natural boundary condition, while the Dirichlet condition defined by equation (3) needs to be prescribed over the boundary Γ_D and is thus often called essential boundary condition.

It should be noted that in MPM the weak imposition of boundary conditions is often required, which is, in particular, for the imposition of Dirichlet conditions a challenging task. Furthermore, for the partitioned coupling approach, introduced in section 4, the weak imposition of Dirichlet conditions in the MPM model is essential. For this purpose, Chandra et al.[4] introduced the Penalty augmentation while in [5], the Lagrange multiplier method to weakly enforce essential boundary conditions in implicit MPM is presented.

To numerically solve the weak equilibrium equation (5), a spatial discretization is introduced, approximating the continuous fields by nodal values and locally defined basis functions. Furthermore, the continuous time domain is divided into discrete time steps, and applying the Newmark-β [6] implicit time integration scheme finally leads to a non-linear equation system which iteratively is solved utilizing the Newton-Raphson method.

3 MATERIAL POINT METHOD

A specification in MPM is that in addition to the computational background grid, which approximates the continuous fields, the body $\mathcal{B}$ is discretized into a finite number n_p of material points representing each a finite volume Ω_p of the body

$$\mathcal{B} \approx \mathcal{B}^h = \bigcup_{p=1}^{n_p} \Omega_p \; . \tag{6}$$

Those material points carry historical information during the calculation procedure. Therefore the classical finite element updated Lagrangian calculation procedure is enhanced by continuous inter- and extrapolation material point information and nodal values of the computational background grid. Therefore the MPM procedure can be categorized into three main phases:

1. **Initialization phase**: A search is performed to define the background grid element, which belongs to each material point, before the necessary variables are mapped via mass projection to the corresponding nodes as initial conditions.

2. **Lagrangian phase**: Solution of the discretized governing equations. Coincides with the solution step for an Updated Lagrangian Element in FEM.

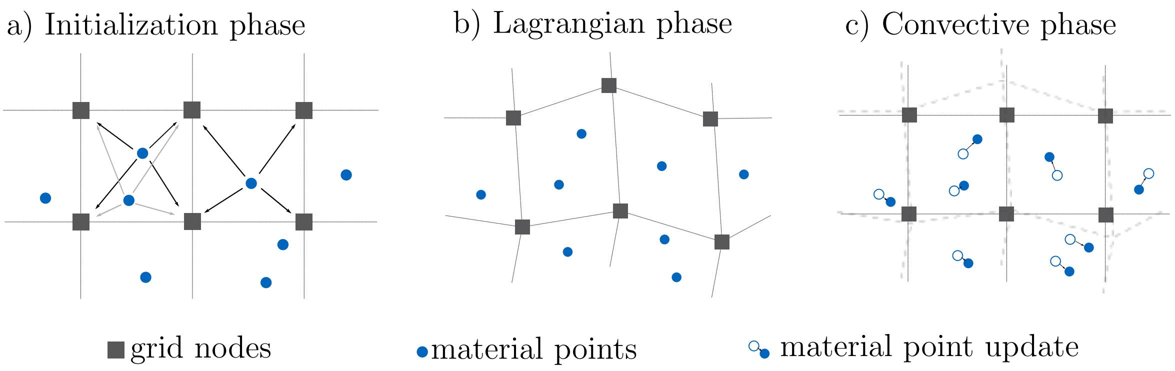

Figure 2: a)Initialization phase, b) Lagrangian phase, and c) Convective phase. Square markers identify the grid nodes, while round markers indicate the material points.

3. **Convective phase**: Solutions obtained at the nodes of the background grid are interpolated back to the material points, resulting in an update of the material point's position and its kinematic values. Then the background grid is reset to its initial position.

Those three phases of the MPM scheme are illustrated in Figure 2.

As the body, discretized by material points, moves through the Eulerian background grid, the imposition of boundary conditions is a crucial part. For a few specific configurations, those conditions can directly be applied to the nodes of the computational background grid, which is equivalent to conventional FEM. However, in general, a weak imposition of boundary conditions is required. For this purpose, Chandra et al. [4] developed the Penalty method for implicit MPM to weakly enforce Dirichlet conditions. To track the outline of the body during the calculation process, boundary particles are introduced, which carry the necessary information for the condition imposition. Further developments for the weak imposition of boundary conditions using the Lagrange multiplier method are presented by the author in [5].

