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Reliability Assessment of Pressurized Pipes with Inclined Defects

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Abstract. *Inclined defects in pipelines can lead to failures in the form of mixed mode fracture. A review of the literature suggests that few studies have been carried out on reliability assessment of pipelines considering mixed mode fracture. This paper intends to present a reliability-based method for assessing fracture failures of pressurized pipes with inclined defects. Richard's criterion is employed in the development of the limit state function for reliability assessment. A Stochastic model of load effects is developed, and first passage method with a developed analytical solution is used to predict the pipe failures. A worked example is provided to illustrate the application of the proposed method. The method presented in this paper can help pipe engineers and asset managers to develop a reliability-based maintenance strategy for better management of pipelines.*

Keywords: *Pressurized Pipes, Inclined Crack, Mixed Mode Fracture, Time-Dependent Reliability, First Passage Probability.*

1 Introduction

Pipelines are widely used as one of the safest and most economical structures to transport gas or liquid (*e.g.*, water; natural gas) in industries (Keshtegar and Miri, 2014). However, most of them have been in service for a long time, and the exposure to the corrosive environment increases the likelihood of failures before the end of the intended design life (Carbal and Kimber, 1997). The consequences of pipeline failures can be economically, socially, and environmentally devastating (Mahmoodian and Li, 2017). Therefore, it is essential to fully understand the failure mechanisms of pipes and accurately predict its service life.

A literature review suggests that most of the previous researches on the pipe failure assessment mainly considered the loss of strength due to the reduction of wall thicknesses of pipes (*e.g.*, Ahammed and Melchers, 1997; Mahmoodian and Li, 2017; Liu *et al*., 2019). Nevertheless, it has been found that pipe failures are more likely to be fracture type due to the stress concentration near the tips of crack-like defects (Li and Mahmoodian, 2013; Fu *et al*., 2019). As the cracked structures often fail unexpectedly at the applied stress much below the material strength (Anderson, 2005), it is necessary to assess the integrity of pipeline based on fracture mechanics. Also, the defects on the pipe walls are often caused by corrosion, which can grow in any directions depending on the heterogeneity of surrounding soil properties (Wang *et al*., 2018; Wang *et al*., 2019). Moreover, the crack-like defects in pipes most likely grow in an inclined manner because of the complex stress state (Li *et al*., 2016). As a result, in these pipes (*e.g*., Figure 1), the failures can be induced by a mixed mode loading interaction. Some previous researches (*e.g*., by Erdogan and Shih, 1968; Chang *et al*., 2006) indicated that the single mode fracture failure criterion is seldom appropriate for assessing the integrity of structural components with inclined cracks. Thus, a method based on mixed mode fracture criteria is more appropriate to assess the fracture failures of pipes. To predict the pipe failures, time-dependent reliability methods have been successfully used since the parameters associated with pipe failures are highly uncertain and changing with time (Fu *et al.*, 2019). Although several researchers, *e.g.*, Li and Mahmoodian (2013); Fu *et al.*, (2019); Wang *et al.*, (2019), have assessed the probability of pipe failures using time-dependent reliability methods for different defect types (*e.g.*, sharp corrosion pit; elliptical corrosion pit), these works only focuses on Mode I (opening mode) fracture failure. The reliability assessment of pipelines with inclined defects considering mixed mode fracture, in comparison, is highly limited.

This paper presents a time-dependent reliability method for pressurized pipe based on first passage probability theory. The cast iron pipe with an external inclined crack-like defect is considered in this paper. A stochastic model of the stress intensity factor is developed by incorporating Richard's criterion into the limit state function. A case study is presented to illustrate the proposed method. The proposed method can be applied to the risk-based maintenance strategy for pipelines.

