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METHODOLOGIC EVOLUTION ASSESSMENT OF LARGE DEFORMATIONS ON ROMANESQUE MASONRY IN THE VAL D'ARAN (XII-XIII), SPAIN

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Abstract. *There is a set of Romanesque churches at Val d'Aran that were built between the eleventh and thirteenth centuries. One of the main features of these buildings is the presence of large deformations and geometrical displacements in their masonry. One of the most deformed buildings is the church of Santa Maria de Arties (XII), which arches' and vaults' have anti-funicular shapes. The assessment is based on a three-dimensional model obtained with a terrestrial laser scanner (TLS). The scanner used is a Leica ScanStation P20. In the study of built heritage, the use of direct measurement techniques for architectural surveying requires a large number of resources. These deformations have caused, in some cases, turn centerlines of the vaults into convex shapes, which have the inverted shape of an arch. Joints appeared because of active and passive thrusts that were performed to keep the structures in equilibrium. Geometrical assessment of the least rigid elements, the pillars, makes it possible to analyze the displacements, which have been the cause of anti-funicular shapes on some vaults. It is possible to deduce the regression plane of the displacements of the pillars of the central nave and to define the deformation vectors over it. Thus, these data all for the directions of the deformations of the vaults to be determined.*

The methodology of this study focuses on the assessment of the geometrical characteristics of the pillars, with the objective of studying the displacements that they have suffered. The point cloud is processed with the software; Cyclone, and the program 3DReshaper (2016), and Google SketchUp (2019). The study allows to analyze the evolution of the treatment of points, concluding that deformations of the pillars are not perpendicular to the axis of the central vault. The methodology used and its results let to understand the nature of the displacements in order to preserve these masonry structures.

1 THE STUDY OF THE GREAT DEFORMATIONS OF THE VAL D'ARAN ROMANESQUE ARCHITECTURE

The churches of Val d'Aran are located at the Spanish Pyrenees and were built between the twelfth and thirteenth centuries. Large deformations have been found on these Romanesque buildings masonry and are one of the main characteristics of them. In the church of Santa Maria Arties (XII-XIII), the most deformed building, some anti-funicular shapes have been found at their arches and vaults. This anti-funicular shape is the inverse of the natural shape of an arch since it is convex in relation to its centerline. One of the causes of these deformations is the displacement of the pillars since these are the least rigid elements of the structure.

Owing to large deformations, these Romanesque constructions were assessed by several authors, such as Emmanuel Viollet-le-Duc (1814-1879), who visited Bossost (1883) [1], and later by Lluís Domènech i Montaner (1850-1923), who was Dean of the Escuela de Arquitectura de Barcelona (1905) [2]. Subsequently, the Institut d'Estudis Catalans organized an expedition with the combined historical and archaeological purpose of visiting the Val d'Aran and La Ribagorça (1907). Two of the participants were the architects Josep Puig i Cadafalch (1867-1956) and Josep Goday i Casals (1881-1936) [3], who suggested that these churches were initially covered with timber structures and later replaced by masonry barrel vaults [4].

Juan Bassegoda i Nonell (1930-2012) stated that formal anomalies are a defining characteristic of the Catalan Romanesque architecture [5]. The case study presented in this paper focuses on the church of Santa Maria Arties, which is the most assessed building of this group. Its huge deformations were identified during the restoration work of the 70s' [6]. Afterwards, José Luis i Villanueva noted the existence of the funicular shapes [7], and in 2009, the structure was assessed by means of finite elements (FEM) by the team of Joan Polo i Berroy [8].

The structure presents a typical deformation pattern: leaning of vertical elements towards the outside because of vaults thrust, and settlement of vaults, which have caused the apparition of funicular shapes, that is the inverse of the natural shape of an arch since it is convex in relation to its axis. One of the causes of these deformations is the displacement of the pillars since these are the least rigid elements of the structure. The investigation focuses on the geometrical parametrization and evaluation of these movements. Previous investigations focused on the assessment of the overall structure, in order to understand the stability conditions, which revealed that masonry is working to the limit through the deformations suffered, while the origin of the movements can be explained by rigidity issues. The assessment is based on a three-dimensional model obtained with a terrestrial laser scanner (TLS). In the assessment of built heritage, the use of direct measurement techniques for architectural surveying requires a large number of resources. The use of massive data capture techniques, such as Terrestrial Laser Scanner (TLS), has recently become prominent in surveying architectural heritage.

Otherwise, the architectural heritage topographic documentation is a key tool for its preservation. Current techniques of Massive Data Capture (MDC), such as digital photogrammetry and terrestrial laser scanner, have become widespread in the assessment of built heritage [9], [10]. Numerous investigations have tested the technique's reliability [11],

[12], and has proven their effectiveness to survey the building's geometry with high precision. The point clouds make it possible to detect and monitor degradation processes and formal anomalies. Other applications go from heritage documentation [13], [14] to deepen in the history and buildings constructions [15], [16] between many others.