4 PARTITIONED MPM-FEM COUPLING APPROACH

The fundamental idea of a partitioned or staggered coupling scheme is that the involved sub-solvers are treated as black-box solvers, and the communication between the domains is shifted to their shared interface. Therefore, considering the soil domain Ω_M modeled by MPM with interface Γ_M and the structural domain Ω_S modeled by classical FEM with interface Γ_S the shared interface is defined by

$$\Gamma_{MS} = \Gamma_S \cap \Gamma_M. \tag{7}$$

To ensure the communication of the partitions, boundary conditions within each sub-solver are introduced along this interface.

A Dirichlet-Neumann partitioning is applied, which is most frequently used for Fluid-Structure interaction problems [7]. Furthermore, it is as well applied for partitioned coupling of MPM with the Discrete Element Method (DEM) [8] to simulate discrete obstacles within a continuous flow or the coupling of FEM with DEM for the numerical investigation of rocks impacting onto highly flexible protective structures [9].

Therefore a Neumann condition is introduced at the FEM interface Γ_S while a Dirichlet condition is imposed at the MPM interface Γ_M. For the solution, both sub-systems are discretized independently, introducing a classical mesh in the FEM system, while in MPM, a background grid is introduced. Furthermore, to impose the boundary conditions in MPM, boundary particles are defined which impose the conditions weakly either by Penalty augmentation according to [4] or the Lagrange multiplier method following [5]. The individual discretized systems and the respective boundary conditions are visualized in Figure 3.