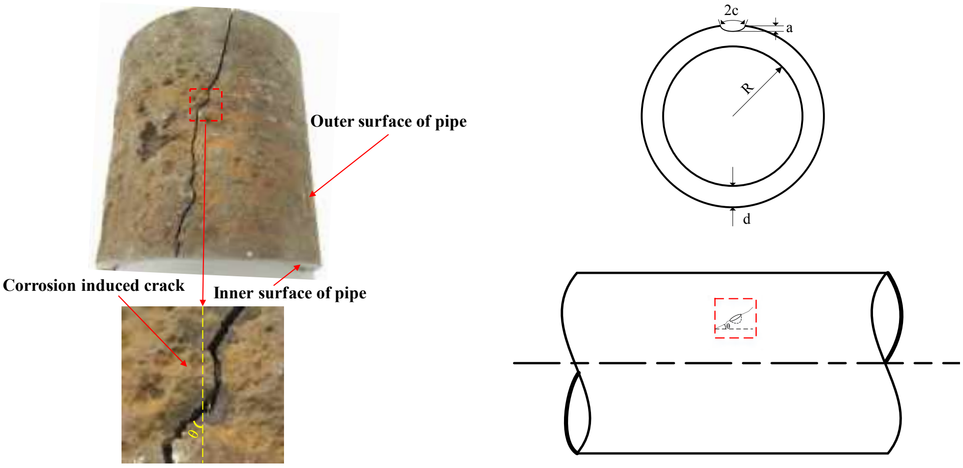


Figure 1. A pressurized pipe with inclined external surface defects.

2 Problem Formulation

To determine the probability of failure of a pipe, a failure criterion must be established. In the structural reliability theory, the criterion can be expressed in the form of a limit state function as follows (Melchers, 1999):

$$G(L, R, t) = L(t) - R(t) \tag{1}$$

where $L(t)$ is the load effect on the structures at time t; $R(t)$ is the acceptable limit (resistance) at time t. As both load effect ($L(t)$) and acceptable limit ($R(t)$) are time-dependent, all or some of the basic variables should be modelled as stochastic processes. For reliability problems involving the stochastic processes, the structural failure event is that the stochastic processes of the load action process $L(t)$ initially pass upwards an acceptable limit (threshold) $R(t)$ during the service life. With the assumption of Poisson processes, the probability of failure can be determined as follows (Melchers, 1999):

$$P_f(t) = 1 - \left[1 - P_f(0)\right] e^{-\int_0^t v\, dt} \tag{2}$$

where P indicates the probability of an event; $P_f(0)$ is the probability of failure at time $t = 0$ and v is the mean rate for the load action process $L(t)$ to cross upwards the threshold $R(t)$. The up-crossing rate v can be determined by the Rice formula (Rice, 1944)

$$v = v_R^+ = \int_{\dot{R}}^{\infty} (\dot{R} - \dot{L}) f_{L\dot{L}}(L, \dot{R})\, d\dot{L} \tag{3}$$

where v_R^+ is the up-crossing rate of the load action process $L(t)$ relative to the threshold $R(t)$; $\dot{R}$ is the slope of R with respect to time; $\dot{L}$ is the time derivative process of L; $f_{L\dot{L}}$ is the joint probability density function for L and $\dot{L}$. Li and Melchers (1993) derived an analytical solution to Equation (3) when $L(t)$ is a Gaussian process and the threshold R is deterministic. It is expressed as follows:

$$v_R^+ = \frac{\sigma_{\dot{L}|L}}{\sigma_L} \phi\left(\frac{R - \mu_L}{\sigma_L}\right) \left\{ \phi\left(\frac{\dot{R} - \mu_{\dot{L}|L}}{\sigma_{\dot{L}|L}}\right) - \frac{\dot{R} - \mu_{\dot{L}|L}}{\sigma_{\dot{L}|L}} \Phi\left(-\frac{\dot{R} - \mu_{\dot{L}|L}}{\sigma_{\dot{L}|L}}\right) \right\} \tag{4}$$

where μ and σ are the mean and standard deviation of random variables represented by subscripts L and $\dot{L}$; $'|'$ denotes the condition; ϕ and Φ indicate the standard normal density and distribution functions. For a given Gaussian stochastic process with mean function $\mu_L(t)$ and auto-covariance function $C_{LL}(t_i, t_j)$, all terms in Equation (4) can be determined as follows:

$$\mu_{\dot{L}|L} = E\left[\dot{L} \middle| L = R\right] = \mu_{\dot{L}} + \rho_L \frac{\sigma_{\dot{L}}}{\sigma_L}(R - \mu_L) \tag{5a}$$

$$\sigma_{\dot{L}|L} = [\sigma_{\dot{L}}^2 (1 - \rho_L^2)]^{1/2} \tag{5b}$$

where $\mu_{\dot{L}} = d\mu_L(t)/dt$; $\sigma_{\dot{L}} = \left[\partial^2 C_{LL}(t_i, t_j)/(\partial t_i \partial t_j)\Big|_{i=j}\right]^{\frac{1}{2}}$; $\rho_L = C_{L\dot{L}}(t_i, t_j)/[C_{LL}(t_i, t_j) \cdot C_{\dot{L}\dot{L}}(t_i, t_j)]^{\frac{1}{2}}$; and $C_{L\dot{L}}(t_i, t_j) = \partial C_{LL}(t_i, t_j)/\partial t_j$.

For Equation (4) to be of practical use, the key is to establish an appropriate limit state function for the mix mode fracture failure of pipes. Richard (2001) developed a fracture criterion to simplify the prediction of crack growth under multiaxial loadings. According to Richard (2001), unstable crack growth in brittle materials would occur if the local loading condition along the crack front reaches a critical value, which described by the following expression (Richard, 2001):

$$K_v = \frac{K_I}{2} + \frac{1}{2}\sqrt{K_I^2 + 4(\alpha_1 K_{II})^2 + 4(\alpha_2 K_{III})^2} \geq K_{IC} \tag{6}$$

where K_v is an equivalent stress intensity factor; K_I, K_{II}, and K_{III} are the Mode I (opening mode), Mode II (in-plane shear mode), and Mode III (out-of-plane shear mode) stress intensity factors, respectively; K_{IC} is the Mode I fracture toughness which quantifies the material resistance to crack extension; $\alpha_1 = K_{Ic}/K_{IIc}$ and $\alpha_2 = K_{Ic}/K_{IIIc}$ with K_{IIc} as the fracture toughness for pure Mode II and K_{IIIc} as the fracture toughness for pure Mode III. With $\alpha_1 = 1.155$ and $\alpha_2 = 1.0$, Equation (6) was found having an excellent agreement with other criteria (*e.g.*, developed by Schöllmann *et al.*, 2002) and the experimental data in brittle materials (Richard *et al.*, 2014). Based on Richard's criterion, Equation (1) can be expressed as:

$$G(t) = \frac{K(t)_I}{2} + \frac{1}{2}\sqrt{K(t)_I^2 + 5.3361 {K(t)_{II}}^2 + 4{K_{III}}^2} - K(t)_{IC} \tag{7}$$

The determination of stress intensity factors is presented in the next section.

3 Model of Load Effects

The stress intensity factor (K) represents the magnitude of the stress fields surrounding the tip of crack or crack-like defect. Its magnitude depends on the far-field stress level σ, the size of crack a, and the geometries of the bodies. For a pipe with an external inclined defect as shown in

Figure 1, the stress intensity factor K can be generalised as follows (Li *et al.*, 2016):

$$\boldsymbol{K} = \frac{pR}{d}\sqrt{\pi\frac{a}{Q}}\boldsymbol{F}\left(\frac{a}{d},\frac{a}{c},\frac{d}{R},\xi,\theta\right) \tag{8}$$

where $\boldsymbol{K} = \{K_I, K_{II}, K_{III}\}^T$; p is the internal pressure; pR/d is the average hoop stress in pipes; ξ is used to define the position of an arbitrary point along the semi-elliptical crack; θ, c, d, and R are defined in Figure 1; Q is the shape function which can be expressed as follows (Shiratori and Miyoshi, 1986):

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}\ ; \quad \frac{a}{c} \le 1 \tag{9}$$