The specific issue of deformation assessment is essential for architectural heritage conservation, and many studies have developed simple procedures to address that issue from the 3D topographical information of the point clouds, i.e. the Cathedral of St. Johannis in Meldorf [17] the churches of Santa Maria in Portonovo [18] and Cantalovo [19] or the Palazzo d'Accursio, in the cathedral of Tortosa [20], and Val d'Aran in Spain [21] [22]. Those investigation focuses on the assessment of vertical elements deformations, except the Portonovo's case, where a simple assessment of the cylindrical barrel vaults is performed from standard primitives.

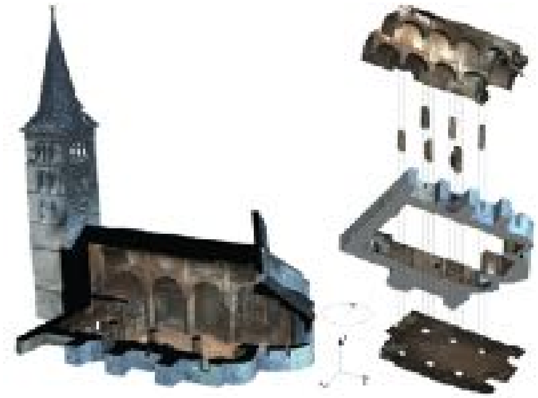

Figure 1. Santa Maria Arties (XII-XIII) [35]

In order to analyse the deformations of the church of Santa Maria Arties (XII-XIII), we used the point cloud extract from the scanner Leica ScanStation P20, with a bandwidth of 808/658, class 1. The scanning ratio is 1,000,000 points, and the noise at 100 is 9.0 mm for black surfaces, 4.30 mm for grey surfaces, and 1.5 mm for white surfaces. The field of view is 360º horizontal and 270º vertical. (Figure 1).

2 METHODOLOGY OF THE GEOMETRICAL ASSESSMENT OF ARTIES PILLARS'

The methodology of this study focuses on the assessment of the geometrical characteristics of the 6 pillars [$P_1 \ldots P_6$] of Santa Maria Arties with the objective of studying the displacements that they have suffered [23]. The pillars of the central nave have deformed in a specific way, namely, through the masonry joints (n_s). These joints are perfectly visible on pillars P_3, P_4, P_5 and P_6, while they are more difficult to see on P_1 and P_2 since these pillars are partially covered by mural paintings. The displacements can be assessed according to the coordinates of the centroid of each row (x_{ci}, y_{ci}, z_{ci}), and the point of reference is taken from the row of the floor plan, which is considered to be undeformable. This establishes the coordinates as (x_{ci}, 0, z_{ci}). These points allow one to define a regression plane P_{ri} for each pillar. Thus, it is possible to define a vector of deformation contained on each plane [$P_{r1} \ldots P_{r6}$]. Finally, these data allow one to determine the general tendency of the deformation of the vaults (Figure 2).

The assessment of shapes can only be understood in three dimensions through an interval (*a, b*), which has to impose the condition of equilibrium according to the elastic theory with the summation of the active thrusts (E_{ba}) of the vaults and the passive thrusts of the walls (E_{mp}) and buttresses (E_{mc}).

$$\sum_{b}^{a} F_{(x,y,z)}\left(E_{ba} + E_{mp} + E_{mc}\right) = 0$$
$$\sum_{b}^{a} M_{(x,y,z)}\left(E_{ba} + E_{mp} + E_{mc}\right) = 0.$$

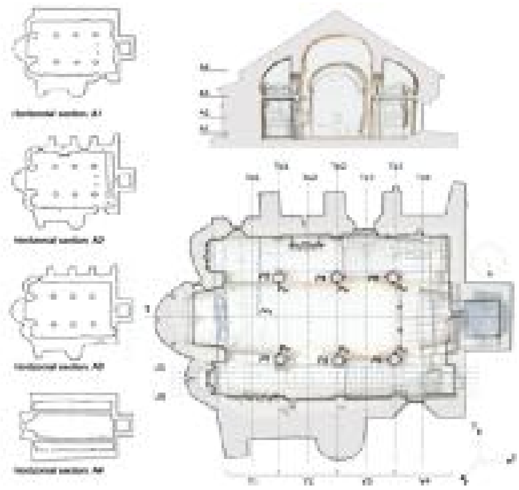

Figure 2. Methodology of the geometrical assessment of Santa Maria Arties pillars'[35]

The assessment of the deformations of the elements under its own weight, especially the central and lateral vaults, makes it possible to deduce that external actions, such as snow or seism. These external actions, together with the loss of lime mortar, inasmuch as humidity or vibrations, cause the primitive function $f_{i(x,y,z)}$ of the vaults. This function tends to deform into the function $f_{f(x,y,z)}$, which is obtained through the topographical survey.