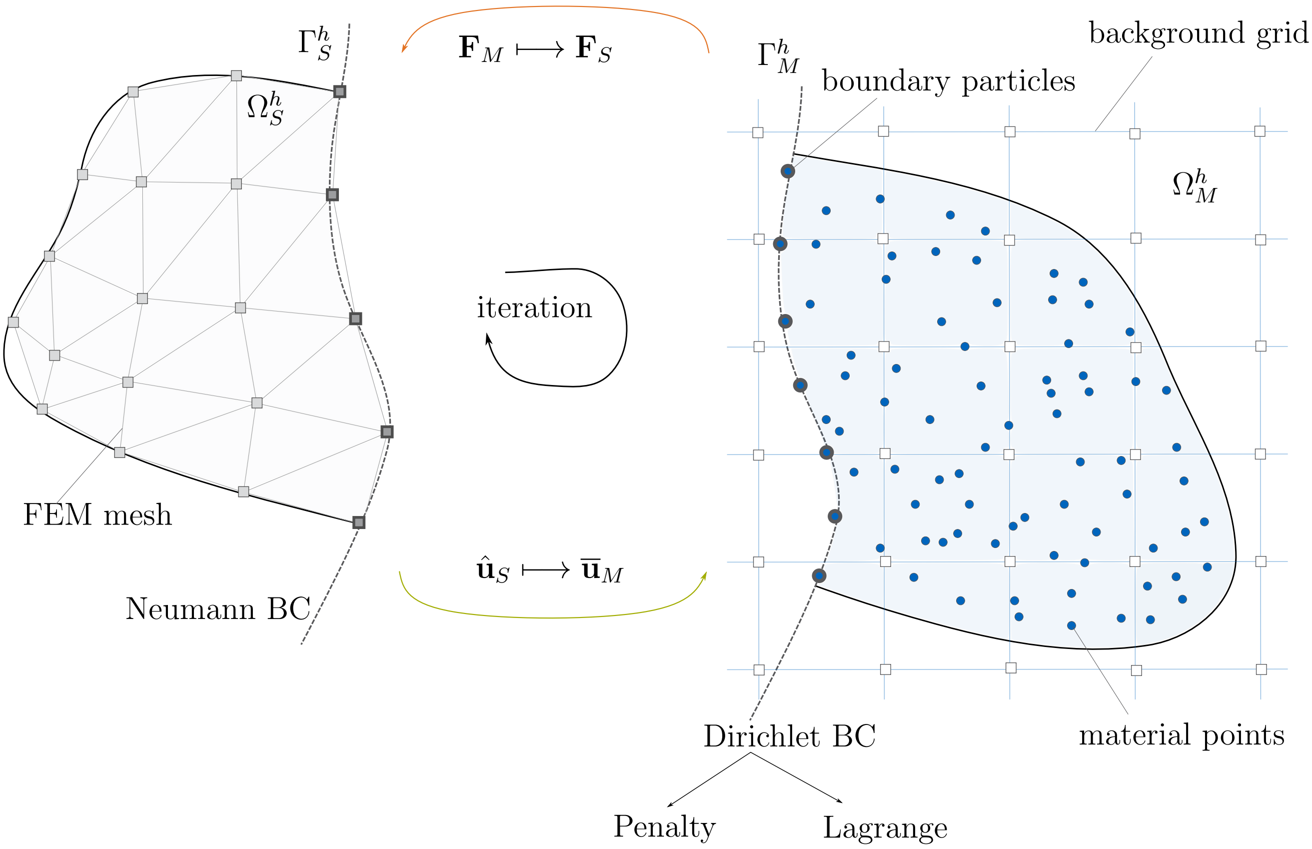

Figure 3: Coupling algorithm of discretized FEM and MPM model

The involved partitions are then solved sequentially, following the classical Gauss-Seidel communication pattern as depicted in Figure 4. Therefore, nodal displacements $\hat{\mathbf{u}}_S$ obtained at the nodes of the interface Γ_S^h are transferred to the boundary particles at the MPM interface as imposed displacements $\overline{\mathbf{u}}_M$ which can be written as

$$\hat{\mathbf{u}}_\mathrm{S} \longmapsto \overline{\mathbf{u}}_\mathrm{M}. \tag{8}$$

Since, in general, the discretization in non-matching a mapping technique [10] is applied for this transfer process.

Then subsequently, the MPM model is solved, resulting in reaction forces $\mathbf{F}_M$ at the boundary particles. Commonly a conservative mapping approach, which results from the conservation of energy at the interface, is applied to transfer the reaction forces $\mathbf{F}_M$ back to the nodes of the FEM interface as external forces $\mathbf{F}_M$ which is expressed by

$$\mathbf{F}_\mathrm{M} \longmapsto \mathbf{F}_\mathrm{S}. \tag{9}$$

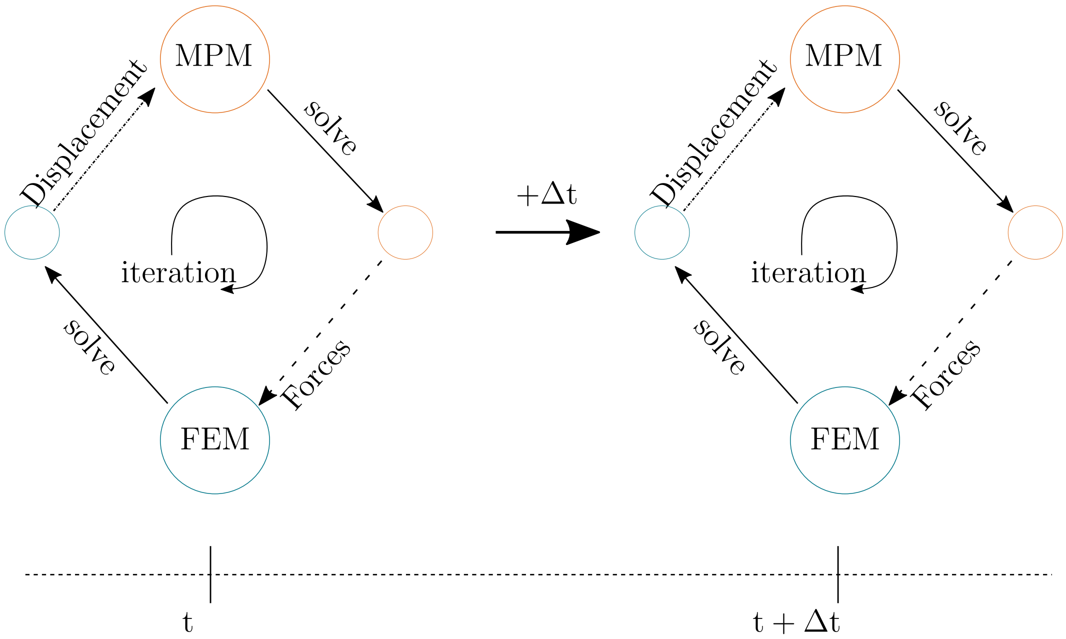

Figure 4: Gauss Seidel communication pattern for strong coupling of MPM and FEM.

