Nonlinear Response of Structures with Sliding Base Isolation

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Abstract. It is accepted that sliding or friction base isolation systems effectively protect structures subjected to severe earthquakes. Precise modeling of the sliding connection is, however, still difficult since various parameters influence the friction coefficient. Simplified models, such as those based on a constant friction coefficient, are convenient to understand the behavior of sliding structures and to identify general trends. However, this article shows that for final structural designs (and in particular if the building enters in the nonlinear range) these trends should be used carefully. The purpose of the article is to discuss the general behavior of structures supported on sliding connections and to describe a simple numeric model to take into account variations in the friction coefficient. The numeric model enables to represent complex force-displacement relationships for the sliding bearings. The model is used to perform a preliminary evaluation of the influence in the response caused by changes in the friction coefficient for elastic and elasto-plastic structures. The variations in the friction coefficient are obtained from experimental testing for a class of teflon-based connections.

Key words: Passive control, Base isolation, Friction isolation, Sliding isolation, Hysteretic isolator

1 Introduction

Base isolation is generally divided in three basic categories: rubber base isolation, sliding or frictional base isolation and a combination of boths. Nowadays, rubber base isolation is the most commonly used base isolation technique, however, due to economic benefits, sliding base isolation systems have gained attention in the recent years (Mokha *et al.* 1988, Bozzo and Mahin 1990, Constantinou *et al.* 1990, Dorka 1994, Tsopelas and Constantinou 1994). Furthermore, the largest and heavier retrofitted building in the US, the San Francisco Court of appealings, is actually being protected using sliding isolation (Keowen *et al.* 1994).

This paper is divided in two parts. The first part describes the general behavior of structures supported on sliding connections. To clearly identify the influence of various structural, isolator and earthquake parameters on response, single degree of freedom elastic structures supported on ideal frictional base isolators are initially studied. Response for different parameters, like total and residual displacements, maximum accelerations and base shear coefficients is discussed. The simple idealization assumed to study the behavior of single degree of freedom sliding structures is, however, not convenient for final designs. The friction coefficient varies mainly as a function of the velocity and axial force during sliding. Previous research proposes rate dependent models which define explicit relations between the friction coefficient, the sliding velocity and the axial force (Mokha *et al.* 1988, Constantinou *et al.* 1990). For a general type of teflon materials, the model gives the friction coefficient as an exponential function of the sliding velocity and pressure. The approach followed in the present work is to perform several tests with the actual sliding connections obtaining sliding velocity-friction coefficient relationships. The tests were performed in the Richmond field station at the University of California at Berkeley. Consequently, rather to define a general rate dependent model, the tests provided velocity-friction coefficient curves representative for the actual teflon-based connection.

The second part of the article proposes a numeric integration scheme to perform the step by step nonlinear analysis considering the variation in the friction coefficient. The scheme is used to perform a preliminary parametric study of the influence on response due to changes in friction coefficient caused by velocity variations during an earthquake. The response of elastic and inelastic single degree of freedom structures supported on sliding connections is evaluated using the velocity dependent model and using the constant friction model.

Usually the base isolation of a new building seeks to maintain the structure in the linear elastic range. The response of old weak buildings or the response of new buildings subjected to extreme earthquakes may not be, necessarily, in the aforementioned ideal elastic range. The main objective of the article is to investigate the relationships between the variation in friction coefficient and the response of sliding base isolated structures, in particular for low strength buildings whose response may be in the nonlinear range. In the inelastic range the influence of the sliding velocity on the friction coefficient may modify significantly the behavior of these weak systems. This is an important aspect which is investigated in the article.

2 Behavior of Structures Supported on Sliding Connections

2.1 Structural Modeling

It has been observed that many base isolated structures, especially those that respond elastically, behave essentially as two degree of freedom systems: one for the structure and one for the isolator. Furthermore, in most cases the mass associated to the sliding degree of freedom is negligible compared to the total supported mass permiting its elimination from the model. This simplification allows parametric studies of the response of single degree of freedom (SDOF) systems enabling significant insights into seismic response (Bozzo and Mahin 1990).

Force-displacement characteristics for sliding connections are essentially rigid-plastic with the initial strength governed by the friction coefficient μ. The deformation hardening slope is characterized by the effective sliding lateral stiffness (K_c). This additional restoring force is provided by metalic springs or by the curvature of the sliding surface in the Frictional Pendulum System (FPS). For the present parametric study, the supported structure responds in the elastic range with a lateral stiffness K. Consequently, the stiffnesses of the connection and structure can be combined as an equivalent serial spring [Fig. 1(a)-(b)]. The resulting equivalent elasto-plastic spring initially has the elastic stiffness K of the structure. The stiffness for the sliding branch is $K_e = KK_c/(K + K_c)$. Thus, K_e and μ along with K and the total supported mass M fully characterize the mechanical properties of the equivalent system. This system can be simplified further in terms of μ, the period of the supported structure T and the equivalent strain hardening $s = K_e/K$, which can be expressed as

$$s = \frac{K_c}{K + K_c} = \frac{1}{1 + (T_c/T)^2} \tag{1}$$

where T_c is the period of a perfectly rigid structure sliding at the connection and given by $T_c = 2\pi\sqrt{M/K_c}$. For the FPS system, $T_c = 2\pi\sqrt{r/g}$ where r is the radius of curvature of the sliding surface. Thus T, T_c and μ become convenient parameters to characterize sliding systems.

To generalize seismic response calculations, the equation of motion for the system is normalized with respect to the peak ground acceleration $\ddot{u}_{g_{\max}}$. The normalized response parameters included in this parametric study are: the normalized total displacement $\lambda(t) = u(t)/\ddot{u}_{g_{\max}}$, the normalized sliding strength coefficient $\eta_{sl} = \mu g/\ddot{u}_{g_{\max}}$ and the normalized restoring force $\rho(t) = R(t)/R_{\mathrm{sl}}$ where $R_{\mathrm{sl}} = \mu M g = \mu W$. The resulting normalized equation of motion is

$$\ddot{\lambda}(t) + 2w\xi\dot{\lambda}(t) + \eta_{sl}\rho(t) = \ddot{u}_g(t)/\ddot{u}_{g_{\max}} \tag{2}$$