From a historiographical point of view, the definition of vaults geometry should be overhaul through the observational technique. Robert Willis (1800-1875) was the first author that studied that fact in its architectural treatise *Remarks on the Architecture of the Middle Ages* (1835), where he makes some references to the Romanesque vaults [24]. He will explain it in more detail in the introductory part of the cylindrical vaults mechanics in 1842 in his book *On the Construction of the Vaults of the Middle Ages* **Error! Reference source not found.**. Other authors that done similar studies are: Emmanuel Viollet-le-Duc (1814-1879), who assessed the barrel vaults and the abutment with timber beams with more precision August Choisy (1841-1909) [26], who did not describe the Romanesque vaults, but he explained the barrel vaults since its Roman construction origins, with brick and concrete [27]. He also referred to the byzantine influence on construction with or without centring [28]. Otherwise, in the work *Historia General del Arte* (1901), Josep Puig i Cadafalch described the great difference between the Romanesque vaults, where elasticity is replaced by stability, which is a concept inherited from roman vaults, those that were significantly more monolithic [29].

Likewise, Puig i Cadafalch determines the characterization of the most important elements in *L'Arquitectura Romànica a Catalunya* (1918-1920), and he also explains the structural and artistic analysis of the masonry, vaults and pillars [30]. In this way, he stablishes vaults' and pillars' evolution and the introduction of the column in the Catalan Romanesque Architecture as one of its main characteristics [31]. From the typological point of view of the canon vaults, it will be Leopoldo Torres Balbás (1888-1960) who tries to explain, in *Bóvedas romanas sobre arcos de resalto* (1946), the transmission of this constructive way from the Roman world, passing through the South of France to The Iberian Peninsula [32]. At the same time, Bonaventura Bassegoda i Musté (1896-1987) will propose, from the mathematical point of view, the elastic theory through the third central part of vaults and domes [33]. Finally, Juan Bassegoda Nonell (1930-2012) explains how the traditional Roman models gave constructive form to medieval vaults in Catalonia [34].

When the geometry of these vaults is regular and the construction is monolithic because of the masonry disposition, the thrust is perpendicular to its centerline. Thus, the thrust of a vault is determined through the following function:

$$E_b\ (E_{bx},\ E_{by},\ E_{bz})$$

If we consider homogeneous vaults with a constant centerline, the vector of this thrust would be, according to Choisy (1873) [27], as follows:

$$E_b\ (E_{bx},\ 0,\ 0)$$

Conversely, barrel vaults of Vall d'Aran churches have specific features. These are conic vaults, as was theorised by Joan Bassegoda (1974) [5], and the stone-cutting is not regular, as stated by Josep Puig i Cadafalch (1901) [29]. Finally, the supports of the vaults, external walls and arches, have a different stiffness, as noted by Luis Villanueva (1974) [7]. Those three conditions are far from the general theory of the thrust of homogeneous vaults with cylindrical axis. In addition, the assessment of the pillar deformations and their stiffness is influenced by the contributions of Josep Puig i Cadafalch (1908) [4].

In any or in a combination of the precedent hypotheses, thrusts work in two directions (x, z) as long as the supports stand still. Thus, the resultant force is not perpendicular to the axis of the vault. The general vectorization in those cases can be defined as the thrust of the vault $E_b\ (E_{bx}, E_{by}, E_{bz})$, and deformations can occur in two directions:

$$f\ (df_x,\ 0,\ df_z)$$

The forces caused by the vaults are transmitted to vertical structural elements. The vaults of the central nave are supported by the walls over former arches, which at the same time are supported by the pillars of the central nave. Thus, the elements can deform over the three planes, pillars $df_p(df_{px}, df_{py}, df_{pz})$, and perimeter walls $df_m(df_{mx}, df_{my}, df_{mz})$.

On the other hand, the deformations of the pillars are a function of their monolithic nature. Thus, the deformations are directly related to the stone cutting of the pillar and the thrust Ep (E_{px}, E_{py}, E_{pz}). According to the internal distribution of forces, the masonry stone cutting and mortar (E_{pz1}), and the irregular geometry of the vault (E_{pz2}), the structure tends towards a state of equilibrium (E_{pz1}- E_{pz2}) or to the opposite state (E_{pz1}+E_{pz2}).

A monolithic, infinitely rigid pillar tends to rotate on its base. The upper part moves over the axis (x) towards the exterior since the main horizontal thrust (E_x) with the consequence that there is also movement in the axis (y) since their extreme upper part declines. Finally, there is also movement over axis (z) due to E_{bz}, which defines the deformations df_p (df_{px}, df_{py}, df_{pz}). If pillars are not monolithic and are built with numerous joints, they tend to deform in the upper part so that there is not Δdf_{py}. If we suppose that the extremes of the vaults have not suffered differential settlements and that the movement of these extremes is thus (Δy=0), then the deformations are df_p (df_{px}, 0, df_{pz}), which is the hypothesis of the present case study. There can be a combination of rotations and translations as a function of the stone cutting, so the general characterization of the displacements should be made through intervals I_1; df_p $(df_{px}, df_{py}, df_{pz})$. Then, in I_2; df_p $(df_{px}, 0, df_{pz})$ displacements (Δy>0) can occur because of the contact between mortar and stones or the friction between stones where there is no more mortar.

