

Active control of building structures under measured seismic loads

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ABSTRACT

The concept of active control has been introduced in recent years in order to reduce the seismic response of structures. A predictive control methodology is used to formulate a discrete-time structural control algorithm. This algorithm computes the control forces by using not only the information provided by the structural response but also that given by the measurement of the seismic ground acceleration. Results for checking the effectiveness of the control algorithm in reducing the structural response are presented.

INTRODUCTION

The fulfilment of appropriate requirements for the seismic analysis and design of structures¹ permits the construction of buildings capable of resisting the effects of strong ground motions within a reasonable safety range. This safety range is restricted to the pre-selected design earthquake or earthquakes with similar characteristics. Moreover, even in that case in which the seismic event is similar to the one considered in the design, the structure could reach a high vibration level and suffer damages, with the subsequent need for repairs. Consequently, one of the objectives of structural engineers is to reduce the response of structures subjected to seismic loads.

As a possible solution for the reduction of the structural response, a new research field using active control techniques has been developed in recent years^{2,3}. Active control systems use the information available provided by the structural response, such as displacements, velocities or accelerations, to manipulate the values of a set of control forces according to a control methodology. The structural response is measured by means of a set of sensors placed in specified points in the structure. A computer receives the information provided by the sensors and calculates the values for the control forces. These forces are applied to the structure by means of appropriate actuators.

The measuring of the response by means of sensors is usual in the experimental dynamics of structures. The aspect dealing with the design of actuators is one of the difficulties in the practical implementation of structural control systems. Almost all the research carried out up to now in the field of structural control is related to the development of the control methodology; but the problems relating to the design of actuators have also begun to be studied recently^{4,5}. As regards the control methodology, the optimum control theory has been the one most invoked for the active control of structures subjected to seismic actions^{6,7}. According to the concepts

of optimum control, the values for the control forces are computed by minimizing a linear quadratic performance index. This operation requires the solution of a matrix Riccati equation with the consequent computational effort. Therefore, this method may be prohibitive in the case of structures with many degrees of freedom, such as tall buildings. In order to reduce the dimension of the matrix Riccati equation, the modal decoupling of the equations of motion has been used⁸. In this case some independent specific modes of vibration of the structure are controlled by using an optimum control law. Within the modal decoupling concept^{9,10} the seismic excitation characteristics are considered to be known and the time evolution of the ground acceleration and velocity is used in the computation of the control forces.

Recently a new methodology for structural control has been proposed¹¹, based on the predictive control concept. From a practical point of view, the mathematical model used in the computation of the control forces has been formulated in discrete-time, which permits a direct implementation on a digital computer.

Different control algorithms can be developed within the predictive control method. An algorithm has been described¹² and applied to reducing the seismic response of a building structure by using a limited number of control forces. The effectiveness of another algorithm in controlling the structural response has been shown¹³ to be un-influenced by the frequency of the seismic excitation. The algorithms used in References 11–13 have considered the seismic excitation to be an unknown perturbation and therefore it has not been included in the computation of the control forces.

In this paper a predictive control algorithm is developed which takes into account the possibility of measuring the ground motion acceleration and uses this information in the computation of the control forces. Its effect on the controlled response is tested by comparing different numerical results.

CONTROLLED STRUCTURAL RESPONSE

Consider a building structure modelled as an n -degrees-of-freedom lumped mass system and subjected to a horizontal ground acceleration $a(t)$. The equations describing its horizontal motion can be written in the form:

$$M\ddot{\mathbf{d}} + C\dot{\mathbf{d}} + K\mathbf{d} = \mathbf{f} - M\mathbf{j}a(t) \quad (1)$$

where M , C and K are, respectively, the $n \times n$ mass, damping and stiffness matrices; \mathbf{d} , $\dot{\mathbf{d}}$ and $\ddot{\mathbf{d}}$ are, respectively, the $n \times 1$ displacement, velocity and acceleration vectors relative to the ground and \mathbf{j} is the $n \times 1$ identity vector. \mathbf{f} is the control force vector of dimension $n \times 1$. Its component f_j is the control force applied in the horizontal direction by an actuator placed on the j th floor. If the number of actuators is $r \leq n$, it can be written:

$$\mathbf{F} = \mathbf{L}\mathbf{u} \quad (2)$$

where \mathbf{u} is a $r \times 1$ vector whose components are the control forces supplied by the actuators mentioned. \mathbf{L} is an $n \times r$ matrix whose elements are equal either to zero or to one depending on the presence or absence of an actuator on the different floors.