Apparent yielding of this equivalent system is related to sliding and not to structural yielding. The minimum normalized strength coefficient η_{st} required for the structure to remain elastic is defined by $\eta_{\mathrm{st}} = R_{\max}/(M\ddot{u}_{g_{\max}}) = Cg/\ddot{u}_{g_{\max}}$ (where C is the base shear coefficient) and can be related by the geometry of the hysteretic loop to the maximum displacement for the equivalent system as

$$\eta_{\mathrm{st}} = \frac{\lambda_{\max}(2\pi)^2 + \eta_{\mathrm{sl}}{T_c}^2}{T^2 + {T_c}^2} \tag{3}$$

Similarly the maximum normalized structural displacement $\lambda_{\mathrm{st}_{\max}}$ can be related to the maximum total displacement as

$$\lambda_{\mathrm{st}_{\max}} = \frac{\lambda_{\max} + \eta_{\mathrm{sl}}(T_c/(2\pi))^2}{1 + (T_c/T)^2} \tag{4}$$

Expressions (2),(3) and (4) permit some general observations to be developed regarding the behavior of sliding structures. For example, Newmark and Hall (1982) have noted that, with the exception of short period structures, maximum lateral displacements of elastic and inelastic systems are similar and tend to increase with increasing structural periods. If we accept this observation, equation (3) indicates that for a system with a given connection period, period of the structure and damping coefficient, the minimum strength coefficient required for

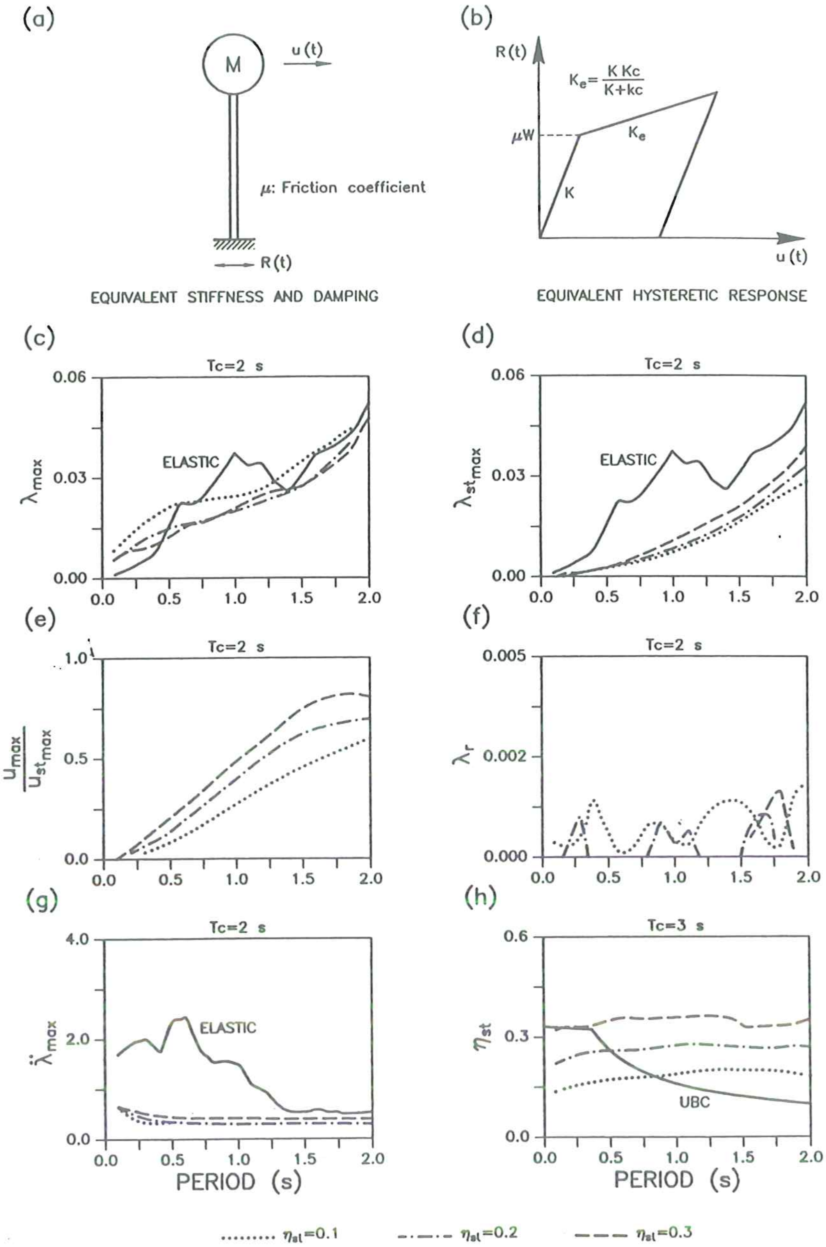


Fig. 1. (a) SDOF system. (b) Equivalent hysteretic response, K: structural stiffness, K_c: sliding stiffness, W: total weigth and μ: friction coefficient. (c) Normalized maximum system displacement ($\lambda_{\max} = u_{\max}/\ddot{u}_{g_{\max}}$). (d) Normalized maximum structural displacement ($\lambda_{st_{\max}} = u_{st_{\max}}/\ddot{u}_{g_{\max}}$). (e) Normalized structural to total displacement ratio ($u_{\max} = u_{st_{\max}}$). (f) Normalized residual displacement (λ_r). (g) Normalized maximum system acceleration ($\ddot{\lambda}_{\max} = \ddot{u}_{\max}/\ddot{u}_{g_{\max}}$). (h) Normalized average structural strength coefficient ($\eta_{st_{\max}} = R_{\max}/M\ddot{u}_{g_{\max}}$) for the ten earthquakes.

the structure to remain elastic is reduced as the sliding strength coefficient decreases. Similarly, from equation (4), it can be observed that, in general, as the period of the connection is increased, the structural displacement is reduced.

Using equation (2) a complete parametric study is performed considering elastic structures with periods ranging between 0.1s and 2.0s. Mass Proportional viscous damping equal to 5% of critical and connection periods of 2s and 3s were assumed. Ten different records, representative of moderate earthquakes on firm ground, were included in the study. However, due to space limits, the results presented in the sections 2.2-2.7 are usually for the El Centro NS 1940 record.

2.2 Maximum total displacements

The total system displacement, regardless of isolator strength, tends to approximate the displacement response for a similar elastic system provided the period of the structure is relatively long. This tendency is observed in Fig. 1(c) for structures with period larger than about 0.5s. In general, the total displacements increase with period. If only periods larger than about 0.5s are considered, the sliding strength coefficient does not influence results too much either. The influence of deformation hardening (or the period of the connection) is not significant, again for intermediate and long periods.