The displacement of the pillar can be deduced through analysis of the displacement of the centroid of n sections (n_s) of the pillar. Thus, coordinates (x_{ci}, y_{ci}, z_{ci}) are set for each section (n_s). The centroid of reference (x_{c0}, y_{c0}, z_{c0}) is set in the section of the floor plan since it would have not suffered any displacement. The final section is located at the impost of the pillar (x_{cs}, y_{cs}, z_{cs}). The obtained points define a function f (c_i), which makes it possible to deduce the regression plane P_{ri}. It characterizes the tendency of the displacement vectors of the structural section supported by the pillar.

The assessment of this displacement vector determines the clearance angle (ω) of the regression plane P_{ri} of the plane over the centerline of the vault (τ). Thus, this angle can be understood as:

- (ω φ) = 90º, perpendicular to the centerline φ1,
- (ω φ) < 90º, displacement towards the apse,
- (ω φ) > 90º, displacement towards the façade.

The reparation and containment of these deformations are the cause of the reinforcement of the perimeter walls by means of the construction of buttresses or strategical placement of bell towers, which are usually built in the façade opposite to the apse. The active thrusts (E_{ba}) of the vaults over pillars and walls have been determined, but to understand the equilibrium of these constructions, it is essential to understand the passive thrusts of the buttressing elements: walls (E_{mp}) and buttresses (E_{mc}). Owing to these thrusts, some vaults have deformed towards anti-funicular shapes.

In the year 2016, the point cloud was processed with the software Cyclone, and the program 3DReshaper, which is used to obtain the three-dimensional mesh with an average distance of points of 0.05 m as well as a measure of the triangle for detecting 0.1 m holes. The model of the interior of the building has 80,582 points and 156,449 triangles, and the exterior has 314,650 points and 609,472 triangles.

Otherwise, in 2019, the point cloud has been processed with the plug-in Undet of the software Google SketchUp (2019). In this occasion, the point cloud has not been converted to a mesh, because this plug-in works with points, not with meshes. The models that we are working with have a grid of points in coordinates (x, y, z) ranging from [0.06 to 0.09 m].

3 DATA TREATMENT WITH THE SOFTWARE CYCLONE (2016)

The use of the specific software Cyclone to process the data enable to visualize the point cloud obtained and to process and join all the scanners done and convert the point clouds to a mesh. This processing occurs through an automatic process with slight manual adjustments, and a complete, depurated point cloud and triangular mesh is obtained. The morphological features assessment of the pillars is made through the visible elements, such as the masonry joints. Non-visible elements are not considered. Thus, data are obtained according to the centroids of each visible row. Thus, pillars have following rows: (P_1=26), (P_2=26), (P_3=27), (P_4= 25), (P_5= 23) and (P_6= 23). Rows are numbered from bottom to top [35]. The identification of pillars' joints was made by means of a manual measurement system, for the reason that the graphical capacity of the software Cyclone, and the program 3DResheaper, does not allow to specify pillar's joints because it constructs it three-dimensionally (Figure 3).

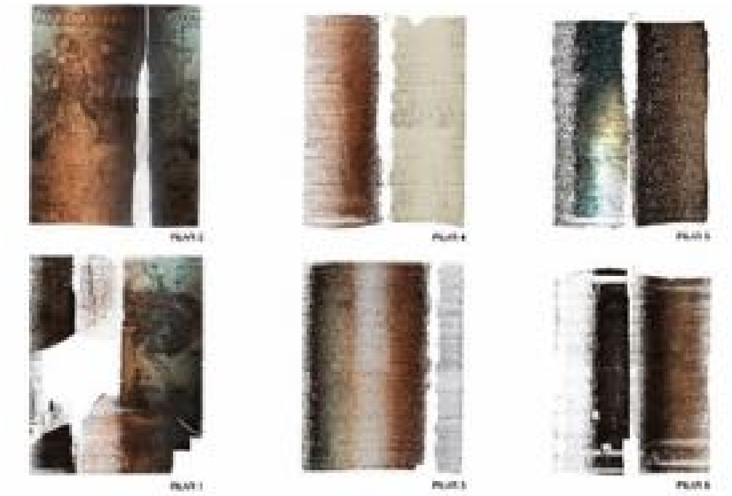

Figure 3. Visualization of pillars' joints with software Cyclone

The greatest displacement is found in pillar P_1, with a range of displacement on each row of [0.270, 0.001]. That displacement is followed by that in pillar P_3, with a range of [0.190, 0.001]. The range of displacements of the rest of the pillars is as follows: P_2 [0.108; 0.002], P_4 [0.109; 0.002], P_5 [0.101; 0.001], and finally, the least deformed pillar, P_6 [0.065; 0.001]. Thus, pillars P_1, P_3 and P_5 have greater deformations than the others. None of the regression planes P_{ri} is perpendicular to the axis of the central vault. Each one is gently sloped. The angle in P_1 is 85.466º, so (ω_{φ}) < 90º (Figure 4). It is also the most inclined pillar (0.270 m) and is the highest (4.170 m). The rest of the pillars have angles (ω_{φ}) > 90º, with a range [103.893º - 126.169º]: P_2 [103,893º], P_3 [108.665º], P_4 [117.245º], P_5 [112.066º] and P_6 [126.169º].