This solution sequence is repeated iteratively until the interface transmission conditions are fulfilled. While the kinematic interface condition ensures a consistent deformation of the involved systems at the interface, the dynamic constraint defines the equilibrium of forces.

To accelerate the interface equilibrium, convergence accelerators can be utilized, which modify the values before mapping them to the subsequent solver. Since the solvers are executed as black-box solvers, the utilization of convergence accelerators, which either modify the mapped values by relaxation or approximate the interface equations by a modified Newton-Raphson scheme, often improve the interface convergence rate significantly.

Once the interface equilibrium is obtained, the solvers advance in time.

It should be mentioned at this point that depending on the considered problem, the interface conditions can alternatively be swapped to run the partitioned scheme. A detailed discussion about that can be found in [11]. However, this approach can only be applied if the interface is not changing during the simulation. Otherwise, further information about the contact needs to be provided at the FEM interface to activate only those nodal Dirichlet constraints which are in contact with the MPM counterpart.

5 NUMERICAL EXAMPLE

The objective of the partitioned MPM and FEM coupling scheme is that the advantages of both methods are combined. While MPM has the strength to model mass-movement hazards like landslides, debris flow, or avalanches flowing down a mountainous region, classical FEM provides the possibility to model highly flexible protective structures considering different element types like trusses, cables, beams, and membranes.

To demonstrate this ability, the impact of granular material on a typical protective structure designed for mud-flow impact is numerically investigated. Typically, these structures consist of a ring net which is attached with shackles to pre-stressed cables, which are in turn spanned between vertical steel profiles. A hinged support at the ground is constructed for these steel profiles, and cables are spanned to keep them in the desired position. This detail is important to absorb the energy during the impact, as the steel profiles can rotate around the support point and thus stretch the cables, which are commonly enriched by braking elements.

The considered protective structure has a total length of 15m and a height of 3.5m and is schematically depicted in Figure 5.