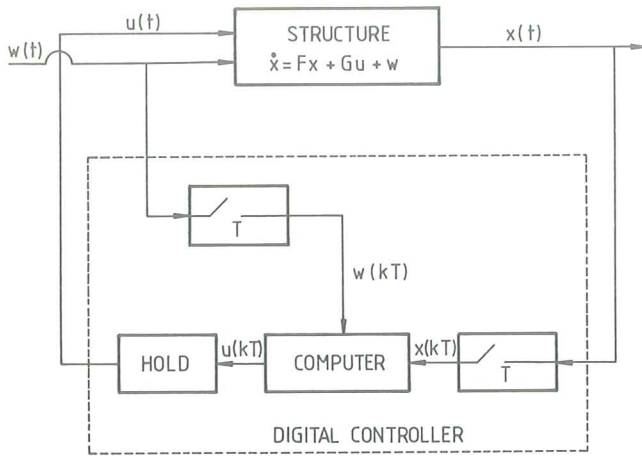


Figure 1 Closed-loop digital control scheme

Defining a $2n \times 1$ state vector x as:

$$x = \begin{Bmatrix} d \\ \dot{d} \end{Bmatrix} \quad (3)$$

and using (2), (1) can be written in the form:

$$\dot{x} = Fx + Gu + w \quad (4)$$

where F is the uncontrolled system matrix of dimension $2n \times 2n$ defined as:

$$F = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad (5)$$

G is the $2n \times r$ control matrix

$$G = \begin{bmatrix} 0 \\ M^{-1}L \end{bmatrix} \quad (6)$$

and w is a $2n \times 1$ excitation vector representing the seismic load defined as:

$$w = \begin{Bmatrix} 0 \\ -j \end{Bmatrix} a(t) \quad (7)$$

Equation (4) describes the continuous time evolution of the controlled structural response in a compact general form.

In order to perform the control operation, the closed-loop scheme of Figure 1 can be considered. At each sampling instant kT (T is the sampling period and k an integer), a set of sensors is used to measure the displacement and velocity response defining the vector $x(kT)$. Another sensor can be placed on the basement of the building to measure the ground acceleration which defines the excitation vector $w(kT)$. The digital computer calculates the value of the control vector $u(kT)$ by means of a given algorithm. A hold device converts the sequence of consecutive vectors $u(kT)$ into a continuous time control action $u(t)$, which is applied to the structure by means of a set of suitable actuators.

Given the discrete-time nature of the diagram of Figure 1, it is useful to formulate a discrete-time mathematical model in order to relate the sampled state vector $x(kT)$ to the control vector sequence $u(kT)$ and the sampled excitation vector $w(kT)$. In order to do this, consider the analytical solution of (4) which, for the initial condition $x(t_0) = x_0$, can be expressed in the form:

$$x(t) = e^{F(t-t_0)}x_0 + \int_{t_0}^t e^{F(t-\tau)}[Gu(\tau) + w(\tau)] d\tau \quad (8)$$

where the exponential matrix is defined as:

$$e^{F(t-t_0)} = I + F(t-t_0) + \frac{F^2(t-t_0)^2}{2!} + \dots \quad (9)$$

By considering (8) between two consecutive sampling instants (making $t_0 = kT$ and $t = kT + T$), it can be written:

$$x(kT + T) = e^{FT}x(kT) + \int_{kT}^{kT+T} e^{F(kT+T-\tau)}[Gu(\tau) + w(\tau)] d\tau \quad (10)$$

The analytical calculation of the integral involved in (10) requires knowing the continuous time evolution of $u(\tau)$ and $w(\tau)$ over the whole sampling interval $[kT, kT + T]$. The form of $u(\tau)$ depends on the type of hold considered. Usually a zero-order hold is used in digital control, which converts the sequence $u(kT)$ into a piecewise function $u(\tau)$ keeping the value of $u(kT)$ constant between kT and $kT + T$. For the excitation, some criterion has to be considered for simulating $w(\tau)$ from its sampled values. A convenient criterion may be, for example, to assume $w(\tau)$ has a linear variation between kT and $kT + T$. In this case, as has been shown in Reference 14, (10) is finally converted into:

$$x(kT + T) = Ax(kT) + Bu(kT) + P_1w(kT + T) + P_2[w(kT + T) - w(kT)] \quad (11)$$

where A , P_1 and P_2 are $2n \times 2n$ matrices and B a $2n \times r$ matrix defined by:

$$A = e^{FT} \quad (12)$$

$$P_1 = \int_0^T e^{F\mu} d\mu \quad (13)$$

$$P_2 = -\frac{1}{T} \int_0^T e^{F\mu} \mu d\mu \quad (14)$$

$$B = P_1G \quad (15)$$

An algorithm for an efficient computation of these matrices has been developed in Reference 14.

A simpler criterion for simulating $w(\tau)$ may consist of assuming $w(\tau)$ to be equal to $w(kT)$ on $[kT, kT + T]$ following the same idea of a zero-order hold device. In this case the discrete-time equation (11) has the form:

$$x(kT + T) = Ax(kT) + Bu(kT) + P_1w(kT) \quad (16)$$

This simplified formulation does not essentially influence the structural response if a small enough sampling period is used.