4 DATA TREATMENT WITH THE PLUG-IN UNDET FOR GOOGLE SKETCHUP

With the plug-in Undet for Google SketchUp (2019) the identification of pillar's joints was made directly with the image of the software. Undet is a plug-in that combine individual scan stations into groups and point clouds from a wide range of scanners, in this case, we have used the data that we extract from the Massive Data Capture (MDC) of the Leica ScanStation P20 scanners. What we have experienced with this plug-in is that the management of the point clouds is more efficient and quicker than with the software Cyclone.

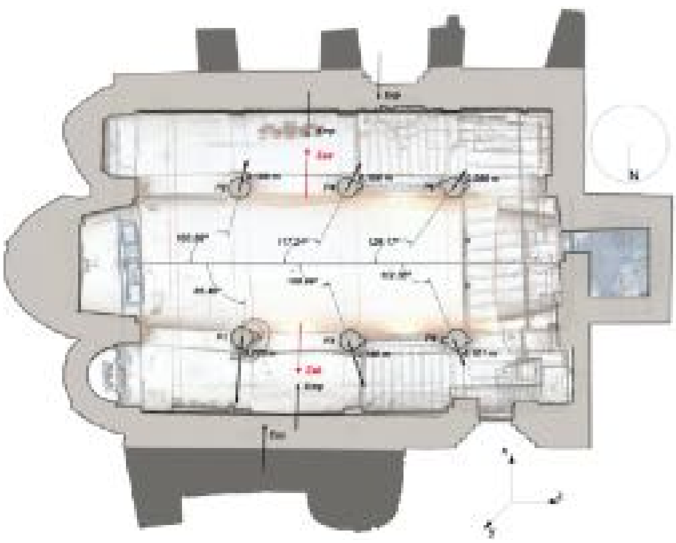

Figure 4. Characterization of the regression planes Pri from the software Cyclone (2016) [35]

Undet for SketchUp (2019) has an interactive colouring and density management, so it let us to adjust transparency, change point cloud and point size and, the most important thing for our research, it we are able to see the point cloud coloured by planes or heights. This function let us to see pillar's joints in detail, so, from now on we will be able to check the manual measurement that we previously have done and to say that the joints measured with Undet range from [0.007 m] to [0.015 m] of width. (Figure 5).

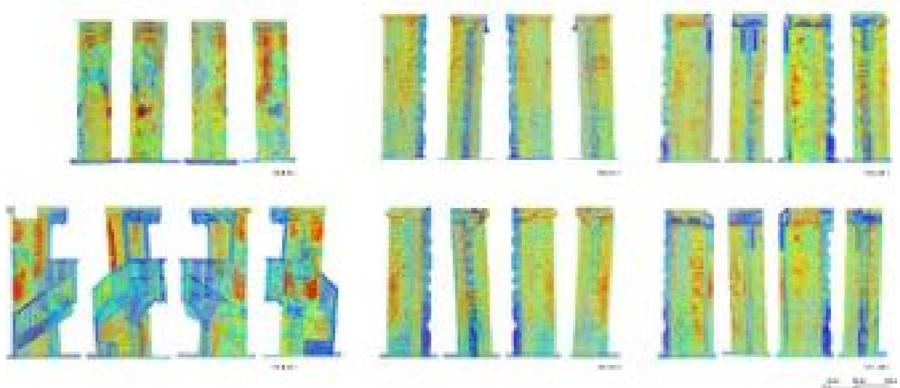

Figure 5. Visualization of pillars' joints with plug-in Undet SketchUp (2019)

We have also analysed the regression planes P_{ri} from the pillars, which, like the previous results obtained from the software Cyclone, they are also not perpendicular to the axis of the central vault. We have observed that there are little differences between the other results and these, being the deformation of P_2 the biggest difference [2.599º] less and the one of P_3 the smallest

[0.003º] less. The angle in P_1 is 85.474º, so (ω_{φ}) < 90º (Figure 6). It is also the most inclined pillar [0.206 m]. The rest of the pillars have angles (ω_{φ}) > 90º, as we have previously seen, with a range [101.291º - 124.756º]: P_2 [103,893º], P_3 [108.657º], P_4 [119.417º], [P_5 124.756º] and P_6 [110.553º]. We have also seen that the deformations of P_4 and P_5 are biggest in the middle of pillars' height than in the top of it. With Cyclone we didn't found that deformations.

