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MACROSCOPIC TRAFFIC MODELING WITH THE FINITE DIFFERENCE METHOD

Prepared by:

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March 15, 1996

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ABSTRACT

A traffic congestion forecasting model (ATOP), developed in the present investigation, is described briefly. Several macroscopic models, based on the solution of the partial differential equation of conservation of vehicles by the finite difference method, were tested using actual traffic data. The functional form, as well as the parameters, of the equation of state which describes the relation between traffic speed and traffic density, were determined for a section of the Long Island Expressway. The Lax method and the forward difference technique were applied. The results of extensive tests showed that the Lax method, in addition to giving very good agreement with the traffic data, produces stable solutions.

Key Words - Macroscopic traffic modeling, Lax, the upwind differencing and forward differencing methods.

Reader Aids:

<u>Purpose:</u> Report on traffic modeling and simulations
<u>Special math needed:</u> Finite difference methods
<u>Results useful to:</u> Traffic engineers

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1. INTRODUCTION

The Intelligent Transportation System (ITS) Program was established to improve the efficiency and effectiveness of surface transportation in the United States. ITS focuses on developing, deploying, and evaluating advanced and emerging technologies. One of several program areas of ITS is the Advanced Traffic Management Systems, or ATMS. Brookhaven National Laboratory (BNL) entered an interagency agreement with the Federal Highway Administration, with the concurrence of the N.Y. State Department of Transportation, to work in the ATMS area for New York State. Specifically, BNL will develop a Congestion Forecasting Model for Long Island's INFORM Management System.

The BNL computer model developed for this project is called ATOP, an acronym for Advanced Traffic Occupancy Prediction. The various modules of the ATOP computer code are written currently in Fortran and run on PC computers faster than real time for a section of the Long Island Expressway under study. The various routines currently contained in the ATOP code are: (1) statistical forecasts of traffic flow and occupancy, using historical data for similar days and time (long-term knowledge), and recent information from the past hour (short-term knowledge), (2) establishment of empirical relationships between traffic speed and traffic density (occupancy) using long-term and short-term information, (3) predictions using macroscopic traffic models, (4) statistical routines for detecting and classifying anomalies and their impact on the highway's capacity which are fed back to previous items, and (5) adaptive corrections and updates to control errors in prediction produced by unanticipated events.

In the present article, emphasis is placed on two aspects of the project: (1) developing, testing, and validating the appropriate macroscopic traffic model with actual traffic data and (2) determining the speed-density relationship and its parameters for a 6.12 stretch of the Long Island Expressway (L.I.E.).

2. MATHEMATICAL FORMULATIONS

Traffic density and flow are predicted on the basis of the solution of the partial differential equation, describing the conservation of vehicles (Eq. 1), by applying initial and boundary conditions. The partial differential equation (Eq. 1) can be solved either by the finite element or the finite difference method. Since the latter can be computed faster than the former, the finite difference method was employed in the present analysis.

The partial differential equation for the conservation of vehicles in one dimension is described by:

$$\frac{\partial}{\partial t}K(x,t) + \frac{\partial}{\partial x}Q(x,t) = G(x,t) - S(x,t) \tag{1}$$

where:

$K(x,t)$ = traffic density (vehicles/mile)
$Q(x,t)$ = traffic flow (vehicles/hour)
$G(x,t)$ = traffic generation term from entrance-ramps (vehicles/hour.mile)
$S(x,t)$ = traffic disappearance term from exit-ramps (vehicles/hour.mile)

It is understood that the above quantities are averaged over the number of lanes.

From the definitions of aggregate flow and density, the following relation holds:

$$Q(x,t) = U(x,t) \cdot K(x,t) \tag{2}$$

where:

$U(x,t) =$ vehicle speed in miles/hr

To solve Eq. 1, another relationship, called the speed-density relation, is required. Various forms for this relation were proposed in the past by different investigators [1]. Subsequent studies [2-4] showed that the different speed-density relations can be derived from the car-following equation developed by General Motors Corporation at Michigan. An extensive, detailed analysis of data carried out in the present study, for the Long Island Expressway (L.I.E.), revealed that the speed-density relation is best represented by the Guassian form (Fig. 1):

$$U(K) = U_f \, e^{-0.5\left(\frac{K}{K_m}\right)^2} \tag{3}$$

where:

$U_f =$ free flow speed
$K =$ traffic density or occupancy
$K_m =$ optimum density or optimum occupancy at which flow attains a maximum value.

The latter parameter, K_m, alternatively known as the critical density, plays a major role in traffic modeling, particularly in the applications of the upwind differencing methods. In traffic measurements, carried out by inductive loop detectors, the occupancy is obtained by determining the percent of the time the detector is occupied. This quantity is related to traffic density (see Eq. 20), and is expressed either as a fraction or a percentage. For the L.I.E., the critical occupancy parameters for the cases under study are determined and range in magnitude from 16% to 34%, while the free-flow speeds range from 61 miles/hr to 74 miles/hr, depending on the zone number (Table 1).

On substituting equations (2) - (3) into Eq. 1, one readily obtains the following partial differential equation for traffic density, K(x,t):

$$\frac{\partial}{\partial t}K(x,t) = V_w(K)\frac{\partial}{\partial x}K(x,t) \tag{4}$$

where:

$$V_w(K) = U(K)\left\{1 - \left(\frac{K}{K_m}\right)^2\right\} \tag{5}$$

Various methods for solving the conservation equation in its discrete form have been published [5]. These techniques were examined, modeled, programmed, and tested in the present study. A final selection of the method was made on the basis of its stability and accuracy e.g., a stable solution with good representation of the traffic data, particularly at high congestion. A brief review of each method follows.

2.1 The Forward Differencing Method

In the forward differencing method, the flow and density at any node, j, are affected by up-stream conditions. In the discrete form, the solution of Eq. 1 is given by:

$$K(j,n+1) = K(j,n) - \frac{\Delta t}{\Delta x}[Q(j,n) - Q(j-1,n)] + \Delta t\,[G(j,n) - S(j,n)] \tag{6}$$

where:

j,n = integers which represent the spatial-increment and time-increment, respectively.
Δx = segment distance
Δt = time increment

The time and distance variables, when defined relative to a prescribed reference time, t_o, and reference distance, x_o, are then given by:

$$\begin{aligned} x(j) &= x_o + j \cdot \Delta x \\ t(n) &= t_o + n \cdot \Delta t \end{aligned} \tag{7}$$

The stability requirement for the various solutions of Eq. 1 was shown [5] to lead to the following condition:

$$C = U_f \frac{\Delta t}{\Delta x} < 1 \tag{8}$$

which is known as the Courant-Friedrichs-Lewy stability criterion, generally cited in the literature as the Courant condition.

2.2 The Lax Method

The forward differencing method (Eq. 6) sometimes leads to instabilities. Lax and Wendroff [6] showed that stability is restored if an average density at node j, calculated from two neighboring nodes j+1 and j-1, is substituted for the density, K(j,n) i.e.,

$$K(j,n) = \frac{1}{2}[K(j-1,n) + K(j+1,n)] \tag{9}$$

and the partial derivative of flow with respect to distance is approximated by

$$\frac{\Delta Q}{\Delta x} = \frac{Q(j+1,n) - Q(j-1,n)}{2\Delta x} \tag{10}$$

In this manner, down-stream and up-stream conditions are weighted equally to estimate the condition at any segment. Then, the solution of Eq. 1 at node, j, becomes:

$$\begin{aligned} K(j,n+1) = {} & 0.5\,[K(j-1,n) + K(j+1,n)] - 0.5\,\frac{\Delta t}{\Delta x}\,[Q(j+1,n) + Q(j-1,n)] \\ & + \Delta t\,[G(j,n) - S(j,n)] \end{aligned} \tag{11}$$

It can be readily shown by a Taylor series expansion that the Lax representation as given by Eq. 11 leads to the following equation for the case where sources and sinks are not present:

$$\partial\frac{K}{\partial t} = -V_w\partial\frac{K}{\partial x} + \frac{\Delta x^2}{\Delta t}(1-C^2)\frac{\partial^2 K}{\partial x^2} \tag{12}$$

As is shown by Eq. 12, an artificial diffusivity is introduced into the problem when applying the Lax method, as demonstrated by the appearance of the second term on the right hand side of the equation. One can view this second term as representing a numerical error in the solution. Therefore, in the application of the Lax method to traffic modeling, it is necessary to dictate that the selected time mesh, Δt, for the particular problem under consideration be such that the Courant limit, C, is close to unity in order to minimize the effect of the diffusive term.