For short period structures there is an initial amplification region. In this region, if the sliding strength coefficient is reduced, total displacements are increased significantly. For example considering a structure with a fundamental period equal to 0.25s, if the sliding strength coefficient is reduced from 0.3 to 0.1, the normalized total displacement is increased from 0.0075 to 0.015. This is consistent with observations by Newmark and Hall that inelastic and elastic systems absorb the same amount of energy. It is proposed to estimate the maximum total displacement in this region by solving the following energy-balance equation:

$$s u_{\max}^2 + 2(1-s) u_{\mathrm{sl}} u_{\max} + u_{\mathrm{el}}^2 + (1-2s) u_{\mathrm{sl}}^2 = 0 \tag{5}$$

where u_{sl} is the displacement at first sliding $\mu g (T/2\pi)^2$ [in normalized terms $u_{\mathrm{sl}}/\ddot{u}_{g_{\max}} = \eta_{\mathrm{sl}}(T/2\pi)^2$]; s is the equivalent strain hardening and u_{el} is the displacement for a similar but linear-elastic structure.

Results presented in (Zayas *et al.* 1989) indicates that this approach can successfully be used to predict total displacements based on elastic response spectrum. Equations (3) and (4) can then be used to estimate structural strength requirements and displacements.

2.3 Maximum structural displacements

The maximum structural displacements tend to vary like a parabolic function of the structural period [Fig. 1(d)]. In general, its shape is smooth and similar for the different earthquakes considered. For short period structures, the displacement is very small. The maximum structural displacement for a structure supported on sliding connections is always smaller than the structural displacement for a similar structure, but without the connections.

If the sliding strength (or friction) coefficient is reduced, the maximum structural displacements are reduced. For example considering a structure with a fundamental period equal to 1s, if the sliding strength coefficient is reduced from 0.3 to 0.1, the structural displacement is reduced from 0.0102 to 0.0072. Similarly, if the period of the connection is increased, structural displacements are always reduced.

2.4 Ratio of structural to total displacements

The ratio of structural to total displacements indicates how much of the total displacements are sustained by the structure. The smaller the value of this ratio, the more effective the friction base isolation connection is in isolating the structure from earthquakes.

This ratio tends to vary like an increasing linear function of the structural period [Fig.1(e)]. For short period structures, the total displacement is mainly due to the sliding displacements. If the friction coefficient is reduced this ratio is reduced and its shape becomes more smooth. If the period of the connection is increased the ratio is always reduced.

2.5 Residual Displacements

The residual displacement of a friction base isolation structure is an important parameter because it provides information on the offset present in the connection after the earthquake. The structure will have zero residual displacement due to its assumed elastic behavior.

¿From Fig. 1(f), it can be observed that the residual displacements are small provided some sliding stiffness is developed by the system. Considering a peak ground acceleration equal to 0.4g and observing from the figure that the maximum normalized residual displacement is about 0.001, the maximum residual displacement for a sliding isolated structure with a connection period $T_c = 2$s is 0.4cm. The residual displacement is an irregular function of the structural period. It was also found to vary considerably between earthquake records. If no deformation hardening is provided, large residual displacements are possible. If the period of the connectice is reduced, residual displacements are usually reduced.

2.6 Maximum Total Acceleration

Total acceleration is an important parameter, as it is an indication of the shear developed in the structure. It also relates to the forces that can be imparted to attached equipment.

The maximum total acceleration tends to be a smooth decreasing function of the structural period [Fig. 1(g)]. For structural periods larger than about 0.6s it is almost constant. It is clear that the maximum total acceleration for structures supported on sliding connections is considerably reduced compared with the total acceleration for a similar structure without the connections. If the sliding strength coefficient is reduced or if the period of the connection is increased, the maximum total acceleration is always reduced. For example considering a structure with a fundamental period equal to 0.5s, if the sliding strength coefficient is reduced from 0.3 to 0.1, the normalized acceleration is reduced from 0.44 to 0.32.

2.7 Structural Strength Coefficient

By definition, the structural strength coefficient is the design force required for the structure to remain elastic. Figure 1(h) shows the mean structural strength coefficient for the ten earthquakes along with the normalized base shear required by the UBC (1991). The design base shear according to the UBC is

$$\eta_{st} = \frac{\alpha Z I C g}{R_w \ddot{u}_{g_{\max}}} \tag{6}$$

where $Z = 0.4$ (high risk), $I = 1$ (normal occupancy), R_w=12 (ductile moment resisting frame), $C = 1.25S/T^{2/3}$ (S = 1 for stiff local soil conditions). The factor α was taken to be 1.5, to account for likely minimum overstrength of UBC designed structures and the difference between working and ultimate strengths. The maximum ground acceleration level selected for the comparison was $\ddot{u}_{g_{\max}} = 0.4g$.

As it is shown in Fig. 1(h), the strength coefficient initially increases with structural period, but subsequently is mainly constant. Like trends for maximum total acceleration, increasing the period of the connection or reducing the sliding strength coefficient always reduces the required structural strength coefficient. For short period structures the forces developed in a structure supported on sliding connections can be substantially smaller than the forces required by the code. In addition, it should be recognized that the isolated structures remain elastic while the conventionally UBC designed structures may suffer substantial damage. For longer period structures, higher design forces are required compared to the UBC forces. However, it should be again recognized that the forces required shown for the sliding structures correspond to no yielding in the structure. Since conventionally designed structures are often substantially stronger than required, strength of structures supported on friction connections need not necessarily be increased over UBC design values. Moreover, ever weaker structures may be constructed if yielding is acceptable in the structure and if details are provided to supply the required structural ductility.

2.8 Torsional response

A very unique response characteristic of friction base isolation systems like the FPS one is that torsional response are negligible (Zayas *et al.* 87). This characteristic is because the center of mass and the center of resistance for sliding base isolated structures tend to coincide. For structures supported on FPS connectors, the center of sliding stiffness also coincides with the center of mass. As a result, friction isolated structures tend to minimize the undesirable effects of torsion.

3 Velocity Dependent Model and Numeric Implementation

3.1 Rate dependent model

All the observations and trends described previouslly were obtained assuming a constant friction coefficient. Experimental evidence indicates, however, that the friction coefficient changes considerably during an earthquake as a function of sliding velocity and axial force. Previous results reported by Constantinou *et al.* (1990) and by Mokha *et al.* (1988) suggest to model the friction coefficient as

$$\mu = f_{\max} - (f_{\max} - f_{\min})\exp(-a|\dot{u}_{\mathrm{sl}}|) \tag{7}$$

where $f_{\max}$ is the friction coefficient at high sliding velocities, $f_{\min}$ is the friction coefficient at velocities near zero, a is a parameter controlling the friction coefficient variation with velocity and $\dot{u}_{\mathrm{sl}}$ is the sliding velocity. The parameters involved in the expression are obtained for a given axial pressure at the sliding surface. This velocity dependent model implies that the friction coefficient is a monotonic increasing function of the sliding velocity, as it is shown in Fig. 2(a). The curve in the figure corresponds to a maximum friction coefficient $f_{\max} = 0.1$, a minimum friction coefficient $f_{\min} = 0.05$, and a parameter $a = 24$s/m corresponding to an axial pressure equal to 5MPa.