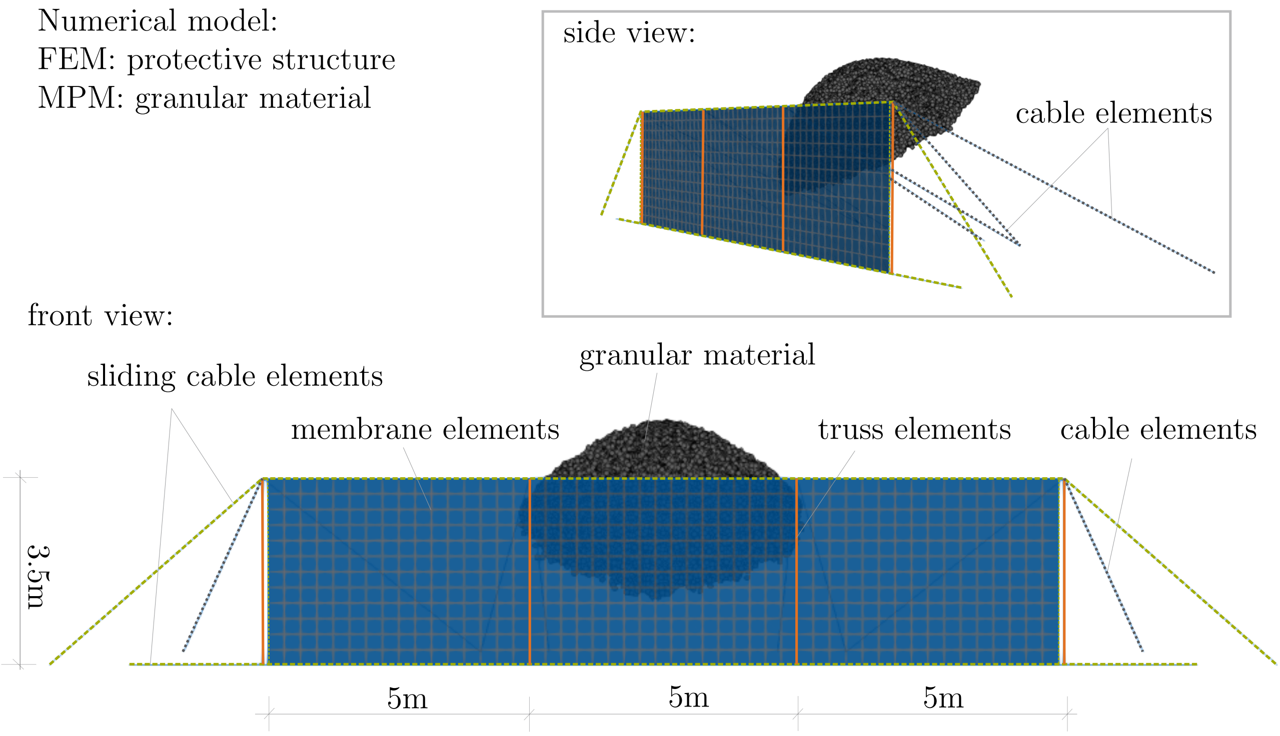

Figure 5: System setup of a highly flexible protective structure modeled with FEM and granular material modeled with MPM, which impacts onto the structure.

It is divided into three equally sized fields with two steel profiles at the outer edges and two intermediate ones. They are highlighted in orange in Figure 5 and are modeled as truss elements in the FEM model. In between, a ring net is spanned, which is connected to all trusses at the upper and lower point. Since detailed modeling of the net, consisting of rings of twisted steel wires which are connected to each other at a few points, would be computationally too expensive, the overall net behavior is modeled with membrane elements using FEM. Therefore the membrane material parameters are adapted to the actual ring net behavior. This simplification is a common approach, accelerating the calculation of the numerical model significantly and still preserving the main characteristics of the net behavior. For nets constructed for rock impacts, this approach is validated and commonly applied [9]. In Figure 5, the net modeled by membrane elements is highlighted in blue.

The net, which will mainly capture the material of the mass-movement hazard during the impact, is attached with shackles to spanned and pre-stressed cables. There are two cables at the outer edges of the barrier spanning between the upper point and the lower point of the respective steel profiles. Furthermore, two cables are spanned along the whole structure in the width direction and are fixed to the ground at the ends. One of these cables is spanned at the bottom of the net, while the other is spanned at the top of the barrier. Those cables are highlighted with dashed green lines in Figure 5. Since the net is attached by shackles to those cables, it can slide along them, allowing larger deformation and thus absorbing the energy of the impact. To capture this behavior in the numerical model, a sliding cable element according to [12, 13] is utilized in the FEM model for those four cables.