In the present study, the average time increment, Δt, satisfying this condition (Eq. 8) is found to be about 15 seconds.

2.3 The First Upwind Differencing Method

This method is used in transport calculations. The solution of Eq. 1 or Eq. 4 is determined by the sign of the wave speed, $V_w(j,n)$, which is determined by the magnitude of the density, K(j,n), relative to its optimum value, K_m, and is described by the following two equations:

$$K(j,n+1) = K(j,n) - \frac{\Delta t}{\Delta x}\,[Q(j+1,n) - Q(j,n)] + \Delta t\,[G(j,n) - S(j,n)] \quad \text{for } V_w(j,n) \le 0 \tag{13}$$

$$K(j,n+1) = K(j,n) - \frac{\Delta t}{\Delta x}\,[Q(j,n) - Q(j-1,n)] + \Delta t\,[G(j,n) - S(j,n)] \quad \text{for } V_w(j,n) > 0 \tag{14}$$

In the upwind differencing method, the conditions at any node are either affected by up-stream or down-stream conditions based on the direction of the wave speed, $V_w(j,n)$, at that node.

The sign of V_w is determined from the slope of the flow-density-density relations (Eq. 2 - Eq. 3):

$$V_w(K) = \frac{\partial}{\partial K} Q(K) \tag{15}$$

For a speed-density relation given by Eq. 3, the wave speed at node, j, and time, n, is represented by:

$$V_w(j,n) = U(j,n) \left\{ 1 - \left(\frac{K(j,n)}{K_m(j)} \right)^2 \right\} \tag{16}$$

Thus, for $K(j,n) > K_m(j)$, then $V_w(j,n) < 0$ and the wave propagation is in the backward direction to the traffic motion, while for $K(j,n) < K_m(j)$, $V_w(j,n) > 0$, and the wave propagation is in the forward direction to the traffic motion.

2.4 The Second Upwind Differencing Method

In this differencing technique, employed in thermal hydrodynamical computations, average values for the interface velocities are considered. The solution for the conservation equation is written in the form [5]:

$$K(j,n+1) = K(j,n) - \frac{\Delta t}{\Delta x} (U_R K_R - U_L K_L) + \Delta t[G(j,n) - S(j,n)] \tag{17}$$

where the magnitudes of K_R and K_L are determined by the sign of the interface velocities, V_R and V_L:

$$\begin{array}{llll} K_R = K(j) & \text{for} \quad U_R > 0 & K_R = K(j+1) & \text{for} \quad U_R < 0 \\ K_L = K(j-1) & \text{for} \quad U_L > 0 & K_L = K(j) & \text{for} \quad U_L < 0 \end{array} \tag{18}$$

The values of the interface velocities are represented by some sort of average, weighted values of V_j, V_{j-1}, and V_{j+1} at the nodal points, j, j-1, and j+1, respectively, such as:

$$\begin{aligned} U_R &= 0.5(U_j + U_{j+1}) \\ U_L &= 0.5(U_j + U_{j-1}) \end{aligned} \tag{19}$$

Other averaging schemes can be considered also [5].

3. MODEL PARAMETER DETERMINATIONS

To test and validate the model, it is necessary to determine, as accurately as possible from the data, the parameters which enter into the various solutions of the conservation equation of vehicles, as represented by equations 6, 11, 13, 14, and 17. Two of the parameters, free-flow speed, $U_f(j)$, and optimum occupancy, $K'_m(j)$, were determined from an analysis of inductive loop-detector data (occupancies and flows), collected at the INFORM Management System in 1993 and 1994 for the L.I.E..

The occupancy and density are related by the following equation [1]:

$$K(j,n) = \frac{52.80\ K'(j,n)}{(L_v(j,n) + L_d(j))} \tag{20}$$

where:

$K'(j,n)$ = traffic occupancy expressed here in percentage units
$L_v(j,n)$ = average vehicle length in a zone, expressed in feet
$L_d(j)$ = loop detector length, expressed in feet

The inductive loop detector length, employed in the present analysis, is assumed to have a constant value of 6.0 feet. The variations of the average vehicle length with zone number and time of day have been studied in detail elsewhere [7]. In the present calculations, a constant value of 17.6 feet is adopted for the present cases.

Table 1. Free-flow speed and Optimum Density Parameters Obtained for the Long Island Expressway for Zones 94 -107

j	ZONE	$U_f(j)$ (miles/hr)	$K_m'(j)$ (occupancy)
1	94	69.0	0.25
2	95	68.2	0.29
3	96	61.2	0.33
4	97	61.2	0.34
5	98	69.2	0.20
6	99	71.6	0.23
7	100	71.6	0.23
8	101	73.2	0.21
9	102	74.4	0.18
10	103	72.0	0.18
11	104	72.1	0.16
12	105	73.1	0.20
13	106	67.0	0.18
14	107	67.0	0.20
K_m' is expressed here as a fraction			

The INFORM loop detector data give occupancies per lane and total flows as a function of time, day, and zone number and were collected during one minute intervals for each day, but were later averaged over 15 minutes intervals.

From the measured averaged occupancies per lane, total traffic flows are calculated by the relation:

$$Q(K',N) = \frac{N.\ 52.80.\ K'.\ U(K')}{(L_v + 6.0)} \tag{21}$$

where:

N = the effective number of lanes
U(K') = average vehicle speed for a zone

A non-linear least squares fit to the data using Eq. 21 can determine the average vehicle length, as well as the effective number of open lanes for a zone. Some of the examples of the many fits to the data, obtained by the two described methods, are shown in Fig. 1 and Fig. 2. In Fig. 1, speed is correlated with occupancy (Eq. 3), while in Fig. 2, flow (or volume) is fit as a function of occupancy (Eq. 21).

Fig. 3 is a representation of a section of the L.I.E. of interest for the present modeling. The designations of zone number, locations of the loop detectors, and the distances between loop detectors are shown in Fig. 3. A value of 17.6 feet was determined as the average vehicle length for the cases under study. Furthermore, to carry out sensitivity analysis, the following additional factors were taken into considerations: (1) the range and variability of these parameters as a function of time and zone, and (2) the quality of the data. This was accomplished by carrying out non-linear least squares fits (LSF) to the data (utilizing a SAS software package installed at the BNL VAX clusters) and observing consistencies, as well as any inconsistencies which may result from this procedure.