PREDICTIVE CONTROL STRATEGY

The control strategy on which the predictive control method is based consists of applying a set of control forces to the structure, at each sampling instant, in such a way that the predicted structural response be equal to a desired one. The practical application of this strategy implies a prior selection of the response variables to be controlled. In order to develop a controller according to this concept, the block scheme of Figure 2 can be considered. This scheme includes:

Driver block. At each sampling instant, the driver block takes into account the final desired state of the

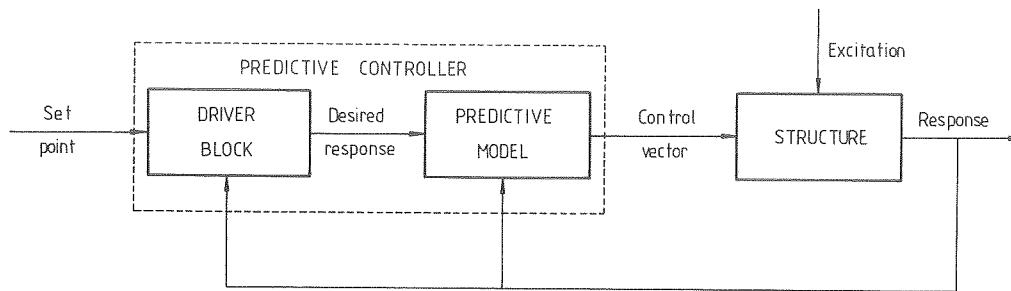


Figure 2 Block scheme of the predictive control method

structure, called set point (generally the equilibrium state of the structure), to define a desired trajectory of the structural response. For example, this trajectory may start from the current structural response and reach the set point according to chosen dynamics. Then the desired structural response, at each sampling instant, is selected from among the points along this trajectory.

Predictive model. At each sampling instant, the predictive model is used to generate a control force vector that makes the predicted structural response equal to the desired one given by the driver block.

This strategy for structural control has been proposed¹¹ and is a subset of an adaptive-predictive control methodology¹⁵⁻¹⁷ used in different control applications¹⁸.

Different control algorithms can be developed, according to the mentioned strategy, depending on the selection of the response variables to be controlled, the definition of the predictive model and the formulation of the characteristics of the driver block¹¹⁻¹³. In all these references the predictive model did not explicitly include the seismic excitation, since this was considered an unknown perturbation. The algorithm developed herein takes into account the measured ground acceleration in the prediction of the structural response.

CONTROL ALGORITHM

The predictive model is considered to be of the following form:

$$\hat{x}(k+1|k) = \mathbf{A}x(k) + \mathbf{B}u(k) + \mathbf{P}_1 w(k) \quad (17)$$

where for notation convenience k denotes the time instant kT and $x(k+1|k)$ is the value for the state vector predicted at instant k for the next instant $k+1$. It can be observed that the model (17) has the same form as (16), but the model is redefined at each instant k , using the measured values for the state vector $x(k)$ and for the seismic excitation vector $w(k)$. With reference to the vector $w(k)$, this model considers a zero-order approximation, as explained previously. This fact permits the prediction of the response for instant $k+1$ by using the excitation measured at the current time instant k . Thus it is not necessary to predict the future time evolution of the seismic ground acceleration based on hypotheses made on its statistical characteristics as is necessary in the optimum control schemes^{9,10}.

The response to be controlled, expressed by a discrete-time vector $y(k)$, can be written as a function of the state vector $x(k)$ in the form:

$$y(k) = \mathbf{H}x(k) \quad (18)$$

where \mathbf{H} is called response matrix. In this case the vector y contains the n horizontal floor displacements, being thus equal to vector \mathbf{d} ; consequently the $n \times 2n$ matrix \mathbf{H} takes the form:

$$\mathbf{H} = [\mathbf{I} \quad \mathbf{0}] \quad (19)$$

According to (17) and (18), the predicted response can be expressed as:

$$\hat{y}(k+1|k) = \mathbf{H}\mathbf{A}x(k) + \mathbf{H}\mathbf{B}u(k) + \mathbf{H}\mathbf{P}_1 w(k) \quad (20)$$

The control vector can be obtained by imposing the condition that the predicted response equal a desired one $y_d(k+1)$, that is¹²:

$$\hat{y}(k+1|k) - y_d(k+1) = \mathbf{0} \quad (21)$$

(21) will now be applied in a generalized form by imposing the condition that the square of the weighted norm of the difference between the predicted and desired responses expressed by:

$$S = [\hat{y}(k+1|k) - y_d(k+1)]^T \times \mathbf{Q}[\hat{y}(k+1|k) - y_d(k+1)] \quad (22)$$

take a minimum value¹³. \mathbf{Q} is an $n \times n$ positive semi-definite weighting matrix.

The control vector $u(k)$ which minimizes S is obtained from the following condition on the gradient of S :

$$\frac{\partial S}{\partial u(k)} = \mathbf{0} \quad (23)$$

By substituting (20) into (22) and by using (23) one has:

$$\mathbf{B}^T \mathbf{H}^T \mathbf{Q} [\mathbf{H}\mathbf{A}x(k) + \mathbf{H}\mathbf{B}u(k) + \mathbf{H}\mathbf{P}_1 w(k) - y_d(k+1)] = \mathbf{0} \quad (24)$$

The control vector $u(k)$ is finally deduced from (24), resulting in:

$$u(k) = \mathbf{R}y_d(k+1) - \mathbf{R}\mathbf{H}\mathbf{A}x(k) - \mathbf{R}\mathbf{H}\mathbf{P}_1 w(k) \quad (25)$$