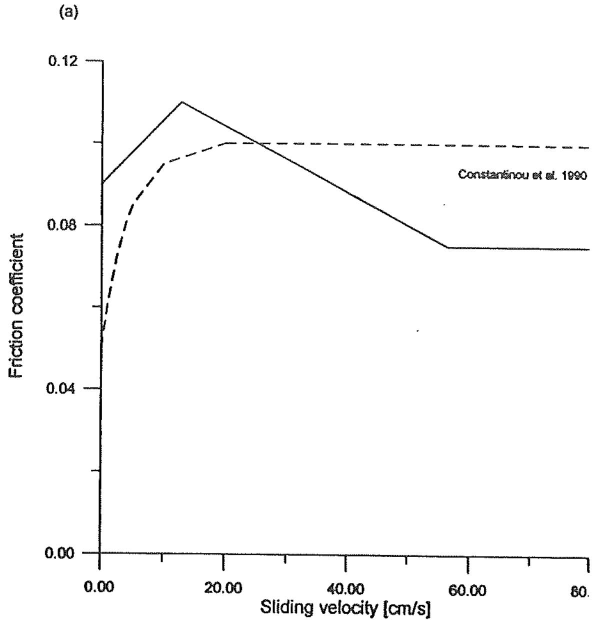
Experimental results obtained at the Richmond Field Station from U.C.Berkeley indicate that the dynamic friction coefficient can be represented as a function of the sliding velocity by a tri-linear model as illustrated also in Fig. 2(a). The proposed relationship was obtained testing individual teflon components (similar to those used in FPS connections). The friction coefficient initially increases with velocity getting a maximum at about 12.5cm/s. Afterward it decreases and it is almost constant for sliding velocities larger than 58cm/s. Consequently, the tests indicated that the relationship friction coefficient-sliding velocity is not necessarily monotonic. In general, the effect of an increment in the axial force is to reduce the friction coefficient. The vertical accelerations during an earthquake affect not only the axial force at the connection but also the friction coefficient.

3.2 Numeric model

The trilinear velocity dependent friction model has been implemented in a nonlinear computer program using the central difference numeric integration scheme. The program models the structure as a lumped mass shear building. The acceleration and velocity at time i are represented by:

$$\ddot{\mathrm{U}}_i = \frac{1}{\Delta t^2}[\mathrm{U}_{i+1} - 2\mathrm{U}_i + \mathrm{U}_{i-1}] \tag{8}$$

and



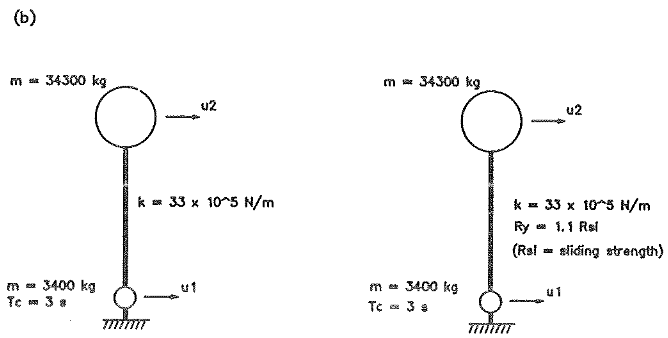


Fig. 2. **(a) Typical curve for the variation of the dynamic friction coefficient and the sliding velocity. (b) Isolated SDOF structures supported on sliding connections. Elastic and elasto-plastic structures.**

$$\dot{\mathrm{U}}_i = \frac{1}{2\Delta t}[\mathrm{U}_{i+1} - \mathrm{U}_{i-1}] \tag{9}$$

The dynamic equilibrium equation at time i for a nonlinear system is

$$\mathbf{M}\ddot{\mathbf{U}}_i + \mathbf{C}\dot{\mathbf{U}}_i + \mathbf{R}_i = -\mathbf{MJ}\ddot{u}_{g_i} \tag{10}$$

Where $\mathbf{R}_i$ is a vector representing the stories nonlinear restoring forces and $\mathbf{J}$ is the unit vector. The damping matrix $\mathbf{C}$ is obtained, as it is usual for base isolation analysis, considering only damping forces due to relative displacements of the structure respecting the sliding connection. The damping coefficient was fixed as 5% equivalent viscous damping.

If the restoring forces are not a function of velocity, the following equation gives the solution at time t_{i+1}:

$$\hat{\mathbf{M}}\mathbf{U}_{i+1} = \mathbf{P}_i \tag{11}$$

Where

$$\hat{\mathrm{M}} = [\frac{1}{\Delta t^2}\mathrm{M} + \frac{1}{2\Delta t}\mathrm{C}] \tag{12}$$

and

$$\mathbf{P}_i = -\mathbf{MJ}\ddot{u}_{g_i} + \frac{2}{\Delta t^2}\mathbf{MD}_i - \mathbf{R}_i + (\frac{1}{2\Delta t}\mathbf{C} - \frac{1}{\Delta t^2}\mathbf{M})\mathbf{D}_{i-1} \tag{13}$$

In this equation, the restoring forces at each step are obtained from the constitutive model.

The friction force at the sliding connection is a function of velocity which is a function of the displacement at t_{i+1}. Therefore, in general, for sliding structures iteration at each step is required. A simple and adequate procedure is to obtain the friction force using an estimate of the sliding velocity at the connection from the displacements at time $i-1$ and i as

$$\dot{\mathrm{U}}_i = \frac{1}{\Delta t}[\mathrm{U}_i - \mathrm{U}_{i-1}] \tag{14}$$

Using this estimate the restoring force at the sliding connection can be obtained from the following equation:

$$\mathrm{R}_i = \mu(\dot{u}_i)W\frac{\dot{u}_i}{|\dot{u}_i|} + K_c u_i \tag{15}$$

In this expresion the first term is a function of the velocity and the second one is a function of the restoring stiffness at the connection. The instantaneous friction coefficient is obtained from the rate dependent model. Clearly, the sliding velocity from the central difference will be different from the initially assumed and, consequently, the friction restoring force will be different too. Using the sliding velocity from the central difference, a new value for the friction force is obtained and iteration is set. The procedure converges after a few cycles thus far the friction coefficient is not precisely known and a relativelly large error in estimating the sliding velocity (and connection restoring force) may be allowed.