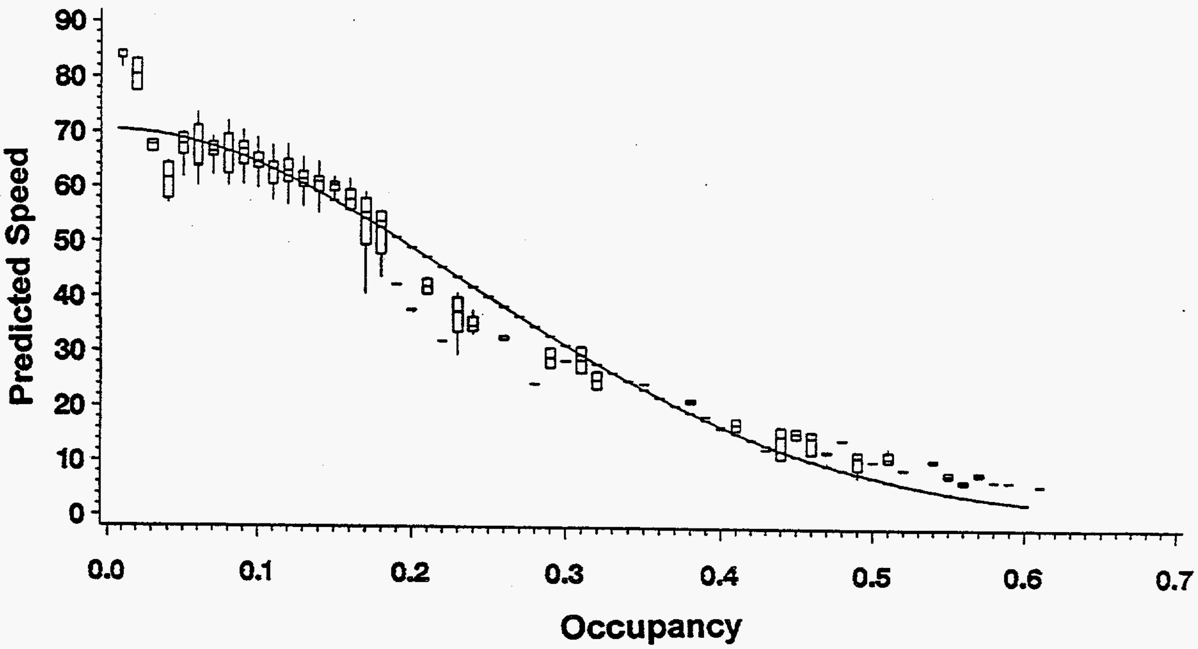

Fig. 1 A semi-Guassian form for the speed-density relation is derived for the Long Island Expressway from a non-linear squares fit analysis. For zone 105, March 5-9, 1994, the free flow speed and the optimum occupancy are 70.46 miles/hr and 23.5%, respectively.

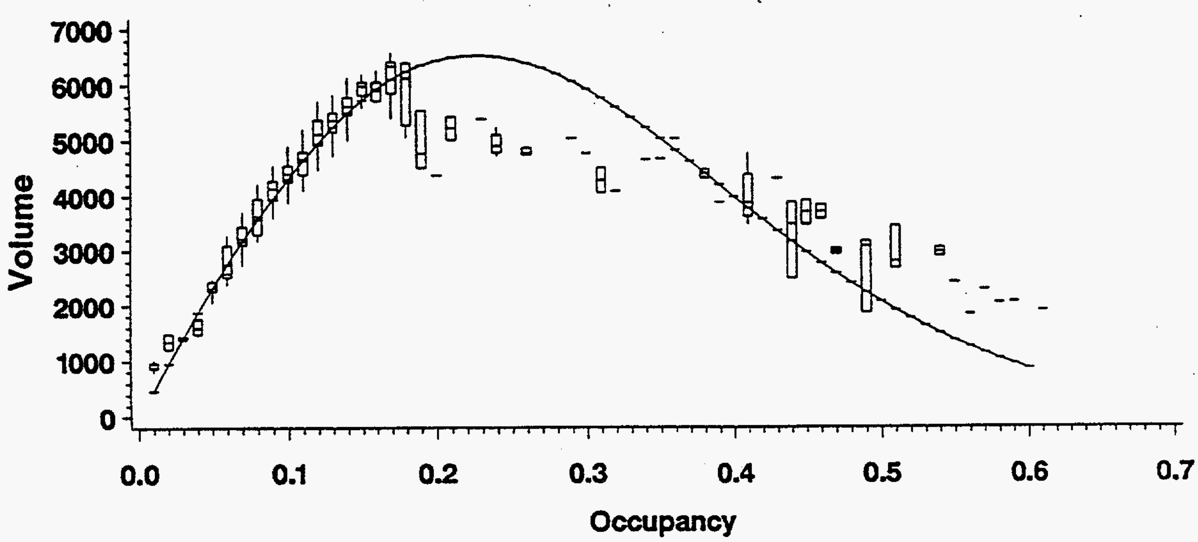

Fig. 2 The speed-density relation can alternatively be derived from an analysis of flow as a function of occupancy. Note that at optimum occupancy of 22.6%, the flow attains its maximum value. An average vehicular length of 18 feet is assumed. Below the optimum density, the slope is positive; while above it, the slope is negative.

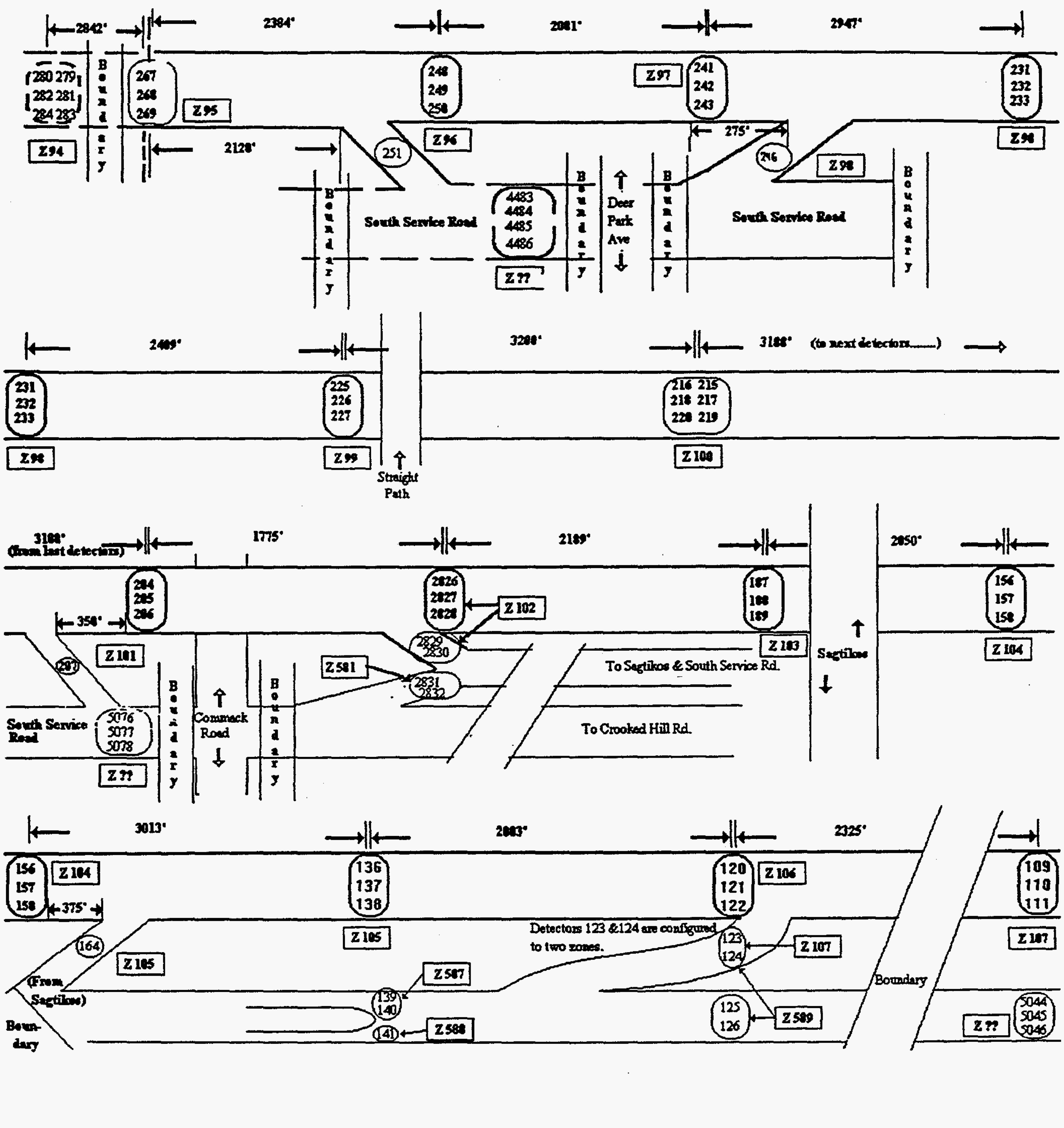

Fig. 3 A configuration of the Long Island Expressway as modeled in the current study. In the analysis, the nodes are positioned at the locations of the loop detectors.

4. TESTS AND VALIDATIONS OF THE MODELS

4.1 General Procedures

Several computer codes were developed for the various methods described by Eq. 6, Eq. 11, Eq. 13, Eq. 14, and Eq. 17. To carry out the computations, the model requires the following information:

- Initial and boundary conditions, as well as exit- and entrance- ramp flows.
- Parameters of the speed-density relation for each segment.
- An average vehicle length for the case under study.