5 DISCUSSION

The obtained results from both methods used are very similar to determine deformations in Romanesque masonry buildings, where the interest of the displacements is more qualitative than the quantitative order magnitude. The graphic precision of the plug-in Undet for Google SketchUp (2019) should be noticed, due to it is much higher and it allows to identify the points that each point that composes the point cloud extract from the Terrestrial Laser Scanner (TLS). To work with this plug-in there is no need to have a computer with a lot of graphic capacity, seeing that you can section the point cloud in few seconds. As for its precision, in a section of [0.005 m] the error is about [0.006 m] (0.030%), unlike the one obtained from the software Cyclone (2016), which was of [0.03 m].

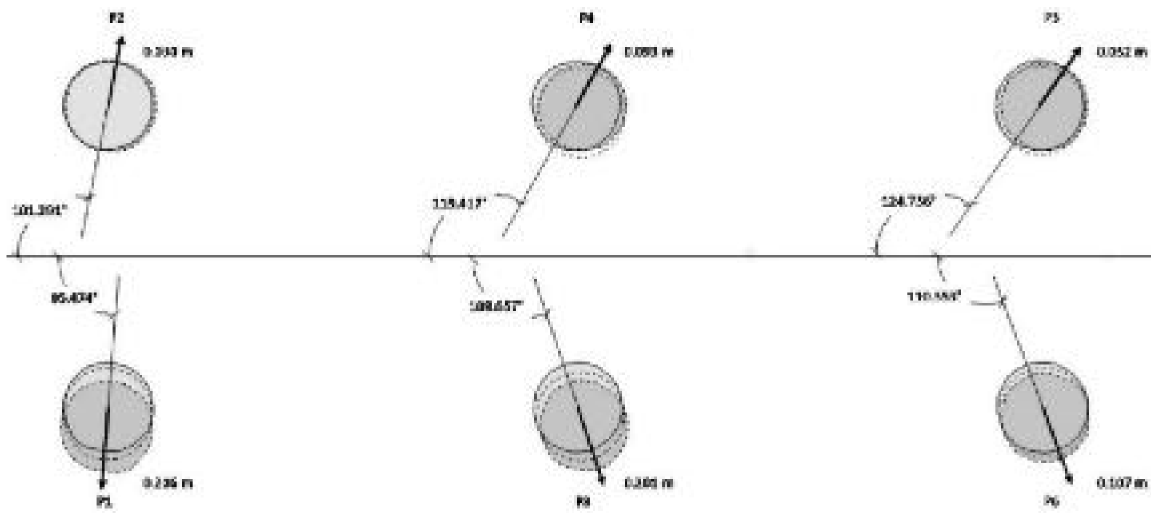

Figure 6. Characterization of the regression planes Pri from the plugin Undet (2019)

6 CONCLUSIONS

The obtained regression planes P_{ri}, which contain the deformations df_p $(df_{px}, df_{py}, df_{pz})$ for each pillar, tend to have the direction of the thrust over the pillar P_i. These displacements are the result of the active thrusts of vaults (E_{ba}) and the passive thrusts of the buttressing system, walls (E_{mp}) and buttresses (E_{mc}). This study revealed that the direction of the displacements of the six pillars P_i is not perpendicular to the central axis of the church φ_1 since (ω_{φ}) ≠ 90º. This results proves the hypothesis that the thrusts of the vaults are not perpendicular to the axis of the church, as was the case in Roman vaults, which Choisy (1873) [27] defined with regular geometry and stone cutting. The direction of displacements is caused by the irregular geometry of the vaults of Santa Maria de Arties as well as the masonry stone cutting and the above mentioned passive thrusts of the walls and buttresses. The last ones were placed to maintain equilibrium during the last millennium.

The displacement of the five pillars ($P_2 \ldots P_5$), where (ω_φ) > 90º, tends to the opposite façade of the apse. In addition, pillars P_5 and P_6, built during twelfth century on that façade, are the least deformed pillars because of two subsequent transformations: the construction of the bell tower over the centre of the façade (XIII-XIV) and the wood choir (XVIII). These elements have a stiffening function.

Pillar P_1 is the most deformed pillar of Santa Maria de Arties and has (ω_φ) < 90º over the main axis. The displacement tends to the apse. This pillar, together with pillar P_3, where (ω_φ) > 90º, have achieved a great balancing through the passive thrust of the walls (E_{mp}) and buttresses (E_{mc}). For a specific weight of more than 24 kN/m^3, the buttressing system weighs 3,144.96 kN. Here is where anti-funicular shapes, inverted, arches, have appeared, so: ff''(x) > 0.

Pillars deformations' df_p $(df_{px}, df_{py}, df_{pz})$ tend to have the same direction of the thrust over pillars P_i, so, regression planes P_{ri}, which contain these deformations, are essential to define any intervention over these masonry buildings, because they show the direction of possible preventive actions.

