

# IMPLEMENTATION OF BAYESIAN MODEL UPDATING IN FIVE-STORY BUILDING

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## ABSTRACT

Numerical modeling of structures often involves simplifications and theoretical assumptions, which may lead to decreased accuracy in simulation results. Model updating techniques have been developed to address this issue by minimizing the discrepancy between experimental data and the modeled structure output. Traditional deterministic approaches aim to obtain a single best estimate for each parameter but fail to capture the full range of variations due to the complexity of mechanical behavior and uncertainties in structural models.

This work presents the application of Bayesian Inference in the parametric updating of a five-story building model and the assessment of its associated uncertainty. A major advantage of this approach is the consideration of uncertainty in the model input parameters, leading to a more accurate representation of the building's actual behavior. The Bayesian framework is employed to update model parameters using experimental data, specifically modal frequencies and mode shapes obtained from a full-scale reinforced concrete building tested on the NEES-UCSD shake table at the University of California, San Diego, USA. In the implementation, it was incorporated an iterative updating approach for the covariance matrix, which provides a more comprehensive understanding of the correlations among modal parameters compared to the traditional use of an identity matrix. Additionally, there is a discussion about the implications of Bayesian modeling, highlighting the importance and consequences of employing a multivariate normal likelihood function in the analysis.

The results demonstrate that the Bayesian model updating approach enables a statistically rigorous fitting of model parameters, facilitating the characterization of uncertainty and enhancing confidence in the model's predictions. This methodology proves particularly valuable in engineering applications where precise model accuracy is paramount. Overall, this study showcases the significance of incorporating Bayesian inference in the parametric updating of structural models, highlighting its ability to account for uncertainties and improve the accuracy of predictions. The findings contribute to the broader field of structural analysis and offer practical insights for engineers and researchers seeking to enhance the reliability and robustness of numerical models.

## PURPOSE OF THE STUDY

This work introduces the implementation of Bayesian Inference for the purpose of parametric updating in structural models, with the aim of capturing the inherent variability present in real-world structural systems and experimental data.

The specific objective is to employ experimental data collected from a full-scale five-story reinforced concrete building subjected to rigorous testing at the University of California, San Diego, to update the input resistance parameters. A comprehensive series of dynamic tests, including Ambient Vibration Tests (AVT), was conducted at the building. These tests were carried out both during the construction phase of the test building and during subsequent base excitation testing phases to characterize the system. In a previous study by Astroza et al. (2016), AVT data were utilized to identify the natural frequencies and damping ratios associated with ten structural mode shapes. From these ten mode shapes, our study focuses on the first three modes and their corresponding frequencies as determined by SSI-DATA. It is noteworthy that Astroza, et al. (2016) conducted a study leveraging the Modal Assurance Criterion (MAC), demonstrating that the mode shapes identified by each method exhibited no significant differences, suggesting that alternative methods would have yielded comparable results.

Uncertainty in modeling complex systems may be categorized as either epistemic or aleatory. Epistemic uncertainty results from a lack of knowledge and may be mitigated through improvements in data and modeling. In contrast, aleatory uncertainty arises from intrinsic variability or randomness and is inherently irreducible. In this study, we aim to quantify the epistemic uncertainty associated with the identified modal data.

## APPLIED METHODOLOGY

In this study, our primary focus is to reduce epistemic errors by incorporating Bayesian inference (BI) into structural models. BI relies on Bayes' theorem, and its integration into structural models is formulated as follows:

$$P(\theta|D, M_j) \cong P(\theta, M_j) \cdot P(\theta|M_j)$$

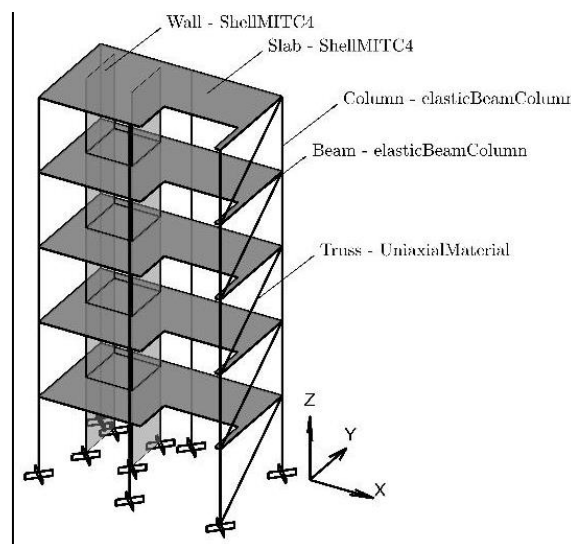
where  $P(\theta|D, M_j)$  represents the posterior Probability Density Function (PDF) of the parameter vector  $\theta$ , given the evidence  $P(D)$ , for the model  $M_j$ . This approach, dealing with an unnormalized posterior distribution, aims to maintain computational efficiency (Beck & Katafygiotis, 1998).

The structural model was created using Opeenseespy, a Python library for constructing finite element (FE) models without a graphical user interface, reducing processing time. The model's representation is displayed in Fig. 1. Following this, a sensitivity analysis is carried out to identify and prioritize parameters that exert the most significant influence on the model's response. These prioritized parameters guide the subsequent updating process. The next step involves establishing a prior distribution for the Bayesian model, which represents our initial understanding of the parameters.

As the next step, defining the likelihood function as a multivariate normal function proves valuable for capturing authentic correlations among output model parameters. To facilitate posterior sampling, we leverage cloud computing resources, specifically Google Cloud's

infrastructure, to provide the necessary computational power. Techniques like Monte Carlo are employed to generate samples. To ensure convergence, we establish convergence criteria and assess goodness-of-fit through various statistical measures and visual comparisons. The convergence of the Markov chains is verified using two methods: the evolution of Effective Sample Sizes (ESS) and the assessment of the Markov Chain Standard Error (MCSE). Finally, a posterior predictive check is executed, enabling the replication of new observations or the generation of synthetic data based on the experimental data.

In the field of Structural Health Monitoring (SHM) and damage identification in civil structures, this Bayesian approach, akin to the framework introduced by Astroza et al. (2017), is employed. This framework utilizes input excitation and dynamic response data from the structure to update a nonlinear FE model. The updated FE model by Bayesian techniques may subsequently be used to detect, localize, classify, and quantify damage, as well as predict the remaining useful life of the structure. While this framework can be applied to various structural and geotechnical systems, different types of FE modeling and analysis techniques, and different loads (e.g., static, quasi-static, time-dependent, dynamic), Model validation studies primarily focused on simple yet realistic nonlinear frame-type structural models with linear geometry and material nonlinearities, considering earthquakes as potential damaging events.



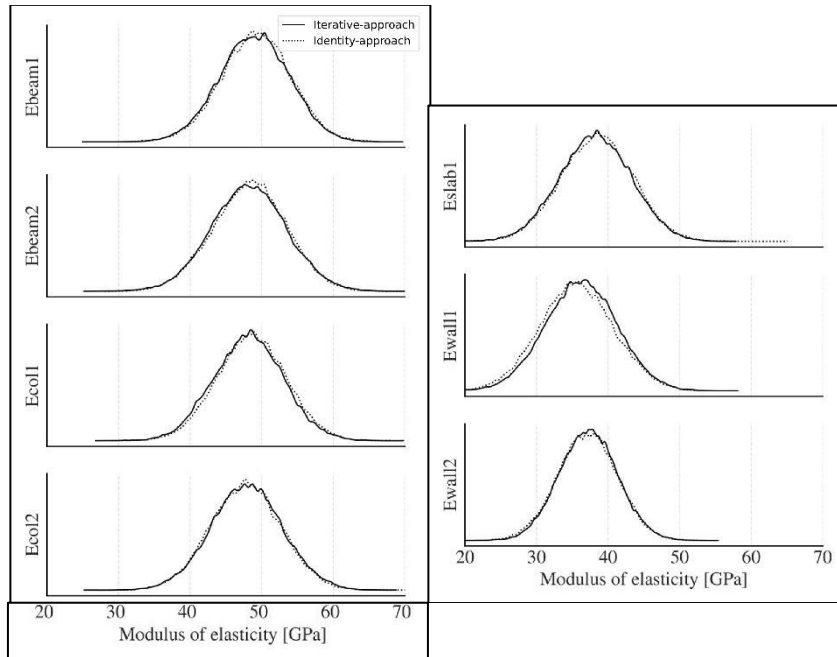
**Figure 1.** FE model developed in Openseespy (Hurtado, 2023).