\mathbf{R} being the $r \times n$ matrix:

$$\mathbf{R} = [\mathbf{B}^T \mathbf{H}^T \mathbf{Q} \mathbf{H} \mathbf{B}]^{-1} \mathbf{B}^T \mathbf{H}^T \mathbf{Q} \quad (26)$$

Equation (25) is the control law which permits the computation of the control forces, at each instant k , by using the measured values for the state vector $x(k)$ and the excitation vector $w(k)$ as well as the desired response vector $y_d(k+1)$ provided by the driver block. The driver block is designed to select the desired response belonging to a desired trajectory defined by:

$$y_d(k+1) = \theta_1 y(k) + \theta_2 y(k-1) \quad (27)$$

This trajectory is redefined, at each sampling instant k ,

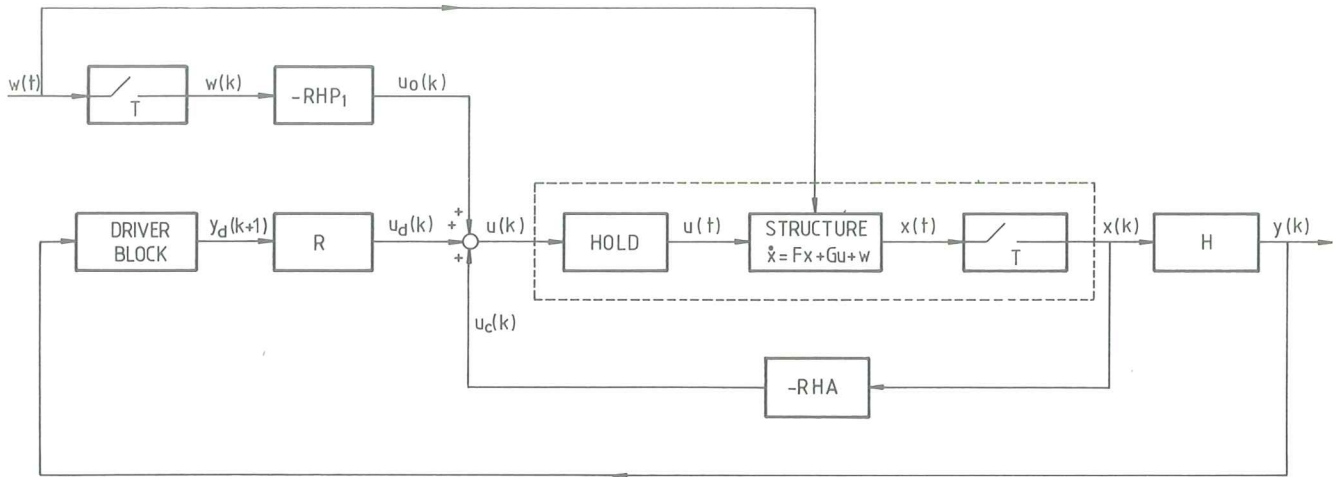


Figure 3 Block diagram of the control law

starting from the previous and the present structural responses. θ_1 and θ_2 are matrices selected in such a way that the trajectory approaches the set point, which in this case is the zero displacement response, with chosen dynamics.

Some decisions prior to the practical application of the control law have to be taken. These decisions refer to:

- (a) the number of the actuators considered to generate the control forces and their allocation, which defines the matrix L of (2);
- (b) the structural response to be controlled, which defines the matrix H and the vector $y(k)$ of (18);
- (c) the sampling period;
- (d) the weighting matrix Q ;
- (e) the parameters defining the desired trajectory of (27).

The practical implementation of the control algorithm requires the application of the following steps at each sampling instant k :

- (1) calculation of the desired response vector $y_d(k+1)$ by means of (27);
- (2) computation of the control force vector $u(k)$ by using (25).

DISCUSSION ON THE CONTROL LAW

Some remarks on the control law described by (25) are made in this section. This equation can be represented by the block diagram of Figure 3, in which the control vector is expanded into the following three terms:

$$u(k) = u_d(k) + u_c(k) + u_o(k) \quad (28)$$

where

$$u_d(k) = Ry_d(k+1) \quad (29a)$$

$$u_c(k) = -RHAx(k) \quad (29b)$$

$$u_o(k) = -RHP_1 w(k) \quad (29c)$$

The term $u_d(k)$ corresponds to the desired response $y_d(k+1)$. The term $u_c(k)$ is obtained by multiplying the state vector by the matrix $-RHA$ which can be referred to as gain matrix. While $u_d(k)$ and $u_c(k)$ represent closed-loop control actions, $u_o(k)$ is an open-loop control action based on the measurement of the excitation vector $w(k)$.

The concept of gain matrix is general in the design of a control law for a system represented by a state equation¹⁹.