Otherwise, the vectorization of the displacements of each row, deduced from the regression plane, makes it possible to parametrize the leaning of each pillar. Second, three deformation modes were identified, so the displacements of the pillars are not uniform. Moreover, on pillars (P_2, P_3, P_5), the displacements are variable and appear to be negative displacements on (x) in relation to the vertical, and the deformation of pillars (P_4, P_5) is biggest in the middle of their height tan in the extremes of pillars.

The ranges of heights where great deformations occur have also been identified, which are: P_1: [1.50 – 2.00 m], P_2 [3.00 – 3.50 m], P_3 [2.00 – 3.00 m], P_4 [1.50 – 2.50 m], P_5 [2.00 – 3.00 m] and P_6 [1.00 – 2.50 m]. There is a difference between the height of the base pillars too, since the inclination of the church floor, being P_1 and P_2 at the same height from their base centre, P_3 is at the same height of P_4, but they begin [0.047 m] upper than P_1 and P_2 and, finally, P_5 is [0.0577 m] upper than P_1 and [0.019 m] lower than P_6. These points are extremely important in order to determine the appropriate actions that needs to be taken for intervention on these buildings and preserve this Romanesque architectural heritage.

REFERENCES

[1] Español, F. (2011). El Pirineu desvetllat: Viatges i descoberta del patrimoni medieval. Annals Del Centre d'Estudis Comarcals Del Ripollès, 7, 13–36.
[2] Granell, E., & Ramon, A. (2006). Lluís Domènech i Montaner. Viatges per l'arquitectura romànica catalana. Barcelona: Colegio de Arquitectos de Cataluña, España.
[3] Alcolea Blanch, S. (2008). La missió arqueológica del 1907 als Pirineus. Exhibition catalog. Barcelona.

[4] Puig i Cadafalch, J. (1908). Les iglesies romàniques amb cobertes de fusta de les Valls De Bohí y d'Aran. Anuari de l'Institut d'Estudis Catalans, MCMVII, 119–136.
[5] Bassegoda, Nonell, J. (1974). Estética expresionista de la arquitectura románica lombarda en Catalunya como consecuencia de las deformaciones formales y estructurales de las iglesias. In De Re Restauratoria (Vol. Vol II, pp. 33–64). Barcelona: Catedra Gaudi UPB.
[6] Saez. (1976). Primera fase del proyecto de restauración de la Iglesia de Santa María de Artíes en el Valle de Arán (Lérida). Cuadernos de Arquitectura Y Urbanismo, 116, 63–71.
[7] Villanueva, J. L. (1974). "Las bóvedas de cañón." In De Re Restauratoria Vol. II, 145–158). Barcelona : UPB.
[8] Polo, C. (2009). Santa Maria d'Arties (Naut Aran, Val d'Aran): Un exemple integrau a nivèu de restauracion d'un monument a compdar des entresenhes arqueologiques. Cicle de Conferències Patrimoni Arqueològic I Arquitectònic a Les Terres de Lleida 2009, Departament de Cultura I Mitjans de Comunicacó.p. 75–103.
[9] Kadobayashi, R., Kochi, N., Otani, H., & Furukawa, R. (2004). Comparison and evaluation of laser scanning and photogrammetry and their combined use for digital recording of cultural heritage. Congress of ISPRS, Istanbul 12-23 July. International Archives of Photogrammetry and Remote Sensign, vol. XXXV, part B5, p. 401-406.
[10] Yilmaz, H. M., Yakar, M., Gulec, S. a., & Dulgerler, O. N. (2007). Importance of digital close-range photogrammetry in documentation of cultural heritage. Journal of Cultural Heritage, 8(4), 428–433. http://doi.org/10.1016/j.culher.2007.07.004
[11] Grussenmeyer, P., Landes, T., Voegtle, T., & Ringle, K. (2008). Comparison methods of terrestrial laser scanning, photogrammetry and tacheometry data for recording of cultural heritage buildings. International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences, 37 (B5), 213–218.
[12] Vendrell, M., Giráldez, P., González, R., Cavallé, F., & Roca, P. (2008). Santa Maria del Mar. Estudi històric-constructiu, materials de construcció i estabilitat estructural. Barcelona: UB-Patrimoni, Veclus, UPC.
[13] Dhonju, H., Xiao, W., Shakya, B., Mills, J., & Sarhosis, V. (2017). Documentation of Heritage Structures Through Geo-Crowdsourcing and Web-Mapping. ISPRS Geospatial Week 2017.
[14] Dhonju, H., Xiao, W., V. Sarhosis, V., Mills, J. P., Wilkinson, S.; Wang, L. Thapa, U. S. P. (2017). Feasibility study of low-cost image-based heritage documentation in Nepal. Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci., XLII-2/W3, XLII-2, 237–242. http://doi.org/https://doi.org/10.5194/isprs-archives-XLII-2-W3-237-2017, 2017.
[15] Benito, M. A. (2011). La Catedral de Ávila: Evolución constructiva y análisis estructural. PhD thesis. Escuela Técnica Superior de Arquitectura de Madrid. Universidad Politécnica de Madrid.
[16] Sternberg, H. (2006). Deformation measurements at historical buildings with terrestrial laserscanners. IAPRS Volume XXXVI, Part 5, Dresden 25-27 September. p. 303–308.
[17] Quagliarini, E., Clini, P., & Ripanti, M. (2016). Fast, low cost and safe methodology for the assessment of the state of conservation of historical buildings from 3D laser scanning: The case study of Santa Maria in Portonovo (Italy). Journal of Cultural Heritage. https://doi.org/10.1016/j.culher.2016.10.006
[18] Bonali, E., Pesci, A., Casula, G., & Boschi, E. (2014). Deformation of Ancient Buildings inferred by Terrestrial Laser Scanning methodology: The Cantalovo church case study