The initial and boundary conditions, as well as the generation and dissipation terms (flows per unit distance for entrance-ramps and exit-ramps) were obtained from the INFORM loop detector data. For testing and validating, it is important to note that actual time-dependent data are used. The initial conditions for this problem consist of the occupancy data for zones 94 - 107 at time t_o. The boundary conditions are the time-dependent occupancy data for zones 94 and 107, bounding the present configuration. The generation and dissipation terms, G(j,n) and S(j,n), are the traffic flows per unit distance for the entrance-ramps at zones 98, 105, and 107 and exit-ramps for zones 96, 101, and 102, Since the main objective of the present study is to predict congestion, the various models were tested under such conditions e.g., for those times of the day where occupancies are close to, or larger than, the optimum value, 0.2 (Fig. 3).

In general, two sets of calculations were carried out for the available daily data during congested conditions:

1. one, covering the time interval from 15:30 to 17:00, for the occupancy data at 15:30 were considered as the initial conditions, and

2. the other, covering the time interval from 17:00 to 18:30, for which the occupancy data at time 17:00 is considered as the initial conditions for this problem.

Occupancies, as well as speeds and flows, were computed by the different models for a selected total time of 90 minutes. Extensive and detailed computations were carried out under a variety of congested conditions.

4.2 The Lax Method

The Lax method was applied extensively in the present investigations because of the successful results of previous macroscopic modeling [9 - 10] efforts which utilized this method.

In all cases considered, the Lax method proved to be quite stable, while the other methods either exhibited unstable behavior or yielded poor fits to the data when dealing with large congestions. Three dimensional temporal profiles of occupancy as a function of distance (in segment units) and time (in minutes), are generated by the model on the basis of the Lax method, and are shown in Figs. 4 and 5. To compare with the data, selected time-slices of Fig. 4 and Fig. 5 are considered, and displayed in Fig. 6 through Fig. 13. As shown, the sensitivity of the predicted occupancies to variations in the value of the average vehicle length, L_v, are determined by varying its magnitude by $\pm$ 20%.

It is particularly interesting to note that the predictions of occupancies for zones 105 and 106 are in reasonably good agreement with the data at 17:00 (Fig.9), depart from the data at 17:45 (Fig. 10), and again are in good agreement with the data at subsequent times, 18:00 - 18:30 (Fig. 11 - Fig. 12). Examples, such as these, may provide indications for the detection of an anomalies (unanticipated events) in the traffic flow, which may be produced by an accident and the subsequent closing of lanes. In this instance, the duration of this anomaly is about half an hour.

On the basis of detailed inspections of Fig. 6 through Fig. 13 and other numerous similar results (not presented here), the following general observations can be made:

- Qualitatively, the predictions of the model, based on the Lax method, reproduce very well the general trend of the data.

- Quantitatively, agreements between predictions and measurements generally are better than 10% for low occupancies and 20% - 30% for the high occupancies.

- On the basis of detailed studies, which were carried out for various days (May 10 through 24, 1993) the Lax method describes the high occupancy regions better than the upwind differencing method when the latter is stable.

4.3 The First Upwind Differencing Method

The mathematical formulation of the first upwind differencing method was described previously and is represented by Eq. 11. To carry out an assessment of this method, its predictions were compared with the experimental data, as well as with the predictions generated by the Lax method. The results of the computations and comparisons are graphically displayed in Fig. 14 through Fig. 18. A detailed examination of these computations reveal the following significant points:

- Very good descriptions of the low occupancy data for zones 95 - 104 are achieved by the predictions of the first upwind differencing and the Lax methods.

- For the high occupancy regions, (zones 105 - 107), the Lax predictions are generally in better agreement with the data than the first upwind differencing method.

- The first upwind differencing method generates unstable solutions at high congestions (Fig. 16, Fig. 18).

4.4 The Second Upwind Differencing Method

The second upwind differencing method was implemented into a traffic model and some exploratory tests were carried out. Additional detailed testing will be required before any definitive conclusion can be reached. At the present, this method is under investigation. The results of these studies will be reported elsewhere [11].

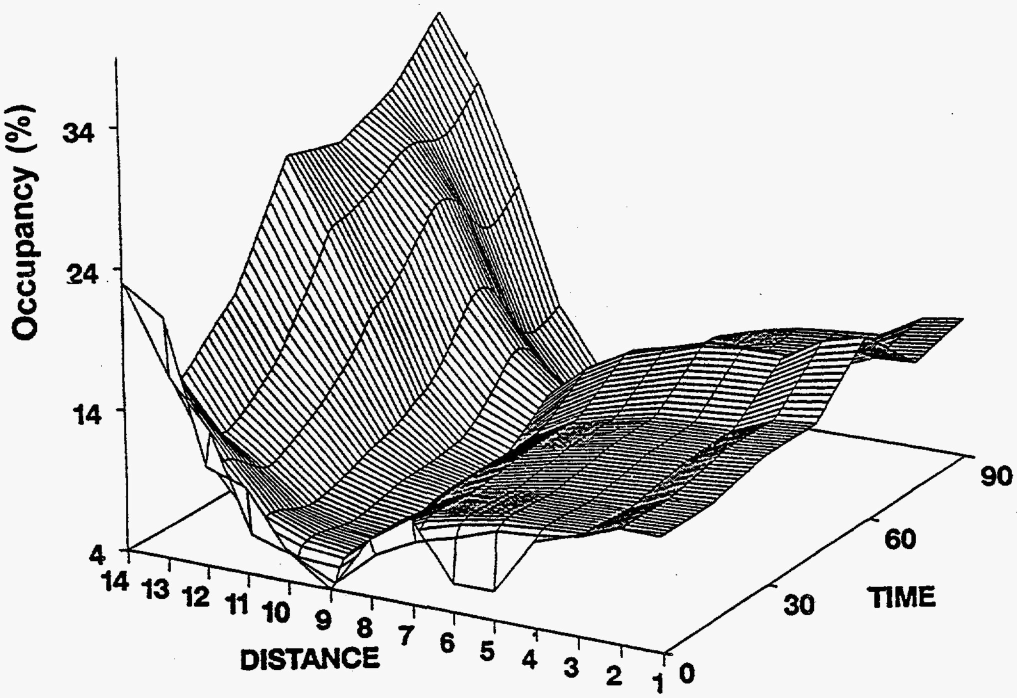

Fig. 4 Occupancy profile for the Long Island Expressway as a function of segment number (zone) and time, generated by the model on the basis of the Lax method. The direction of traffic is from right to left. Segments 1 and 14 correspond to zones 94 and 107, respectively and are considered as the boundary conditions for the problem. Time is expressed in units of minutes. Data collected May 24, 1993 15:30 to 17:00.

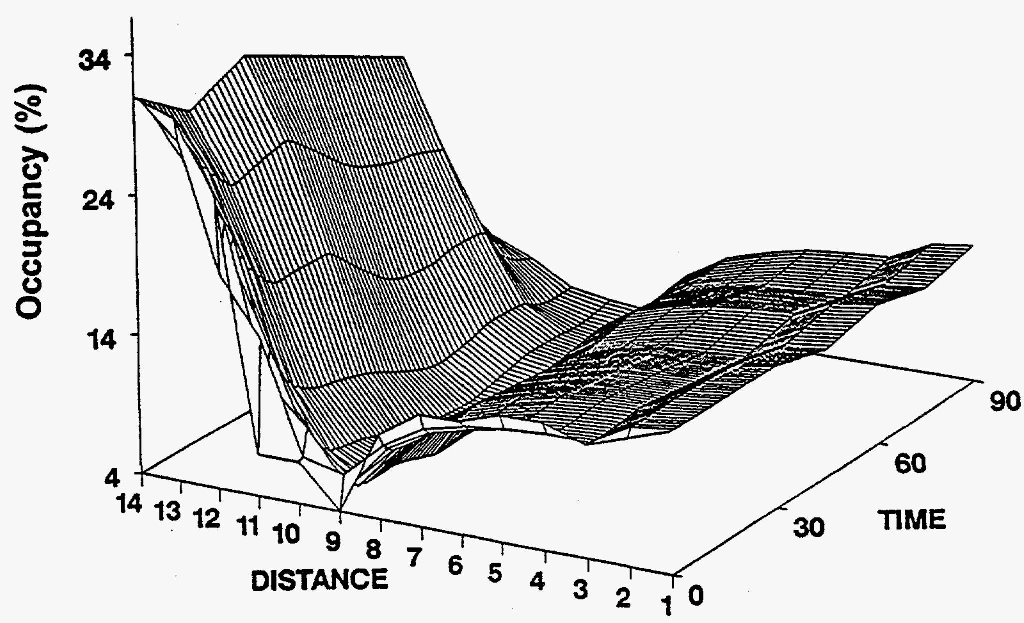

Fig. 5 Occupancy profile for the Long Island Expressway as generated by the model on the basis of the Lax method for May 24, 1993 from 17:15 to 19:00. See Fig. 4.