It is convenient to observe that using this procedure, it is relatively simple to include variations in the friction coefficient due to changes in axial forces. The unique modification is to include another trilinear model according to each axial force level. Thus at each time step the velocity and axial force at the connection are estimated.

3.3 Structural response

Using the rate dependent model and the proposed numeric method, this section presents a preliminary study on the response of sliding structures taking into account variations in the friction coefficient. In order to illustrate the rate dependent model and the resulting hysteretic response consider the structures in Fig. 2(b). The structural mass is 34300kg and the isolator mass is 3400kg. The structural stiffness is 33×10^5 N/m which corresponds to a 0.55s nonisolated period. The structural damping coefficient is 5%. Two cases were considered, one corresponds to an elastic structure and the other corresponds to an elasto-plastic structure. The assumed strength in the latest case is 10% greater than the strength required to start the sliding and thus $R_y = 1.1R_{\text{sl}} = 25 \times 10^3$ N. This case corresponds to a weak structure supported on sliding connections, such as a building being retrofitted. The connection period in both cases is 3s. The constant friction coefficient selected corresponds to the minimum value in Fig. 2(a) and the earthquake ground motion is the NS component of the El Centro 1940 register.

Figures 3 and 4 show the response for the linear elastic structure and the elasto-plastic structure, respectively. Each figure presents comparisons between the constant friction model and the velocity dependent model. Figure 3(a)-(d) indicates that the shear forces are not significantly varied between the constant and rate dependent models. For example, the maximum shear force in the structure for a constant friction model [Fig. 3(c)] is about 30×10^3 N and the similar value for the rate dependent model [Fig. 3(d)] is about 31×10^3 N. Figure 3(b) shows that, in general, for long sliding displacements the force decreases as compared to the constant friction model. This seems to be explained since long sliding diplacements are asociated to higher velocities compared to short reversals, and the velocity dependent model gives lower friction coefficients for larger velocities.

Figures 3(e)-(h) show that the time history of the structural displacements is composed of two different motions. One corresponds to short period oscillations —the structure— and the other corresponds to a much longer period oscillation —the isolator—. It can also be observed that the time history response for the constant friction model [Figs. 3(e) and 3(g)] and the rate dependent model [Figs. 3(f) and 3(h)] is very similar.

Figure 4 presents the response for the elasto-plastic system. In this case there is a significant difference in response between the constant and velocity dependent friction models. For example, the maximum structure interstory drift displacement for the constant friction model [Fig. 4(c)] is 1.25cm and the similar value for

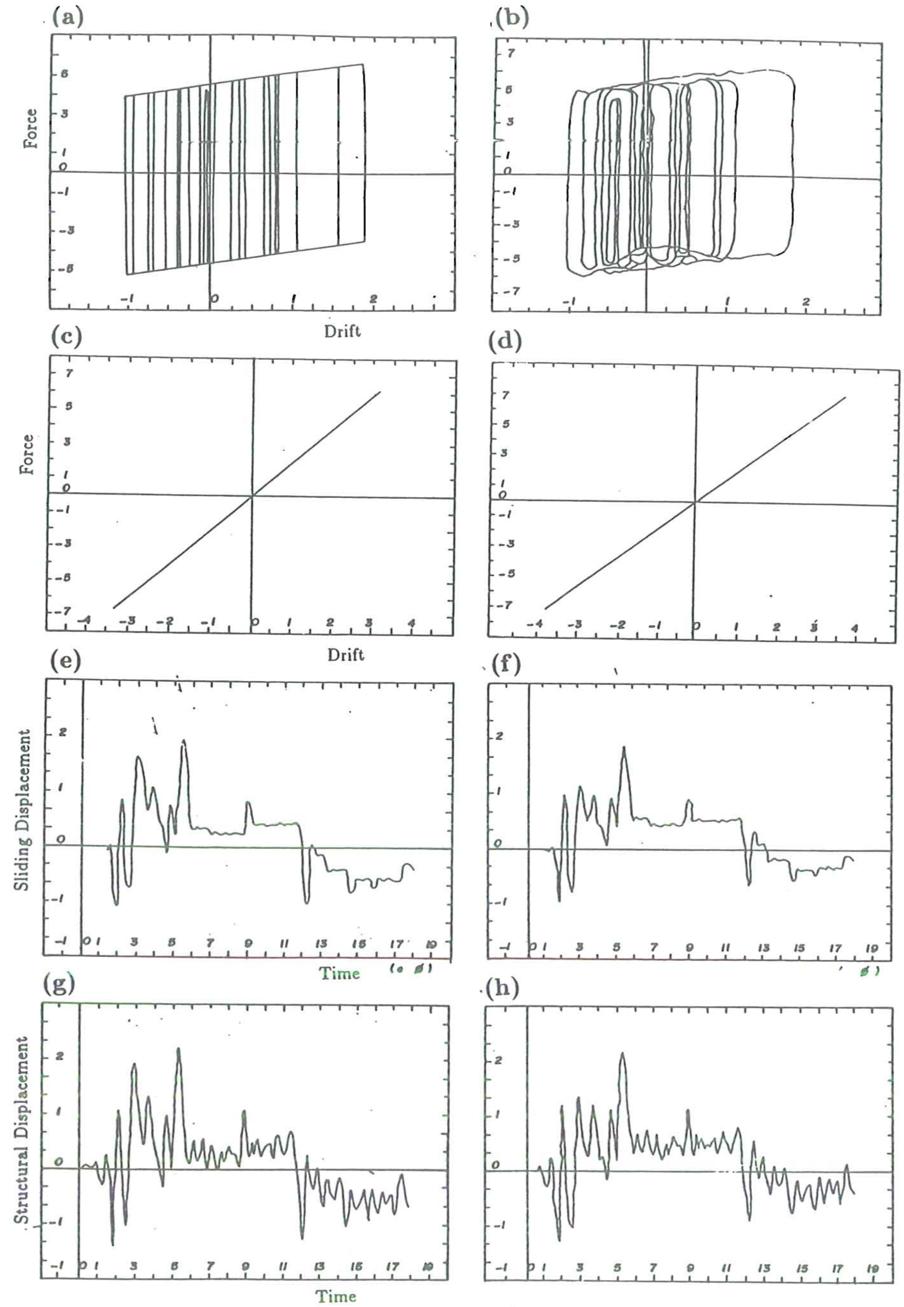


Fig. 3. **Elastic response for the NS component of the El Centro 1940 earthquake, $\xi = 5\%$. [Units: 1Force = 4400N; 1Displacements = 2.54cm]. (a)-(d) Constant friction model. (a) Sliding connection hystereric response. (b) Structure hysteretic response. (c) Time history for the sliding connection. (d) Time history for total structural displacements (sliding plus drift). (e)-(h) Rate dependent model. (e) Sliding connection hystereric response. (f) Structure hysteretic response. (g) Time history for the sliding connection. (h) Time history for total structural displacements(sliding plus drift).**