(Northern Italy). Archaeometry, 56(4), 703–716. http://doi.org/10.1111/arcm.12028

[19] Pesci, A., Bonali, E., Galli, C., & Boschi, E. (2012). Laser scanning and digital imaging for the investigation of an ancient building: Palazzo d'Accursio study case (Bologna, Italy). Journal of Cultural Heritage, 13(2), 215–220. http://doi.org/10.1016/j.culher.2011.09.004.

[20] Lluis i Ginovart, J., Costa-Jover, A., Coll-Pla S., & Puche Fontanilles, J. M. (2016). Techniques of massive data capture: A comparative approach for the assessment of complex spaces in architectural heritatge. Revista de la Contrucción, 15(1), 42–50.

[21] Costa-Jover, A., Lluis i Ginovart, J., Fortuny-Anguera, G., & Coll-Pla, S. (2014). Large deformations on a XII Century Romanic church at Val d'Aran (Spain). 9th International Masonry Conference 2014 in Guimarães. Book of Abstracts, 221.

[22] Costa Jover, A., & Coll Pla, S. (2017). Mecánica de bóvedas de obra de fábrica en las Iglesias del Valle de Arán. Actas Del Décimo Congreso Nacional y Segundo Congreso Internacional Hispanoamericano de Historia de La Construcción, 1.

[23] Lluis i Ginovart, J., Costa-Jover A., Coll-Pla, S. (2016). La determinación del concepto de homogeneidad versus resistencia en los pilares de una catedral gótica mediante técnicas no invasivas. Informes de la Construcción. Julio-septiembre 2016. Vol. 68, 543, 155. doi: http://dx.doi.org/10.3989/ic.15.119.

[24] Willis, R. (1835). Remarks on the Architecture of the Middle Ages, especially of Italy., Cambridge: Pitt Press.

[25] Willis, R. (1842). On the Construction of the Vaults of the Middle Ages. Transactions of the Royal Institute British Architects, 1: 3-13.

[26] Viollet-le-Duc, E. E. (1868). Dictionaire raisonné de l'architecture française du XIe au XVIe siècle (Vol. III). Paris: B. Bance (A. Morel).

[27] Choisy, A. (1873). L'Art de batir chez les Romains. Paris : Ducher et Cie. Retrieved from http://www.academia.edu/9953727/AugusteChoisy

[28] Choisy, A. (1883). L'art de batir chez les Byzantins. Paris : Librairies de la Sociètè Anonyme de Publications Périodiques.

[29] Puig i Cadafalch, J. (1901). HIstoria General del Arte. Escrita y editada en vista de los monumentos y de las mejores obras publicadas hasta el día, bajo la dirección dels arquitecto Don José Puig i Cadafach. Barcelona: Montaner y Simón Editores.

[30] Puig i Cadafalch, J., et alt. (1918). L'Arquitectura Romànica a Catalunya. Vol III. Barcelona: Institut d'Estudis Catalans.

[31] Puig i Cadafalch, J. (1920). L'Arquitectura Romànica a Catalunya. Barcelona: Ricard Durant i Alsina.

[32] Torres Balbás, L. (1946). Bóvedas romanas sobre arcos de resalto. Archivo Español de Arqueología, 64: 217-262.

[33] Bassegoda Musté, B. (1944). Racionalismo a ultranza en la arquitectura ojival. Separata Memorias de la Real Academia de Ciencias y Artes de Barcelona. Tercera época, núm. 504, vol. XXVII, núm. 4. Barcelona: Real Academia de Ciencias y Artes de Barcelona.

[34] Bassegoda Nonell, J. (1977). Bóvedas medievales a la romana. Separata Memorias de la Real Academia de Ciencias y Artes de Barcelona. Tercera época núm. 782. Vol. XLIII Núm. 8: 287-382. Barcelona: Real Academia de Ciencias y Artes de Barcelona.

[35] Lluis i Ginovart, J. Coll-Pla, S., Costa-Jover, A., López-Piquer (2017). Evaluation of large deformations on Romanesque masonry pillars: The case of Santa María de Arties http://dx.doi.org/10.7764/rdlc.16.3.468.