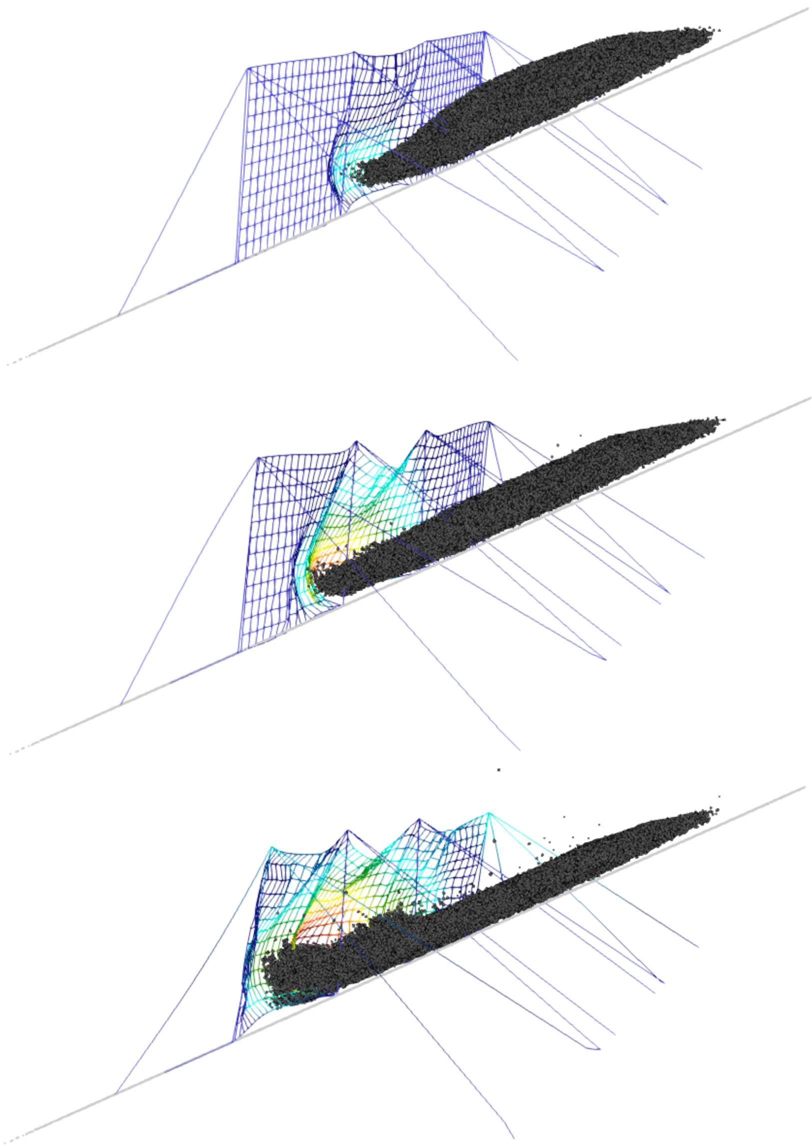

Figure 6: Granular material impacting highly flexible protective structures. Numerical results for specific times.

Finally, the vertical steel profiles, with hinged support at the bottom, are braced in space with two additional cables each. They are spanned uphill and connect the upper points of the steel profiles with the ground. Usually, they are not pre-stressed and get activated at the very moment of the impact and the subsequent deformation of the barrier. In the numerical model, they are modeled as cable elements and are highlighted with gray dashed lines in Figure 5.

For the impacting mass, 25m^3 granular material is considered, which is sliding down an inclined plane due to gravity before impacting the flexible protective structure. To model the dry granular material with a density of $2085\frac{\text{kg}}{\text{m}^3}$, MPM is applied using Mohr Coulomb material model.

In Figure 6, three snapshots of the numerical simulation for different times are presented.

Due to the impacting material, the protective structure deforms tremendously to absorb the energy. Although the material mainly crashes into the barrier in the middle part of the structure, the deformation of the hole barrier can be observed, which fits very well with conducted experiments and real-case impact scenarios. Furthermore, the steel profiles are rotating as expected around their supports at the bottom, activating the cables which are spanned uphill.

For further developments, the membrane model should be enhanced to consider wrinkling effects and the penetration of material through the barrier. Moreover, the developed MPM-FEM coupling scheme should be validated against experimental results, and thus, further verification and validation tests are planned to be performed in the near future.

6 CONCLUSIONS

In this work, a partitioned coupling scheme to simulate soil-structure interaction problems is presented by using a coupled implicit material point and finite element method. This approach allows us to combine the advantages of both methods for advanced modeling of mass-movement hazards impacting flexible protective structures.

To capture the effects of the mass-movement hazards flowing down a mountainous region, MPM is utilized to model the large strain event, including history-dependent material laws. Due to its continuum-based approach, it can be applied to real-scale events, such as landslides, with reasonable computational effort.

Therefore, MPM is coupled to classical FEM, which in turn allows us to model the highly flexible protective structures. These structures are often installed due to their high energy absorption ability and usually consist of a protective net spanned between steel profiles and braced by steel cables. To model those advanced engineering structures, FEM is the most appropriate numerical method.

Coupling MPM and FEM in a partitioned manner allows to preserve the modularity of the involved sub-solvers, and therefore the most suited solvers can be selected each, while the interaction among them is shifted to their shared interface. The numerical example of granular material modeled with MPM flowing down due to gravity of an inclined slope and impacting a typical protective structure modeled with FEM demonstrates the applicability of the proposed method for such scenarios. The impact process and the subsequent significant deformation of the structural model are properly captured by the numerical method and agree well with expected results.

Nevertheless, some future works are necessary to improve and validate the proposed coupling scheme regarding numerical stability and computational efficiency. For this purpose, further verification and validation tests are planned to be performed in the near future. Finally, this will allow us to predict the impact of real-scale mass-movement hazards involving complex multi-phase flows onto complex engineering protective structures. Knowledge of the impact forces and their evolution over time enables the dimensioning and optimization of such structures so that they can protect infrastructures and human lives in vulnerable areas.

Code Availability

For this work, the open-source multiphysics software *KRATOS*[14, 15, 16] has been used, which is written in C++ and offers a Python interface.

ACKNOWLEDGMENTS

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