Depending on the methodology used in the development of the controller, the gain matrix takes different forms. In the optimum control methods, a Riccati equation has to be solved to evaluate the gain matrix⁶⁻¹⁰. In the pole placement methods, the gain matrix is selected in such a way that the roots of the closed-loop characteristic equation have *a priori* specified values^{20,21}. The gain matrix $-RHA$ which results from the proposed predictive algorithm is calculated in a simpler way by using (26). In those cases in which the excitation is considered in the formulation of an optimum control scheme^{9,10}, the future complete time history of the ground acceleration has to be predicted prior to starting the control algorithm since this time history has to be used in the computation of the control forces. The way in which the predictive algorithm proposed herein is formulated allows the inclusion of the earthquake acceleration measured online at each time instant.

The use of a desired response, which gives the term $u_d(k)$ in (28), is one specific point in the predictive control concept. The design of the driver block based on (27) is one particular case within a more general design procedure based on optimization criteria^{22,23}.

ILLUSTRATIVE EXAMPLE

The 14 degrees-of-freedom building structure pictured in Figure 4a has been used as an application example of the control algorithm. It is subjected to a horizontal seismic ground acceleration and is modelled by the lumped mass system given in Figure 4b. Its horizontal motion is described by (1), in which the mass matrix M is diagonal:

$$M = \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & m_{14} \end{bmatrix}$$

and the stiffness matrix K takes the form:

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 & \cdots & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & -k_{14} & -k_{14} \end{bmatrix}$$

The mass and stiffness characteristics of the structure are given in Table 1.

The damping matrix corresponds to a damping ratio of 0.05 for each mode of vibration, and is calculated by using a direct modal evaluation procedure²⁴.

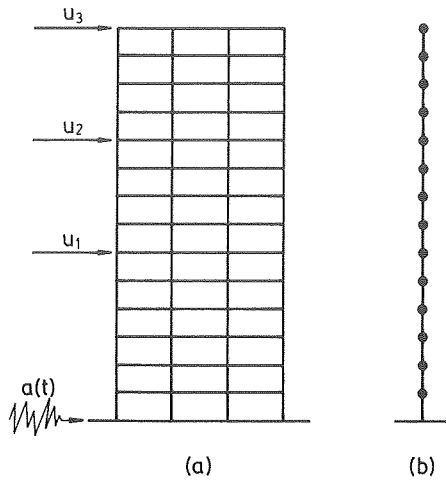


Figure 4 (a) Building structure; (b) dynamic model

Table 1 Mass and stiffness characteristics of the structure

Floor	Mass (N sec ² /cm)	Stiffness × 10 ³ (N/cm)
1	1280	2100
2	1280	1900
3	1280	1700
4	1130	1500
5	1130	1300
6	1130	1200
7	1010	1000
8	1010	900
9	890	900
10	890	750
11	760	750
12	760	470
13	690	470
14	690	300

A synthetic accelerogram generated by Ruiz and Penzien's method²⁵ has been used as seismic excitation of the structure. An acceleration record with a duration of 20 sec and an expected maximum acceleration of 0.25 g has been generated and filtered through a filter with a frequency of 2 Hz and a damping ratio of 0.3 (see Figure 5).

A set of numerical tests has been carried out by using a different number of control forces. At the same time the effect of the inclusion of the measured ground acceleration in the computation of the control forces has been analysed. Table 2 summarizes some of the results obtained. It includes the maximum values for the displacement response of each floor both for the controlled and the uncontrolled structure. Three cases have been distinguished for the controlled structure, corresponding respectively to the application of 14, 10 and 3 control forces. For each of these cases, computations have been performed considering the seismic excitation to be, respectively, an unknown perturbation (A columns) and a measured variable (B columns).

In the application of the control algorithm, the following characteristics have been considered:

- (1) the displacement response of each floor has been selected as the response to be controlled;
- (2) a sampling period of 0.05 sec has been fixed;
- (3) the weighting matrix **Q** has been chosen to be equal to the 14 × 14 identity matrix;