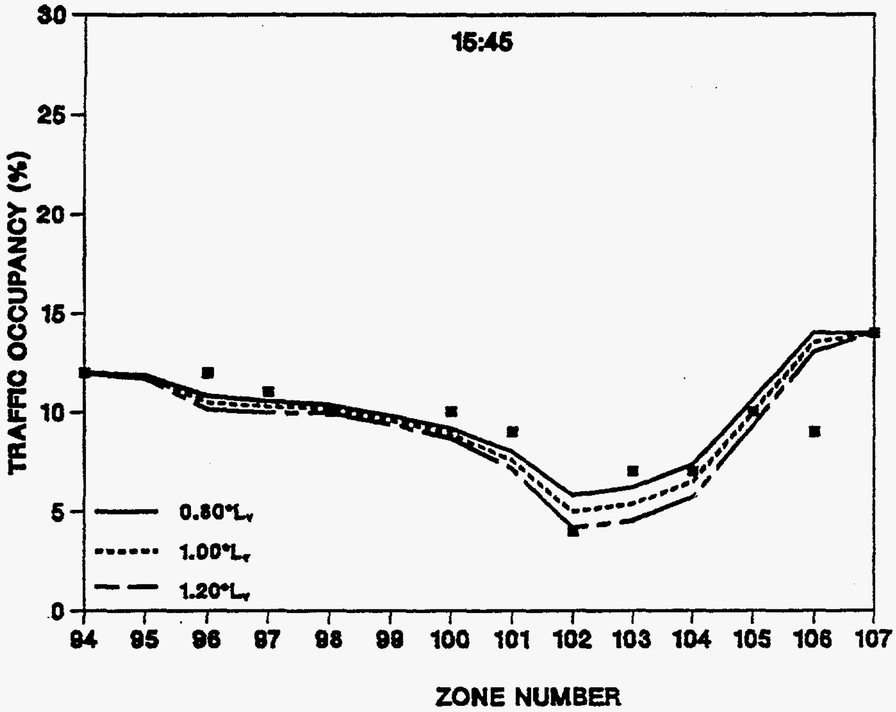

Fig. 6 A comparison between the predicted (Lax method) and measured occupancies as a function of zone number and L_v at 15:45 on May 24, 1993.

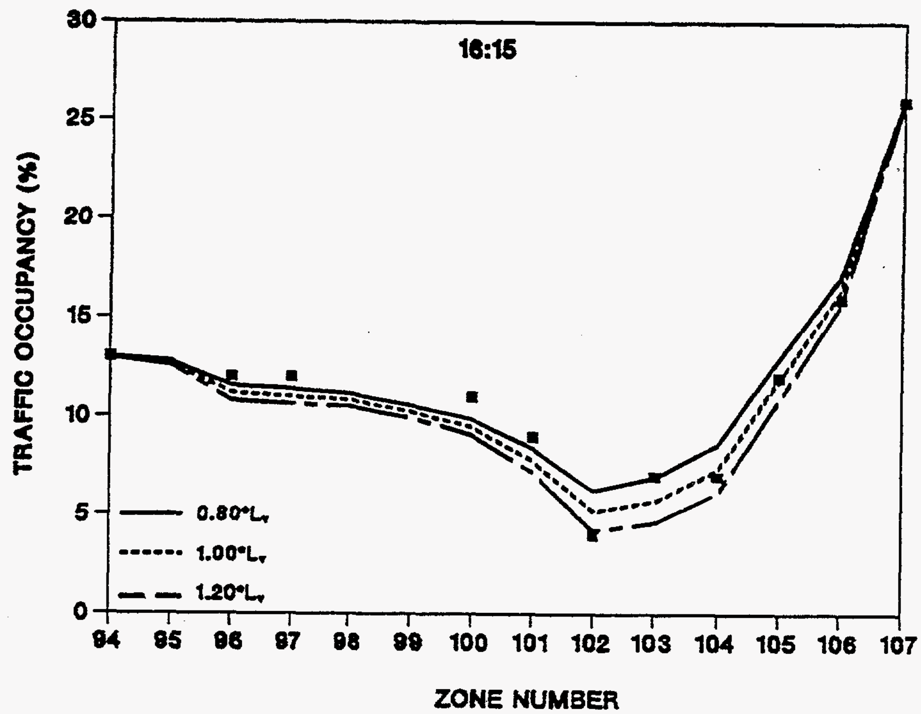

Fig. 7 A comparison between the predicted (Lax Method) and measured occupancies as a function of zone number and L_v at 16:15 on May 24, 1993.

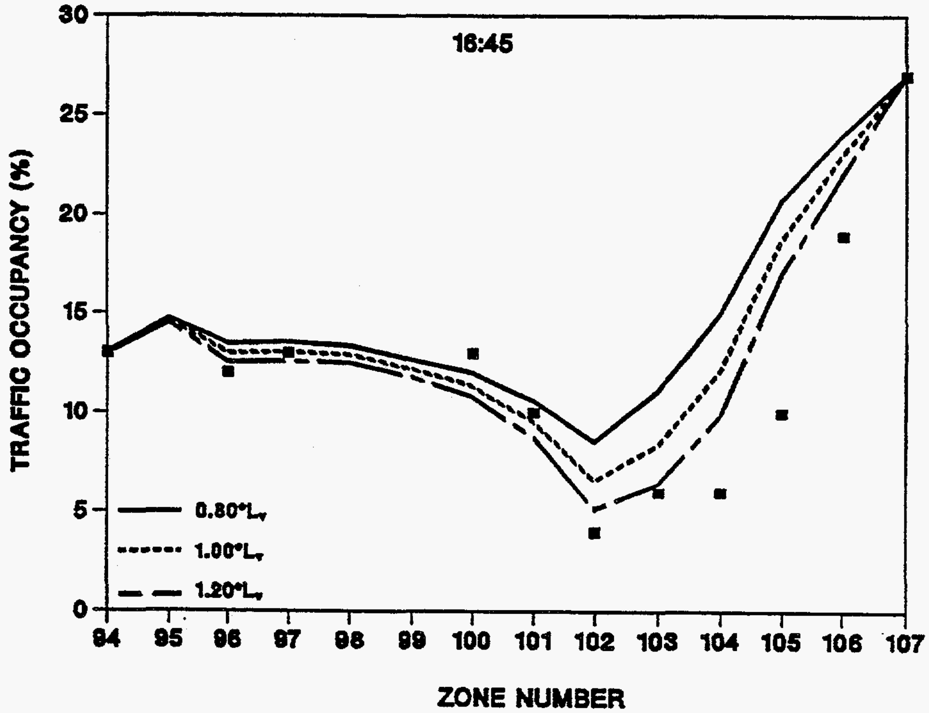

Fig. 8 A comparison between predicted and measured occupancies as a function of zone number and L_v at 16:45 on May 24, 1993. The predictions are based on the Lax method.