## RESULTS

As mentioned earlier, two distinct approaches were employed for handling the covariance matrix within the likelihood function: the iterative approach, involving the calculation of the covariance matrix at each iteration, and the identity approach, utilizing an identity matrix as the covariance matrix. Fig. 2 illustrates the final distribution for each parameter obtained using both approaches. It visually represents the Probability Density Function (PDF) generated by each method, enabling a comparative analysis of the results. Table 1 provides the primary statistical parameters of the updated parameters. The mean values reveal two distinct groups: one centered around 48 GPa, corresponding to the modulus of elasticity for beams and columns, and another group with a modulus of elasticity around 37 GPa, relating to slabs and walls. By implementing a modification in the source code of the BI library, the preservation of the covariance matrix for each sample becomes feasible.

Fig. 3 presents the results obtained from the iterative updating of the covariance matrix, offering a graphical representation of correlations between the experimental parameters. In this context, it demonstrates how correlated the frequencies associated with the first three

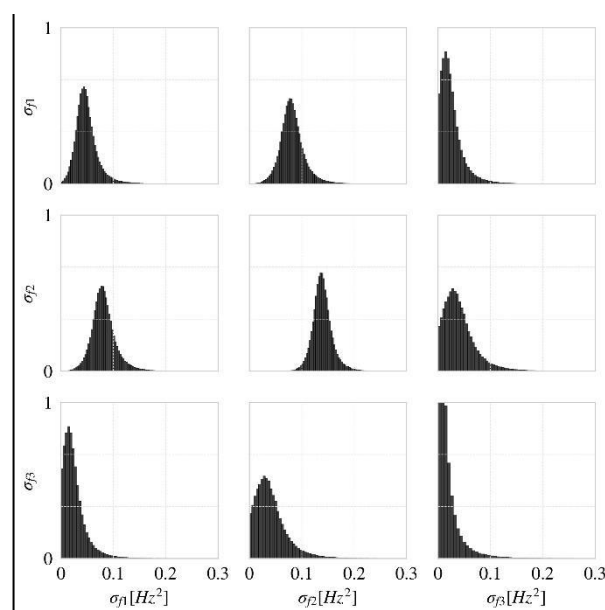
shape modes are. This information is significant in the field of SHM as it provides insights into how damage may affect a structure's seismic performance in terms of resonance avoidance by estimating only one shape mode.



**Figure 2.** Posterior probability density function and tracing values (Hurtado, 2023).

**Table 1.** Summary of posterior distribution of model parameters. Iterative-approach (Hurtado, 2023).

Element Group	Mean [GPa]	Std. Dev. [GPa]	HDI 3% [GPa]	HDI 97% [GPa]
Beam 1	48.9	4.9	39.7	58.0
Beam 2	48.2	5.3	38.4	58.2
Column 1	48.1	4.9	38.9	57.2
Column 2	47.9	5.0	38.6	57.7
Slab 1	38.2	5.0	28.7	47.5
Wall 1	36.2	5.0	28.7	45.7
Wall 2	37.2	4.1	29.5	44.8



**Figure 3.** First three rows and columns on the Covariance matrix representing the first three frequencies of the building (Hurtado, 2023).