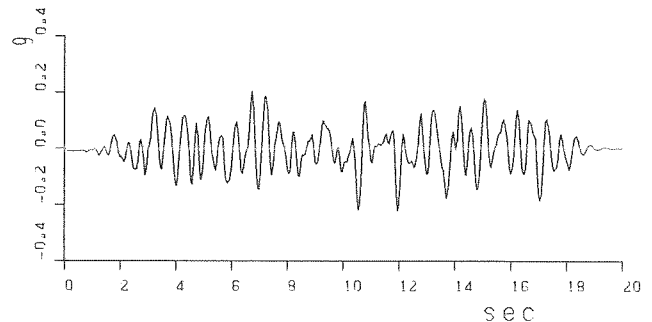


Figure 5 Artificial earthquake acceleration

Table 2 Summary of numerical results

Floors	Without control	Case 1 (14 control forces)				Case 2 (10 control forces)				Case 3 (3 control forces)			
		A		B		A		B		A		B	
		D	F	D	F	D	F	D	F	D	F	D	F
1	1.27	2.086	3148.16	0.54	684.36	0.68	—	0.60	—	0.73	—	0.87	—
2	2.68	2.786	1222.83	0.70	276.14	1.25	—	1.03	—	1.37	—	1.61	—
3	4.19	2.906	264.49	0.73	305.40	1.67	—	1.20	—	1.88	—	2.04	—
4	5.74	2.93	154.01	0.73	286.31	2.16	—	1.03	—	2.26	—	1.99	—
5	7.29	2.93	165.95	0.73	291.19	3.22	1941.23	0.70	1253.21	2.53	—	1.40	—
6	8.66	2.94	168.99	0.74	293.29	2.90	375.26	0.74	195.01	3.15	1807.40	0.68	2140.62
7	9.89	2.94	152.57	0.74	263.52	2.95	155.29	0.73	279.26	2.98	—	1.11	—
8	10.84	2.94	153.56	0.74	264.57	2.95	153.01	0.74	261.78	2.90	—	1.42	—
9	11.44	2.95	135.87	0.74	233.79	2.95	134.87	0.74	232.71	2.89	—	1.21	—
10	12.22	2.95	136.33	0.74	234.37	2.95	135.53	0.74	233.39	2.95	715.63	0.61	1385.57
11	13.09	2.95	116.67	0.74	200.48	2.95	116.11	0.74	199.67	3.01	—	1.24	—
12	15.22	2.95	116.93	0.74	200.86	2.95	116.43	0.74	200.13	3.31	—	2.00	—
13	17.05	2.95	106.31	0.74	182.58	2.96	105.90	0.75	181.94	3.25	—	1.89	—
14	18.76	2.95	106.41	0.75	182.74	2.96	106.03	0.75	182.13	2.99	311.52	0.66	564.33

D = maximum displacement (cm)
F = maximum control force (KN)

A = unknown ground acceleration
B = measured ground acceleration

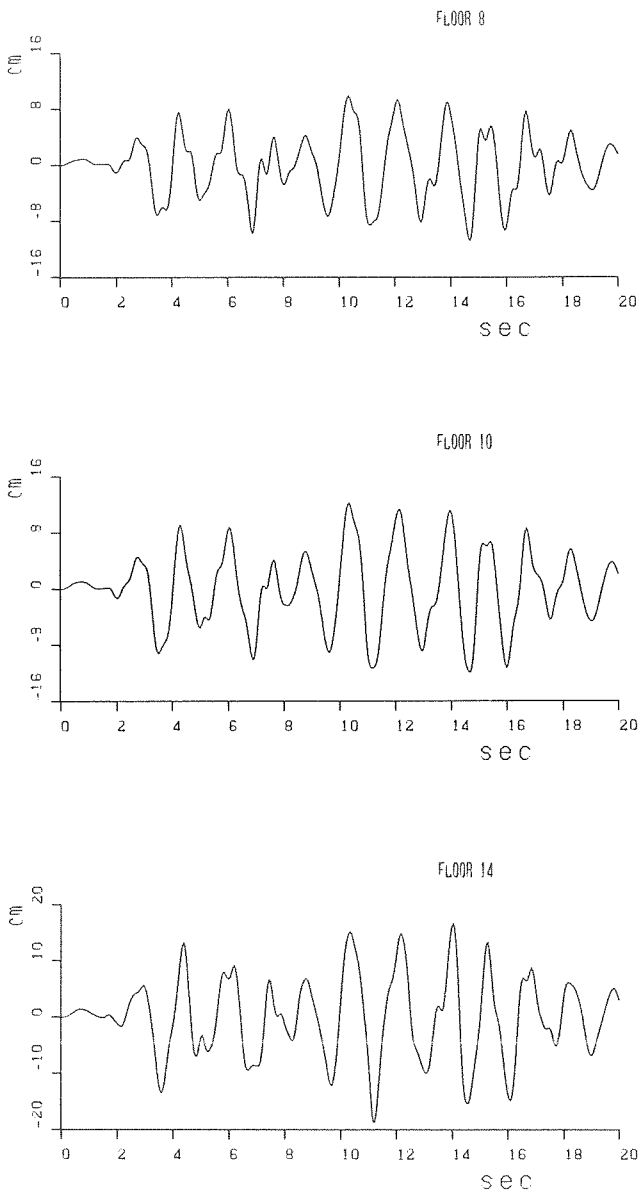


Figure 6 Uncontrolled seismic displacement response

- (4) the matrices θ_1 and θ_2 , defining the desired trajectory of (27) have been selected to be diagonal, corresponding to 14 scalar trajectories, one for each controlled displacement. The parameters of these trajectories correspond to a second-order model with unit damping coefficient and a natural frequency of 5 rad/sec.

A global comparison between the displacement responses corresponding to the three control cases given in Table 1 and that of the uncontrolled structure, shows a significant reduction in the response of the controlled structure. A comparison between the A columns of the three cases shows that a decrease in the number of control forces does not significantly influence the controlled response. In each case, by comparing column A with column B, it can be observed that the measuring of the ground acceleration and its use in the computation of the control forces provides a more notable reduction in the displacement response with an increase in the control forces acting on the majority of the floors.