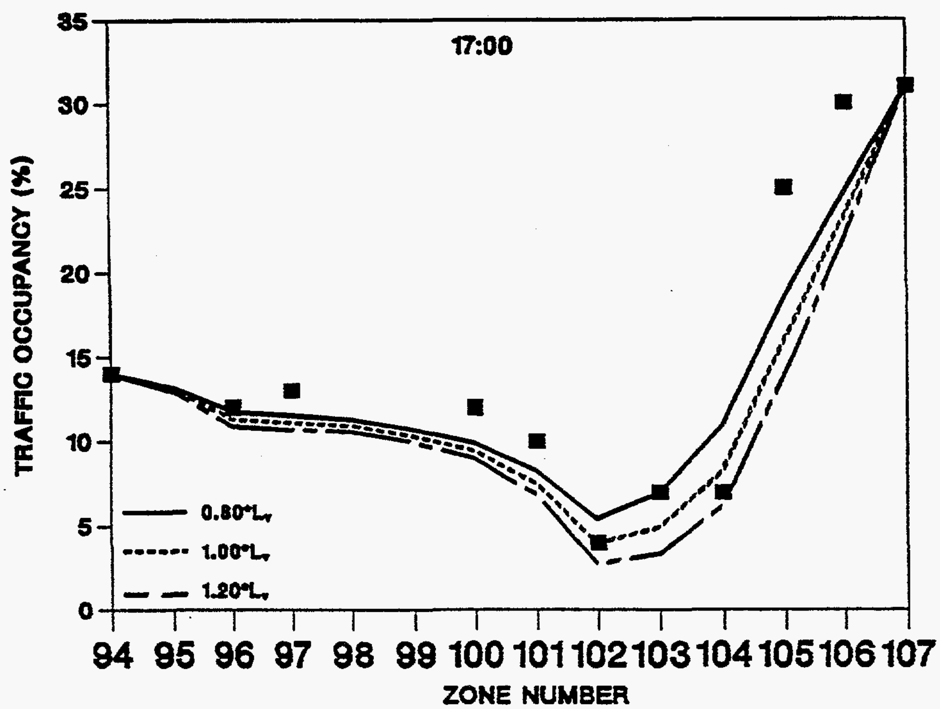

Fig. 9 A comparison between the predicted and measured occupancies as a function of zone number and L_v at 17:00 on May 24, 1993. The predictions are based on the Lax method.

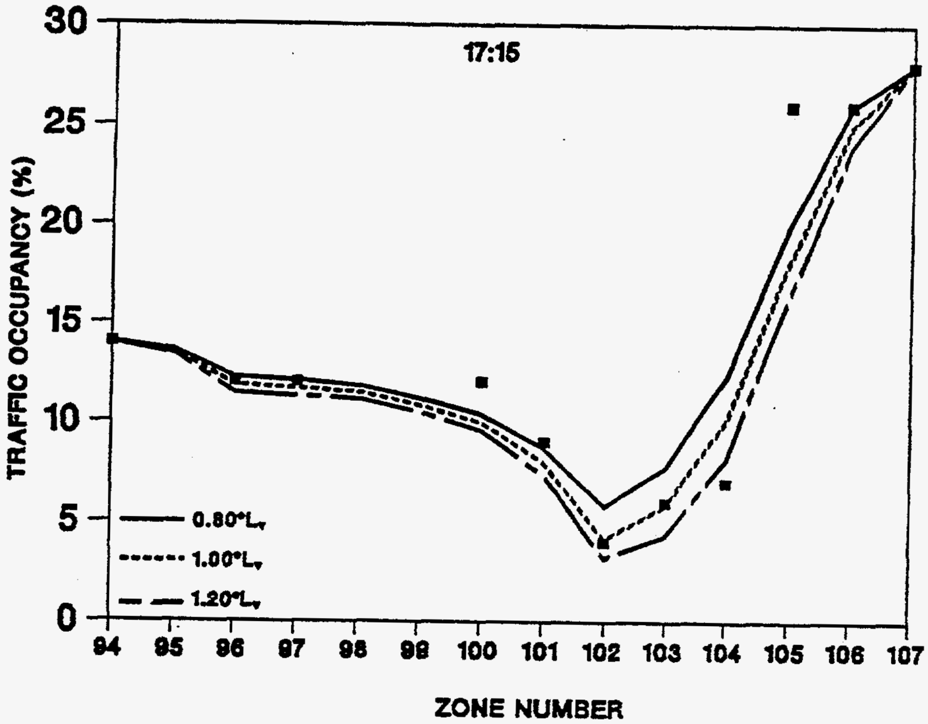

Fig. 10 A comparison between the predicted and measured occupancies as a function of zone number and L_v at 17:15 on May 24, 1993. The predictions are based on the Lax method.

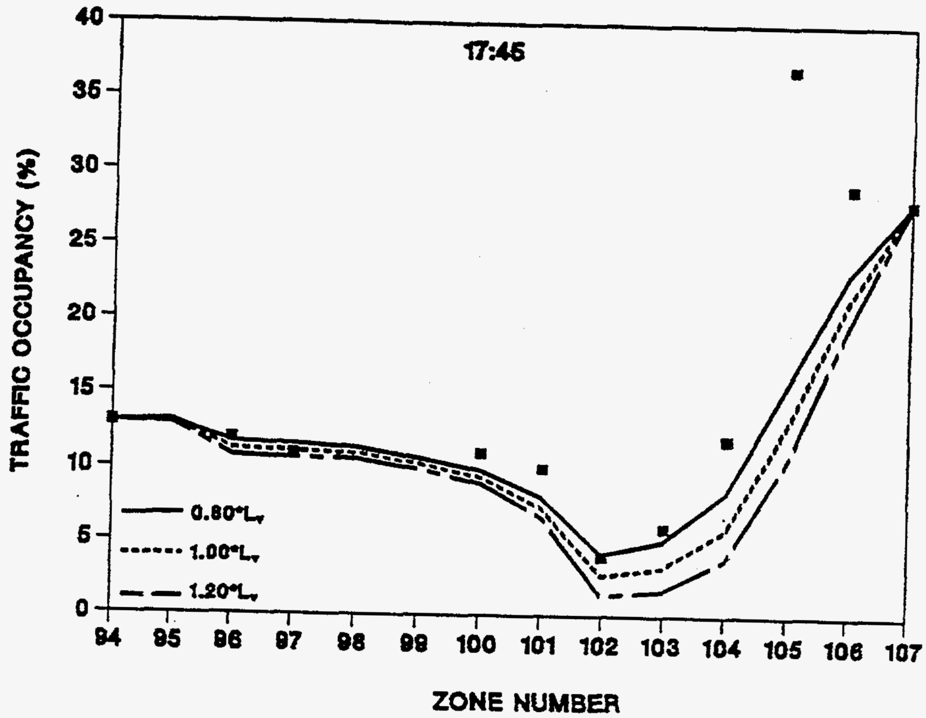

Fig. 11 A comparison between the predicted and measured occupancies as a function of zone number and L_v at 17:45 on May 24, 1993. The predictions are based on the Lax method.

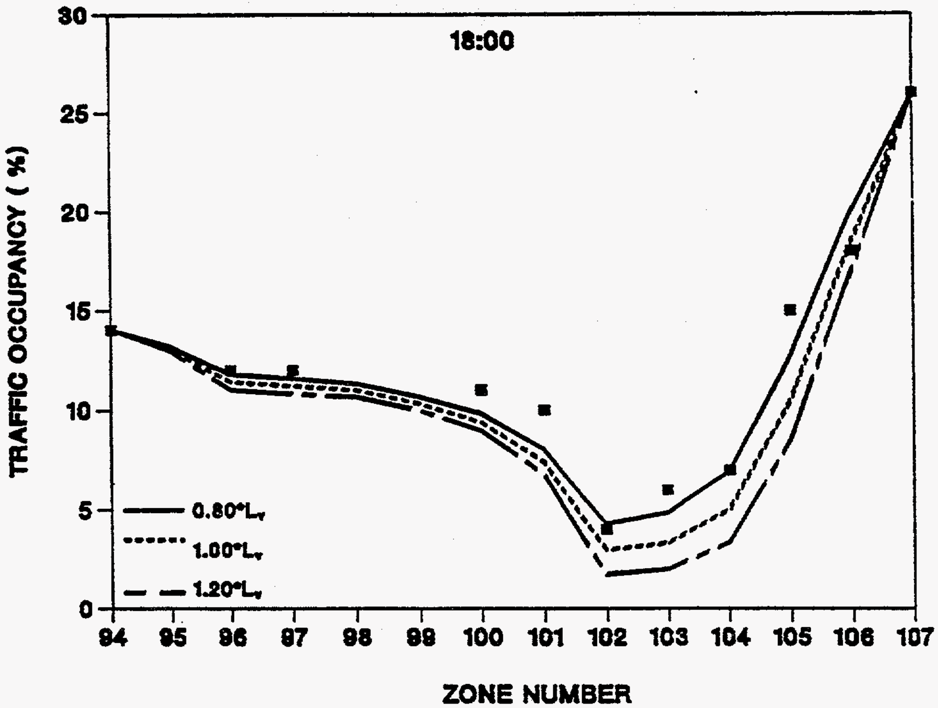

Fig. 12 A comparison between predicted and measured occupancies as a function of zone number and L_v at 18:00 on May 24, 1993. The predictions are based on the Lax method.