## CONCLUSIONS

This study introduced a methodology for enhancing the precision of finite element (FE) structural models by employing Bayesian inference. Modal properties obtained from a full-scale five-story reinforced concrete (RC) building were utilized to update the model parameters. The study elucidated the process of quantifying parametric uncertainty, achieved through a multivariate normal likelihood function, outlining the essential role of the updating algorithm in computing posterior probabilities, and determining the covariance matrix of observations. Interestingly, it was observed that two distinct approaches for integrating the covariance matrix, the identity-approach, and the iterative-approach, produced similar posterior distributions and convergence rates, offering a practical advantage in terms of computational efficiency.

The resulting updates of posterior parameters revealed smaller standard deviations in comparison to the prior distributions, signifying reduced errors in both model and parameter values. Furthermore, the study highlights the importance of assessing the reliability of structural models, particularly in the context of mode shapes, which are crucial in SHM for understanding structural behavior and identifying potential damage. It underscored the significance of quantifying the uncertainty associated with frequency, a task accomplished through Posterior Predictive Checks and compared against experimental data. In summary, this methodology shows great potential for enhancing the reliability of structural models, thereby providing valuable support for decision-making in structural design and assessment.

## BIBLIOGRAFÍA

Argyris, C., Papadimitriou, C., Panetsos, P., & Tsopelas, P. (2020). Bayesian model-updating using features of modal data: Application to the metsovo bridge. *J. Sens. Actuator Netw.*, 27.

Armstrong, C. (1994). Modelling requirements for finite-element analysis. *s. Comput.-Aided Des*, 573–578.

Astroza, R., Ebrahimian, H., Conte, J.P., Restrepo, J.I., and Hutchinson, T.C. (2016). System identification of a full-scale five-story reinforced concrete building tested on the NEES-UCSD shake table. *Structural Control and Health Monitoring*, 23(3), 535–559.

Astroza, R., Ebrahimian, H., Conte, J.P., Restrepo, J.I., and Hutchinson, T.C. (2016). Influence of the construction process and nonstructural components on the modal properties of a five-story building. *Earthquake Engineering & Structural Dynamics*, 45(7), 1063–1084.

Beck, J., & Katafygiotis, L. (1998). Updating models and their uncertainties. I: Bayesian statistical framework. *J. Eng. Mech.-Pro ASCE*, 455–462.

Chen, M., Pantoli, E., Wang, X., Astroza, R., Ebrahimian, H., Hutchinson, T., . . . et al. (2016). Full-scale structural and nonstructural building system performance during earthquakes: Part I—specimen description test protocol, and structural response. *Earthq. Spectra*, 737–770.

Ghurye, S., & Olkin, I. (1962). A characterization of the multivariate normal distribution. *Ann. Math. Stat.*, 533-541.

Ebrahimian, H., Astroza, R., Conte, J.P., and de Callafon, R.A. (2017) Nonlinear finite element model updating for damage identification of civil structures using batch Bayesian estimation. *Mechanical Systems and Signal Processing*, 194-222.

Gutierrez, M. (2020). Bayesian Finite Element Model Parameter and Covariance Estimation. Master's Thesis, Universidad de los Andes.

Hurtado, O. D. (2023). Bayesian Model-Updating Implementation in a Five-Story Building. *Buildings*, 1568.

Kampenes, V., Dybå, T., Hannay, J., & Sjøberg, D. (2007). A systematic review of effect size in software engineering experiments. . *Inf. Softw. Technol.*, 1073–1086.

Pantoli, E., Chen, M., Hutchinson, T., Astroza, R., Conte, J., Ebrahimian, H., . . . Wang, X. (2016). Landmark data set from the building nonstructural components and systems (BNCS) project. *Earthq. Spectra*, 1239–1259.

Sehgal, S., & Kumar, H. (2016). Structural dynamic model updating techniques: A state of the art review. *Arch. Comput. Methods Eng*, 515–533.

Tong, Y. (2012). *The Multivariate Normal Distribution*; Springer Science & Business Media: Berlin/Heidelberg.

Yan, W., & Katafygiotis, L. (2015). A novel Bayesian approach for structural model updating utilizing statistical modal information from multiple setups. *Struct. Saf*, 260–271.