Figure 6 gives the displacement time history of the structure without control, subjected to the seismic excita-

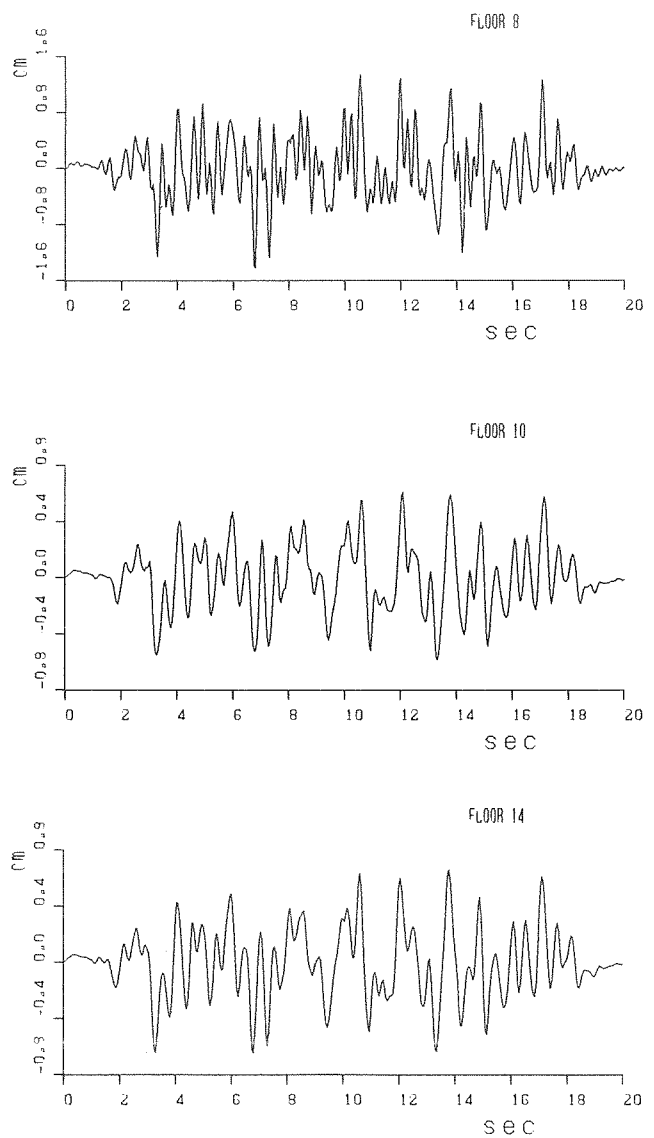


Figure 7 Controlled seismic displacement response

tion of Figure 5. For case 3B, which corresponds to the application of three control forces to the structure, these forces being computed by using the measured seismic acceleration, Figure 7 shows the controlled displacement response time history for floors 8, 10 and 14. In Figure 8 the time evolution of the corresponding control forces acting on floors 6, 8 and 14 are represented.

CONCLUSIONS

Within the predictive control methodology, a structural control algorithm has been developed. The computation of the control force vector is carried out by means of a control law including two closed-loop terms and one open-loop term. The closed-loop terms are related, respectively, to the state vector and to the desired response vector. The open-loop one uses the ground acceleration measured in real-time. The gain matrix is computed in a simple way without requiring either the solution of a matrix Riccati equation or a specific pole location. The control law has been formulated in discrete-time in order to implement it directly on a digital computer. Its effectiveness in significantly reducing the structural response has been numerically shown.