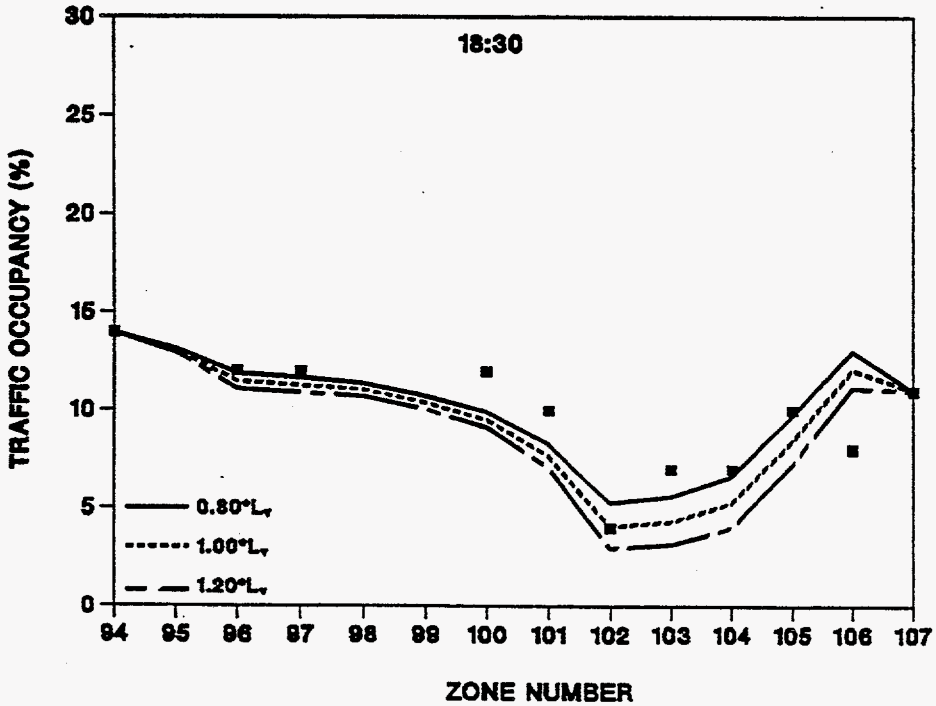

Fig. 13 A comparison between predicted and measured occupancies as a function of zone number and L_v at 18:30 on May 24, 1993. The predictions are based on the Lax method.

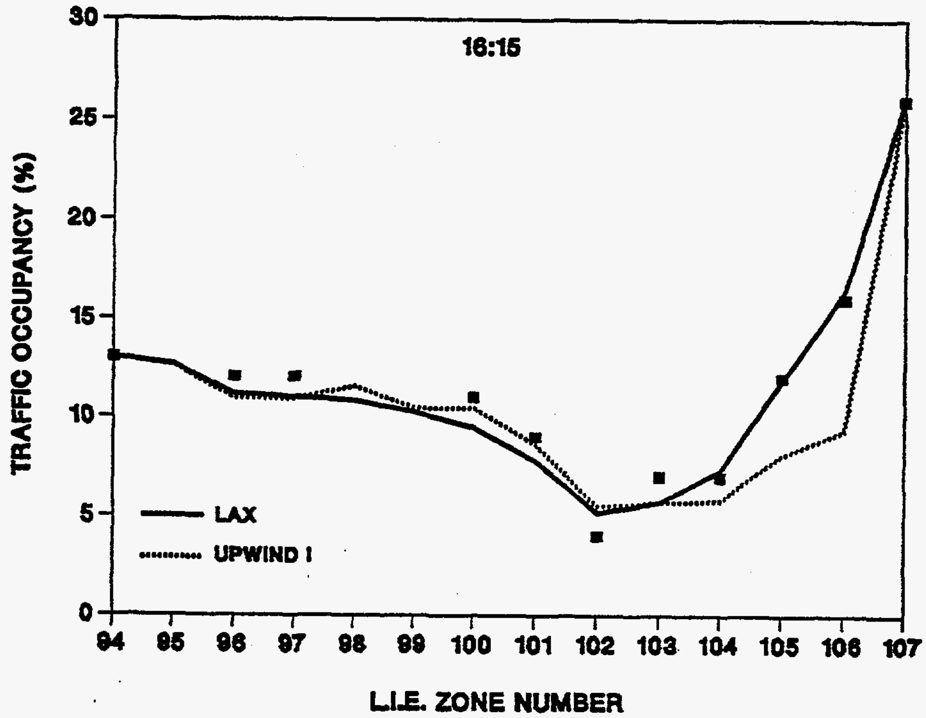

Fig. 14 A comparison between the predicted and measured occupancies as a function of zone number at 16:15 on May 24, 1993. The predictions are based on the first upwind differencing method (dotted line) and are compared with the Lax predictions.

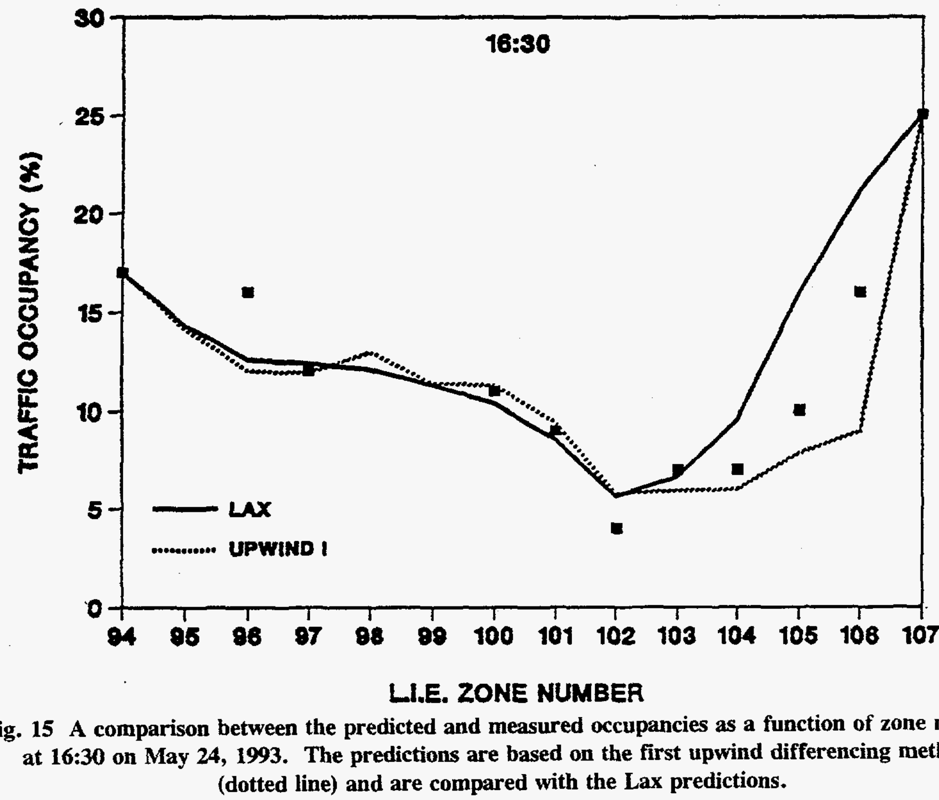

Fig. 15 A comparison between the predicted and measured occupancies as a function of zone number at 16:30 on May 24, 1993. The predictions are based on the first upwind differencing method (dotted line) and are compared with the Lax predictions.