the rate dependent model [Fig. 4(d)] is increased to 3.8cm. Consequently, the structural ductility demand using the rate dependent model is three times the ductility demand using the minimum friction model. This can be explained since as the friction coefficient is increased the sliding displacement is reduced. Taking into account that sliding base isolation reduces forces through energy dissipation, as the sliding displacement is reduced, larger ductility requirements in the structure should be expected.

The sliding and structural time history responses for the constant and rate dependent models are also quite different [Fig. 4(e)-(h)]. The sliding displacement for the constant friction model is consistently larger compared to the rate dependent model.

In order to understand better the aforementioned observations, response spectrums were generated for periods ranging from 0.1 to 1.0s. The period of the connection is 3s and the structural viscous damping is 5%. A typical register corresponding to stiff local soil conditions, the El Centro NS 1940 register, is considered. The masses and strengths correspond to the values shown in Fig. 2(b). The period variations are achieved modifying the structural stiffness.

Figures 5 and 6 present spectrums for linear elastic and elasto-plastic structures, respectively. In all the figures the response is compared between isolated and similar —either elastic or inelastic— non-isolated structures and between the constant and rate dependent models. From Fig. 5(a) and 6(a), it seems that, in general, the total structural displacements are similar regardless structural strength since the aforementioned parameter has the same order of magnitude whether the structure responds in the elastic or in the inelastic range. For example considering a structure with a fundamental period equal to 0.4s and considering the rate dependent model, the total displacement for the elastic structure is 4.8cm and the similar value for the elasto-plastic structure is 5cm.

In the initial amplification region the response is sensitive regarding the constant or rate dependent friction models. Considering again the elastic structure with fundamental period equal to 0.4s the total displacement [Fig. 5(a)] using the constant and rate dependent friction models is 5.8cm and 4.8cm, respectively. For periods between 0.5s and 1s and in absolute terms, the total displacement is similar regardless of the friction model. As expected, it is clear that the minimum friction coefficient model provides an upper bound to the total structural displacements.

Figure 5(b) shows that for elastic structures the influence of the frictional model on the structural drift is not important. This is certainly not the case for elasto-plastic structures which are significantly affected by the friction model [Fig. 6(b)]. For example considering a inelastic structure with a fundamental period equal to 0.6s the structural drift is increased from 1.5cm to 4.3cm for the constant and rate dependent models.