In Equation (8), $\boldsymbol{F}\left(\frac{a}{d},\frac{a}{c},\frac{d}{R},\xi,\theta\right) = \{F_I, F_{II}, F_{III}\}^T$, where F_I, F_{II}, and F_{III} are the influence coefficient functions for Mode I, II, and III, respectively. Li *et al.*, (2016) have developed the influence coefficients of stress intensity factors for inclined cracks in pipes under internal pressure using three-dimensional finite element analysis. The approximations of the influence coefficients can be expressed as follows:

$$F_I = M_I N_I sin^2(h_I^{11} + h_I^{12}\theta) \tag{10a}$$

$$F_{II} = M_{II}[h^6(\xi) + h^7(\xi)^3]\, sin(h_{II}^8 + h_{II}^9\theta) \tag{10b}$$

$$F_{III} = M_{III}N_{III}\, sin(h_{III}^{11}\theta) \tag{10c}$$

where $M_i = h_i^1 + h_i^2\left(\frac{a}{d}\right) + h_i^3\left(\frac{a}{d}\right)^2 + h_i^4\left(\frac{a}{d}\right)^3 + h_i^5\left(\frac{a}{d}\right)^4$; $N_i = h_i^6 + h_i^7(\xi)^2 + h_i^8(\xi)^4 + h_i^9(\xi)^6 + h_i^{10}(\xi)^8$ with the coefficients, h_i^n $(n = 1 - 12)$ are given in Table 1.

The crack size a can be represented by the corrosion pit depth. A widely accepted model for corrosion pit depth is employed in this paper, which is expressed as follows (Kucera and Mattsson, 1987):

$$a(t) = kt^n \tag{11}$$

where k and n are empirical coefficients to be determined from field data.

Apart from the growth of corrosion pit, the operating internal pressure and pipe wall thickness also change with time. Therefore, it is justifiable to model the load effect $L(t)$ as a stochastic process. The randomness of the load effect can be taken into account by introducing a random variable, ξ_L. This variable is defined in such a way that its mean is unity, i.e., $E(\xi_L) = 1$, and its coefficient of variation, λ_L, is a constant (Mahmoodian and Li, 2017). The load effect can be expressed as follows:

$$L(t) = L_c(t) \cdot \xi_L \tag{12}$$

where $L_c(t)$ is treated as a pure time function determined by the load effect function. The mean and auto-covariance functions of $L(t)$ can be expressed as follows (Li and Melchers, 2005):

$$\mu_L(t) = E[L(t)] = L_c(t) \cdot E[\xi_L] = L_c(t) \tag{13}$$

$$C_{LL}(t_i, t_j) = \lambda_L^2 \rho_L L_c(t_i) L_c(t_j) \tag{14}$$

where ρ_L is the auto-correlation coefficient for $L(t)$ between two points in time t_i and t_j.

Table 1. Values of h in Equation (10) with $d/R = 0.1$ and $a/c = 0.4$ (Li *et al.*, 2016).

Constant	i = I	i = II	i = III
h_i^1	0.957	-0.427	0.742
h_i^2	0.504	-0.123	0.076
h_i^3	0.256	0.114	-0.070
h_i^4	0.155	-0.019	0.073
h_i^5	0.088	-0.346	0.309
h_i^6	0.983	0.454	0.618
h_i^7	-0.059	0.186	-0.101
h_i^8	-0.044	3.142	-0.604
h_i^9	0.023	-2.001	1.279
h_i^{10}	-0.166		-1.016
h_i^{11}	1.589		1.997
h_i^{12}	0.994		