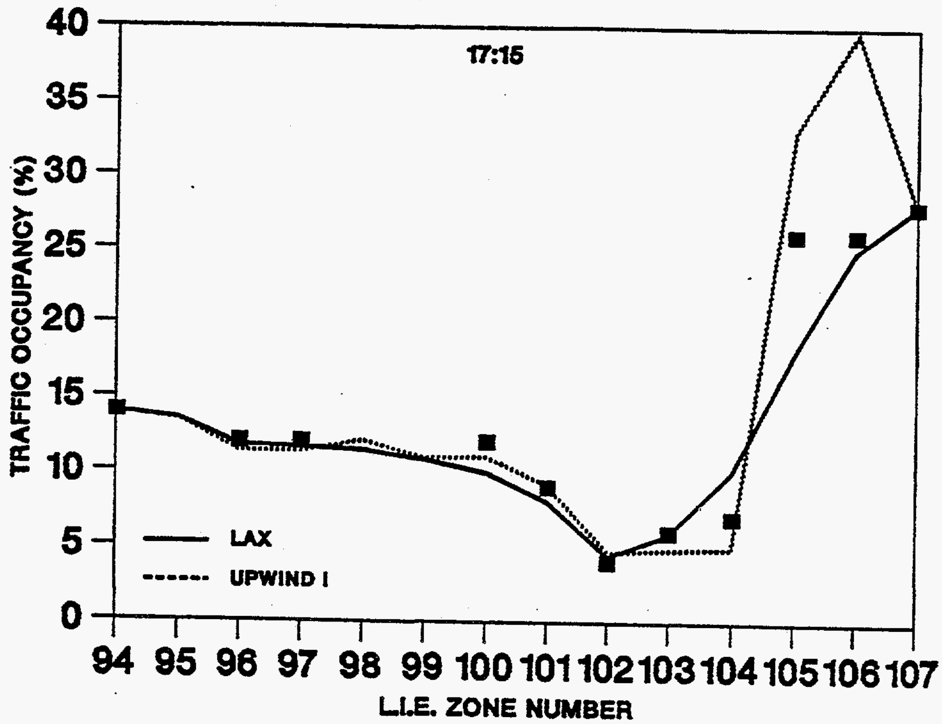

Fig. 16 A comparison between the predicted and measured occupancies as a function of zone number at 17:15 on May 24, 1993. The predictions are based on the first upwind differencing method (dotted line) and are compared with the Lax predictions.

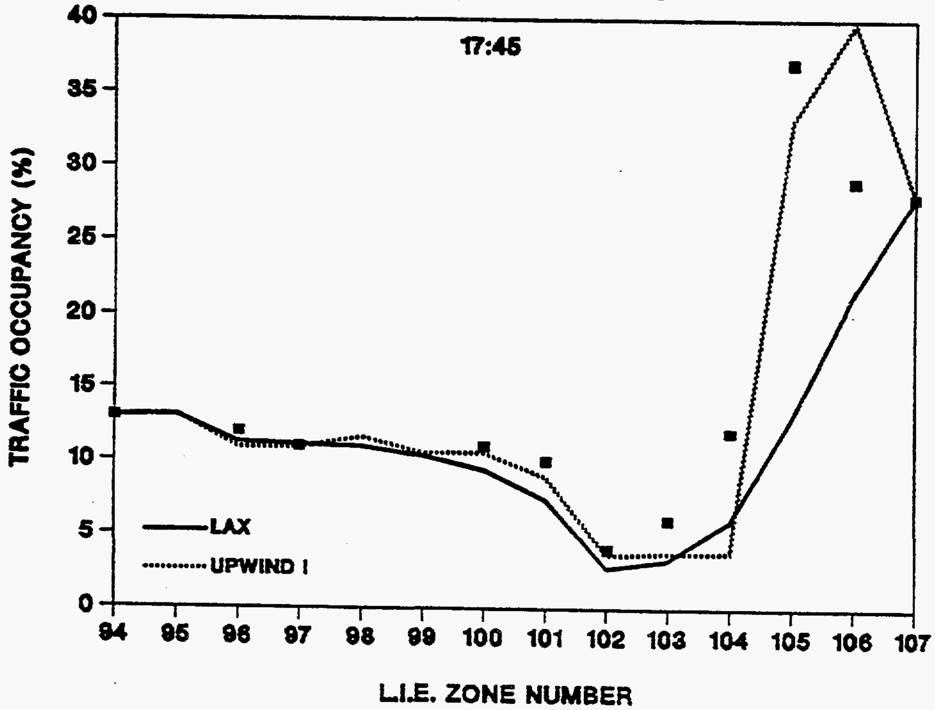

Fig. 17 A comparison between the predicted and measured occupancies as a function of zone number at 17:45 on May 24, 1993. The predictions are based on the first upwind differencing method (dotted line) and are compared with the Lax predictions.

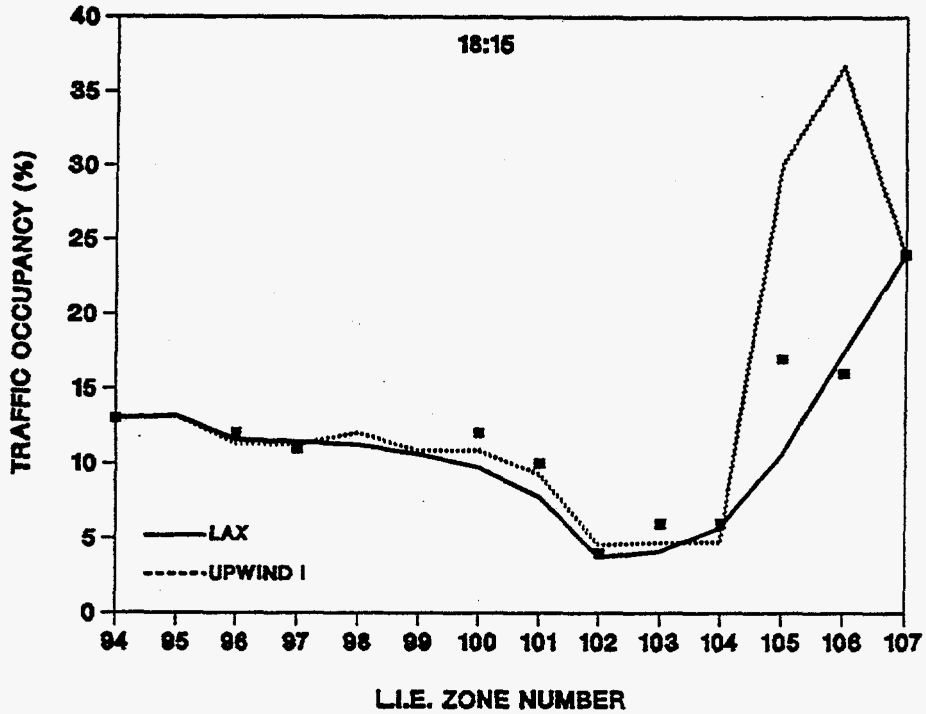

Fig. 18 A comparison between the predicted and measured occupancies as a function of zone number at 17:45 on May 24, 1993. The predictions are based on the first upwind differencing method (dotted line) and are compared with the Lax predictions.

5. SUMMARY AND CONCLUSION

Various speed-density relations were applied to a 6.14 mile stretch of the Long Island Expressway. The semi-Guassian form was found to represent adequately the traffic data. Free-flow speeds and optimum occupancies were determined for fourteen zones of the Long Island Expressway and are used in the traffic modeling. Four macroscopic traffic models, based on the forward differencing method, the Lax method, the first upwind differencing method and the second upwind differencing method are described and were tested. The Lax method was very stable under a variety of congested conditions and yielded reasonable agreement between predictions and data.

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