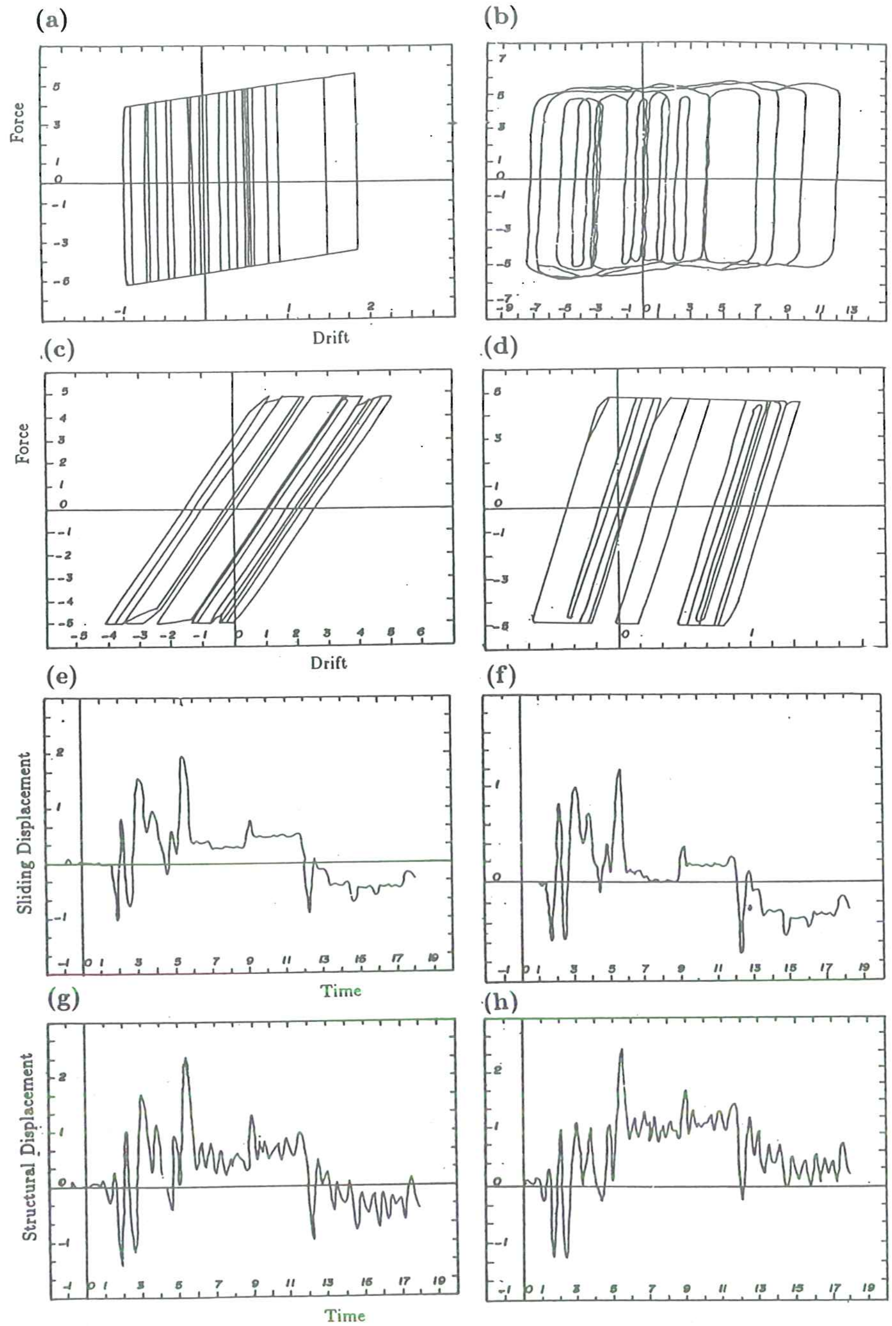


Fig. 4. Elasto-plastic response for the NS component of the El Centro 1940 earthquake, $\xi = 5\%$. [Units: 1Force = 4400N; 1Displacements = 2.54cm]. (a)-(d) Constant friction model. (a) Sliding connection hystereric response. (b) Structure hysteretic response. (c) Time history for the sliding connection. (d) Time history for total structural displacements (sliding plus drift). (e)-(h) Rate dependent model. (e) Sliding connection hystereric response. (f) Structure hysteretic response. (g) Time history for the sliding connection. (h) Time history for total structural displacements(sliding plus drift).

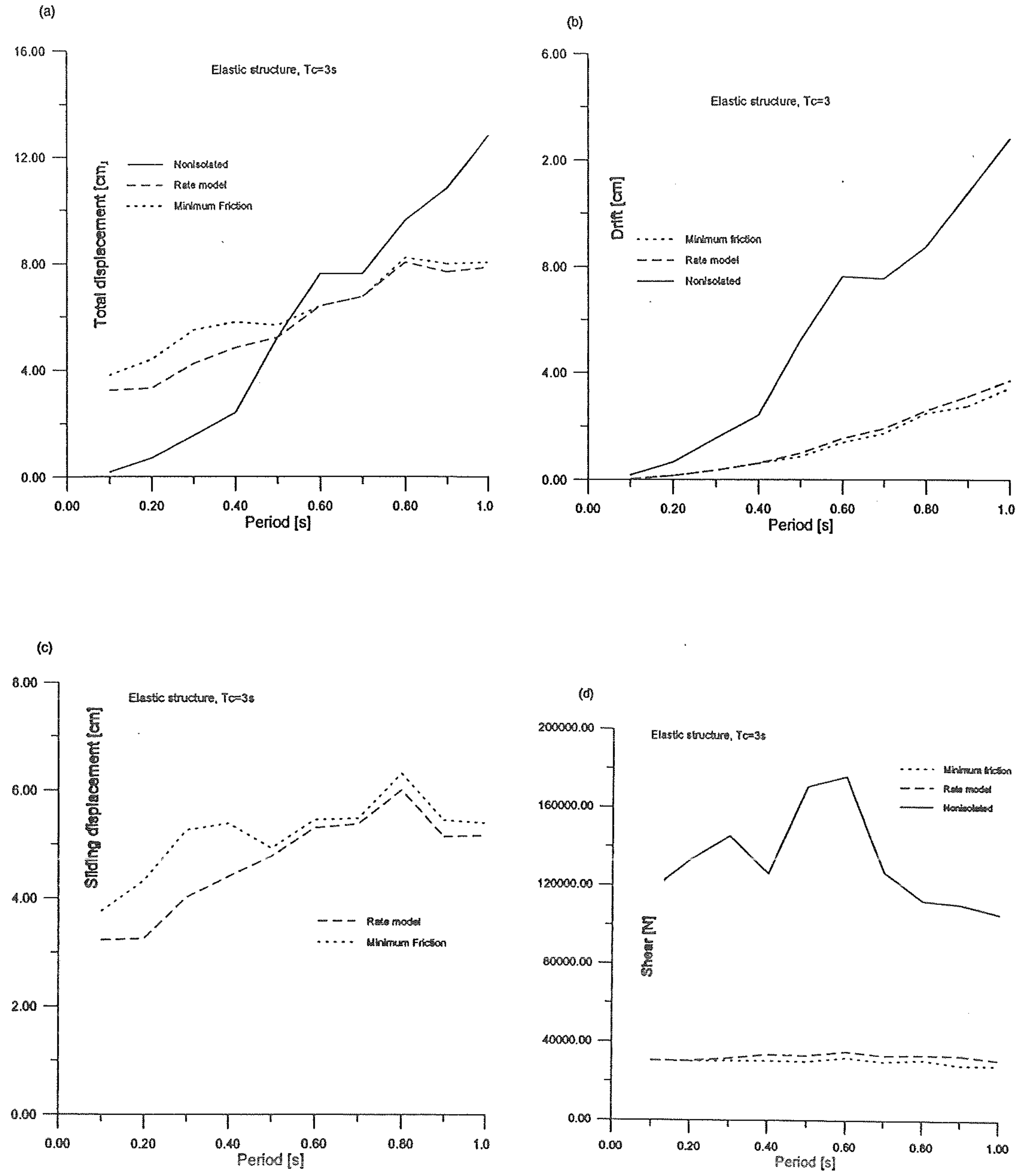


Fig. 5. Response spectrums for elastic structures subjected to the NS component of the El Centro 1940 earthquake. $\xi = 5\%$. (dotted line : Minimum Friction; dashed line: Rate Model; solid line : Nonisolated.) (a) Total structural displacement. (b) Structural drift. (c) Sliding displacement. (d) Base shear

For elastic structures the sliding displacements [Fig. 5(c)] are significantly affected by the friction model for short period structures. Since the total displacements are equal to the sliding displacements plus the structural drifts, it is clear that the variations observed in total displacements for elastic structures are caused by the variation in the sliding displacements. For inelastic structures the sliding displacements are significantly affected by the friction model for the whole range of periods included in the analysis [Fig. 6(c)].

It is observed that the shear forces for elastic structures are always increased using the rate dependent model compared to the minimum friction coefficient model [Fig. 5(d)]. This increment is expected because the friction coefficient for the constant model is generally smaller than the friction coefficient for the rate dependent model. Nevertheless, the shear forces are always considerably smaller compared to those obtained in a linear structure and the difference between the friction models is not significant. For example considering a elastic structure with a fundamental period equal to 0.6s, the base shear for a non isolated structure, and two isolated structures modeled using the constant and rate dependent models is 18×10^4 N, 3.2×10^4 N, and 3.4×10^4 N, respectively.