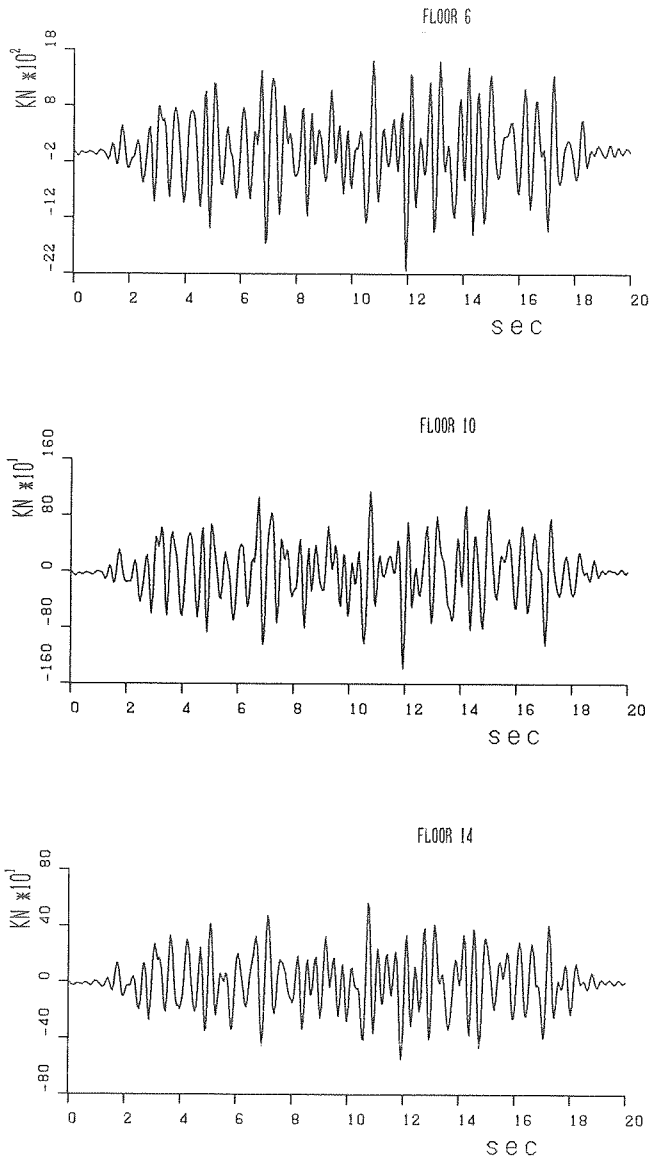


Figure 8 Control forces

REFERENCES

- 1 Newmark, N. M. and Rosenblueth, E. *Fundamentals of Earthquake Engineering*, Prentice-Hall, Englewood Cliffs (1971)
- 2 Zuk, W. Kinetic structures, *Civil. Eng. ASCE*, **38**(12), 62-64 (1968)
- 3 Yao, J. T. P. Concept of structural control, *J. Struct. Div. ASCE*, **98**(ST7), 1567-1574 (1972)

- 4 Safford, F. B. and Masri, S. F. Development and use of force pulse train generators, *Proc. Dynamic Response of Structures, Instrumentation, Testing Methods and System Identification, ASCE Specialty Conf., Atlanta*, 72-86 (1981)
- 5 Masri, S. F., Bekey, G. A. and Caughery, T. K. Optimum pulse control of flexible structures, *J. Appl. Mech.*, **48**, 619-626 (1981)
- 6 Yang, J. N. Application of optimal control theory to civil engineering structures, *J. Eng. Mech. Div. ASCE*, **101**(EM6), 819-838 (1975)
- 7 Wolf, J. P. and Madden, P. M. An assessment of the application of active control to reduce the seismic response of nuclear power plants, *Nuclear Eng. Design*, **66**, 383-397 (1981)
- 8 Meirovitch, L. and Silverberg, L. M. Control of structures subjected to seismic excitation, *Eng. Mech. Div. ASCE*, **109**(2), 604-618 (1983)
- 9 Yang, J. N. and Lin, M. J. Optimal open-loop critical-mode control of building under seismic load, *J. Eng. Mech. Div. ASCE*, **108**(EM6), 1167-1185 (1982)
- 10 Yang, J. N. and Lin, M. J. Building critical-mode control: non-stationary earthquake, *J. Eng. Mech. Div. ASCE*, **109**(6), 1375-1389 (1983)
- 11 Rodellar, J. and Martín Sánchez, J. M. Predictive structural control, *Proc. 2nd Int. Symp. Structural Control, Univ. Waterloo, Canada* (1985)
- 12 Rodellar, J. and Barbat, A. H. Digital control of structures subjected to seismic actions, *Proc. Numerical Methods in Engineering: Theory and Applications*, A. A. Balkema Publ., 281-290 (1985)
- 13 Rodellar, J. and Barbat, A. H. Predictive digital control of the seismic response of structures, *Rev. Int. Métodos Numéricos para Cálculo y Diseño en Ingeniería*, **1**, 53-66 (1985)
- 14 Rodellar, J. and Barbat, A. H. Numerical analysis of the seismic response. A state space approach, *Proc. Numerical Methods in Engineering: Theory and Applications*, A. A. Balkema Publ., 273-279 (1985)
- 15 Martín Sánchez, J. M. Contribution to model reference adaptive systems from hyperstability theory, *Doctoral Dissert.*, Universidad Politécnica de Cataluña, Barcelona (1974)
- 16 Martín Sánchez, J. M. A new solution to adaptive control, *Proc. IEEE*, **64**(8), 1209-1218 (1976)
- 17 Martín Sánchez, J. M. Adaptive predictive control method, USA Pat. No. 4,197,576 (August 4, 1976)
- 18 Martín Sánchez, J. M. and Shah, S. L. Multivariable adaptive-predictive control of a binary distillation column, *Automatica* **20**(5), 607-620 (1984)
- 19 Franklin, G. and Powell, J. D. *Digital Control of Dynamic Systems*, Addison-Wesley (1980)
- 20 Abdel-Rohman, M. and Leipholz, H. H. Structural control by pole assignment method, *J. Eng. Mech. Div. ASCE*, **104**(EM5), 1159-1175 (1978)
- 21 Abdel-Rohman, M. Active control of large structures, *J. Eng. Mech. Div. ASCE*, **108**(EM5), 719-730 (1982)
- 22 Martín Sánchez, J. M. Reply to 'Comments on a new solution to adaptive control', *Proc. Lett. Proc. IEEE* **65**, 587-588 (1977)
- 23 Rodellar, J. Optimal design of the driver block in the adaptive-predictive control system, *Doctoral Dissert.*, Universidad de Barcelona (1982)
- 24 Wilson, E. L. and Penzien, J. Evaluation of orthogonal damping matrices, *Int. J. Num. Meth. Eng.*, **4**, 5-10 (1972)
- 25 Ruiz, P. and Penzien, J. PSEQGN, artificial generation of earthquake accelerograms, *Report*, College of Engineering, University of California, Berkeley (1969)