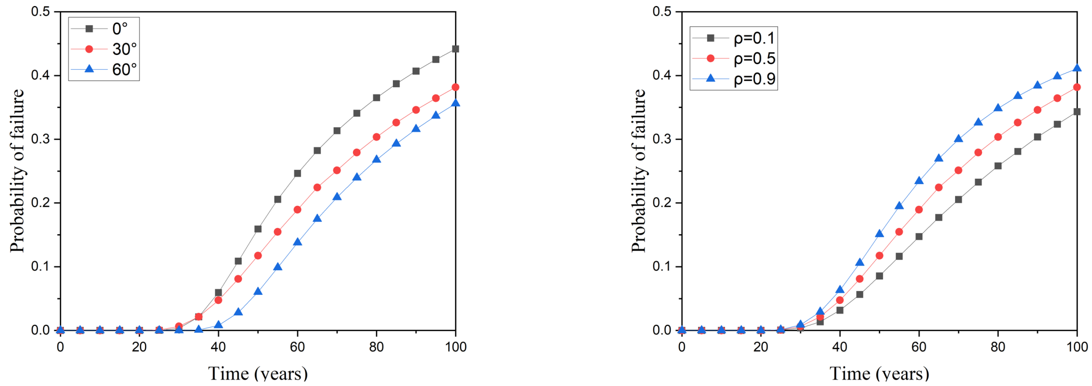
4 Worked Example

The proposed methodology is applied to a case study in which a cast iron pipe with inclined corrosion pits is considered. To calculate the stress intensity factors, the influence coefficient functions for Mode I, II, and III are determined by means of the constants in Table 1 and the assumption of aspect ratio $a/c = 0.4$. According to Li *et al.*, (2016), for $a/c = 0.4$, the critical stress intensity factor occurs at the deepest points along the crack front. The crack front normalized coordinate ξ is assumed to be 0.1. The pipe characteristics and the geometric information of corrosion pit are summarised in Table 2.

Table 2. Values of variables for reliability analysis in the worked example.

Basic variables	Mean	C.O.V	Reference
k	2.540	0.197	Mahmoodian and Li (2016)
n	0.320	0.188	Mahmoodian and Li (2016)
R	127 mm	-	Li and Mahmoodian (2013)
d	16 mm	-	Li and Mahmoodian (2013)
P	0.45 MPa	0.27	Sadiq *et al.*, (2004)
K_{Ic}	7.66 MPa/m$^{0.5}$	-	Marshall (2010)

The load effect is calculated by Equation (8), (9) and (10). The mean of function μ_L and standard deviation σ_L can be calculated as a function of time using Monte Carlo simulation together with the proposed model of pit depth. The up-crossing rate v_R^+ can be obtained from Equation (5) for a given auto-correlation coefficient, and the probability of failure can be calculated by Equation (2).



(a) (b)

Figure 2. Probability of pipe failure: (a) for different inclination angles with $\rho\rho = 0.5$; (b) for different auto-correlation coefficients with $\theta\theta = 30°$.

The calculation results of the probability of failure with different inclination angles and auto-correlation coefficients are shown in Figure 2. The probability of pipe failure increases with the exposure time due to the growth of pit depth. Also, it can be seen that the probability of failure decreases with the increase of the inclination angles since the equivalent stress intensity factor KK_{vv} in Equation (6) drops markedly with the increase of angles $\theta\theta$. Moreover, the auto-correlation coefficients affect the pipe fracture failure significantly since the fracture failure of pipes depends on many factors. These factors, such as pipe geometries, internal pressure, and corrosion pit depth, are interrelated at different time points in service life. As a result, the load effect is correlated in the time domain. The considerable difference in the probability of failure under different $\rho\rho$ justifies the necessity of using a time-dependent reliability method based on the concept of first passage probability theory.

5 Conclusion

In this study, the time-dependent failure probability of pipes with inclined crack-like defects was calculated, in which a mixed mode fracture criterion was used to establish the limit state function. First passage probability theory was employed to predict the pipe failures. A case study was provided to illustrate the proposed method. From the results of the case study, it has been found that the stress intensity factors increase rapidly with the increase of corrosion pits depth, and the probability of pipe failure increases for longer service time. Moreover, the probability of failure is highly sensitive to both the auto-correlation coefficients and inclination angles. The smaller the inclination angle is, the higher the probability of failure. As such, for engineering assessment of pipes with inclined defects, more attention should be paid to the defects with smaller inclination angles. The proposed method can be applied to the development of rehabilitation strategies for existing pipe networks.

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