For elasto-plastic structures it is found that the friction model is an important parameter regarding the ductility demands [Fig. 6(d)]. For example, the ductility demand for a structure with a period equal to 0.6s is increased from 1.9 to 4.7 whether the model is based on a minimum friction or a rate dependent model. Furthermore, the benefits of sliding isolation in reducing ductility demands with respect to nonisolated structures is, in general, considerably reduced for elasto-plastic structures. Figure 6(d) shows that a nonisolated inelastic structure with a fundamental period larger than about 0.8s has similar ductility demands than isolated structures modeled using the rate dependent model.

Apparently, the variation in the friction coefficient is important for weak elasto-plastic structures because the mass and damping forces are small at the connection level. By dynamic equilibrium the equality between the shear force at the connection and at the columns is required. However, for these limited strength systems, the restoring force capacity is bounded by R_y and consequently the connection force and the sliding displacements are limited by R_y. In other terms, if the isolated structure enters the inelastic range, the structure tends to stop the sliding, consequently increasing the ductility requirements and structural drifts for the isolated building. This observation is conservative since there is generally at least some deformation hardening in the columns.

4 Conclusions

Friction isolation provides an effective and reliable method for seismic resistant design. Studies indicate that response is well behaved and governed by simple interrelationships. For example, the total response of elastic and inelastic struc-

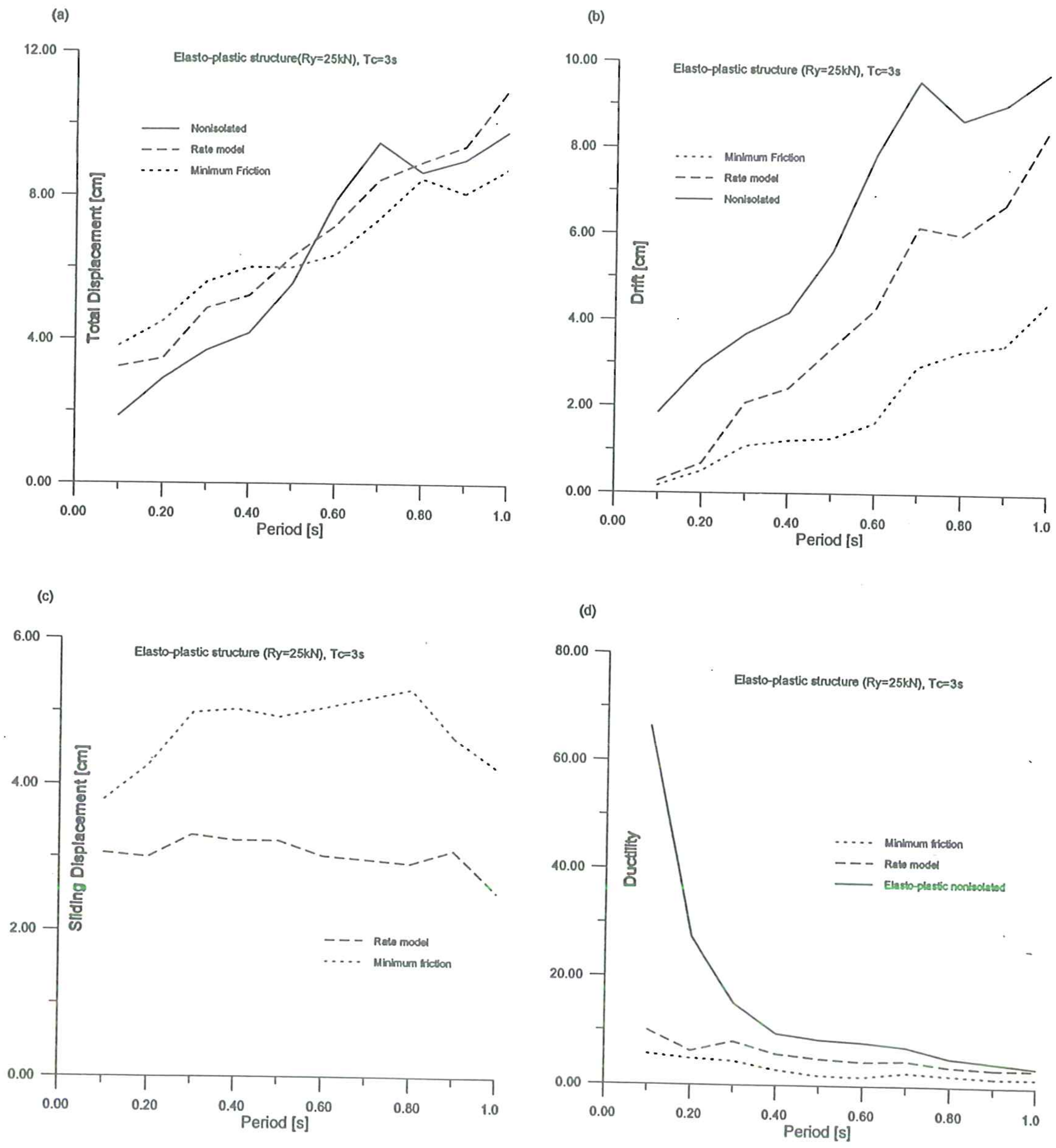


Fig. 6. Response spectrums for elasto-plastic structures subjected to the NS component of the El Centro 1940 earthquake. $\xi = 5\%$. (dotted line : Minimum Friction; dashed line: Rate Model; solid line : Nonisolated.) (a)Total structural displacement. (b) Structural drift. (c) Sliding displacement. (d) Structural ductility demands.

tures supported on sliding connections is well estimated by the elastic displacements of the nonisolated structure, so long as the period exceeds 0.5s. For shorter periods, it is proposed to estimate the total displacements using an energy balance amplification procedure. Similarly, simple relationships are developed to estimate structural and sliding displacements. Where elastic structures are represented as SDOF systems, the response is conveniently represented by normalized nonlinear response spectra.

Sliding stiffness was found to be important in limiting permanent and total displacements. Response trends observed were consistent with simple intuitive and theoretical concepts.

Structural yielding was found to have a predictable effect on response. The response of these systems was found to be consistent with good seismic performance. By taking advantage of the energy dissipation characteristics of the structures and of the sliding connection economic benefits are possible relative to conventional design, without significant increases in ductility demands.

The minimum friction coefficient model generally provided an upper bound to total sliding displacements and a lower bound to structural drifts. For shear forces —or ductilities— this model provided a lower, unconservative, bound. The difference is considerably more important for weak structures —such as retrofitted ones— compared to ideally elastic ones. Consequently, this article shows that for sliding base isolation, it is fundamental to consider a realistic friction model, particularly for weak structures whose response may be in the inelastic range. Variations in ductility demands due to changes in friction coefficient are not linearly dependent.

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