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ADAPTIVE MESH REFINEMENT TECHNIQUES FOR STRUCTURAL PROBLEMS

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SUMMARY

In this paper some adaptive mesh refinement (AMR) strategies for finite element analysis of structural problems are discussed. Two mesh optimality criteria based on the equal distribution of: (a) the *global error*, and (b) the *specific error* over the elements are studied. It is shown that the correct evaluation of the rate of convergence of the different error norms involved in the AMR procedures is essential to avoid oscillations in the refinement process. The behaviour of the different AMR strategies proposed is compared in the analysis of some structural problems.

1. INTRODUCTION

The evaluation of discretization errors and the design of suitable meshes via adaptive mesh refinement (AMR) are nowadays two of the challenging issues in the analysis of structures using the finite element method (FEM).

The topic of error estimation and mesh adaptivity in the FEM is by no means new. For a comprehensive review of the topic see the reference list of Chapter 14 of Volume I of [20]. Zienkiewicz and Zhu [1-5] have introduced a successfull AMR strategy for elasticity problems using a simple error estimate based on the difference of the discontinuous finite element stress (or strain) field with an "improved"smooth solution. These authors and others have extended this AMR procedure for plate bending and shell analysis[6-10], [19]. This AMR strategy has also been successfully used for fluid flow problems [2], [19].

In this paper we present an overview of different AMR strategies based on the Zienkiewicz and Zhu error estimator [1-6] for structural analysis. We will show first how the standard AMR algorithm based on equal distribution of the error over all the elements requires a careful identification of the rate of convergence of the different error terms involved in the design of the new element size to avoid oscillations in the refinement process.

An alternative AMR strategy based on the equal-distribution of the "specific error", i.e. the *error per unit area* (or volume) in all the elements in the mesh will be presented next. We will see in the examples shown that this strategy allows to concentrate more and smaller elements in zones where stress concentrations occur, as it should be expected from the engineering point of view.

In next section some basic concepts of the error estimation process are given.

2. BASIC CONCEPTS OF ERROR ESTIMATION

In dealing with adaptive mesh refinement the following two concepts should be clearly defined:

(a) *Error estimator.* Since the "exact"solution is not known, a method to approximately evaluate the error of the finite element solution should be defined.

(b) *Acceptable solution.* A finite element solution is "acceptable" if the estimated error satisfies some prescribed *global* and *local* conditions.

Both concepts (a) and (b) are further explained in next sections.

2.1 Error estimator

One of the most popular error estimators for structural problems is based on the error energy norm expressed as [1–6]

$$\|e\| = [\int_\Omega [\sigma - \hat{\sigma}]^T \mathbf{D}^{-1} [\sigma - \hat{\sigma}] d\Omega]^{1/2} \tag{1}$$

where σ are the exact stresses, $\hat{\sigma}$ are the stress values obtained from the finite element solution, $\mathbf{D}$ is the constitutive matrix and Ω is the structure domain. For plate and shell problems σ should here be interpreted as the "resultant stresses" [10], [16].

Since the exact stresses are usually not known they are approximated by

$$\sigma \simeq \sigma^* = \mathbf{N}\bar{\sigma}^* \tag{2}$$

where $\bar{\sigma}^*$ are nodal values obtained by simple nodal averaging of the finite element values, least squares local and global smoothing, or other adequate projection methods [1-5]. A simple approach is to use a global nodal smoothing with a lumped "mass" matrix giving the nodal smoothed values, $\bar{\sigma}^*$, as

$$\bar{\sigma}^* = \mathbf{M}_D^{-1} \int_\Omega \mathbf{N}_\sigma \hat{\sigma} d\Omega \tag{3}$$

where $\mathbf{N}_\sigma$ are the chosen stress interpolating functions giving a smooth stress field in terms of the nodal stress values obtained [1-5] and $\mathbf{M}_D$ is a diagonal matrix with $M_{D_{ii}} = \int_\Omega N_{\sigma_i} d\Omega$. Eq. (3) can be obviously applied to solve independently for each individual stress component. It can be verified that eq.(3) yields an accurate smoothed stress field for elements with linear interpolation and is adequate for quadratic elements providing correction factors are used [1]. Recently much improved methods have been devised by Zienkiewicz and Zhu [23] to obtain accurately recovered σ^* and these can be used as alternatives.

The strain energy of the exact solution is estimated as

$$\|U\| = [\int_\Omega \sigma^{*T} \mathbf{D}^{-1} \sigma^* d\Omega]^{1/2} \tag{4}$$

Both $\|e\|^2$ and $\|U\|^2$ can be evaluated as sum of their respective element contributions so that

$$\|e\|^2 = \sum_{i=1}^{n} \|e\|_i^2 \quad , \quad \|U\|^2 = \sum_{i=1}^{n} \|U\|_i^2 \tag{5}$$

where n is the total number of elements in the mesh.

2.2 Definition of acceptable solution

There is unfortunately no general consensus of what accuracy is acceptable. However, it is usually accepted that a solution is "correct" if the two following conditions are satisfied:

(a) The global error in energy norm is less than a specified percentage value of the total strain energy, i.e.

$$\|e\| \leq \eta \|U\| \tag{6}$$

where η is the user's specified value of the permissible relative global error.

Eq.(6) allows to define a global error parameter, ξ_g, as

$$\xi_g = \frac{\|e\|}{\eta \|U\|} \tag{7}$$

Clearly the values $\xi_g \leq 1$ denote satisfaction of the global error criterion, whereas $\xi_g > 1$ indicates that further refinement is necessary.

(b) The distribution of the elements in the refined mesh satisfies a "mesh optimality criterion". This local condition can be expressed as

$$\|e\|_i = \|e\|_{r_i} \tag{8}$$

where $\|e\|_i$ is the actual error norm in each element i and $\|e\|_{r_i}$ is the "required" error norm in the element.

From eq.(8) we can define a local error parameter $\bar{\xi}_i$ for each element i as

$$\bar{\xi}_i = \frac{\|e\|_i}{\|e\|_{r_i}} \tag{9}$$

Note that a value of $\bar{\xi}_i = 1$ defines an "optimal" element size, whereas > 1 and $\bar{\xi}_i < 1$ indicate that the size of element i needs refinement and -refinement, respectively.

The definition of the required error in each element $\|e\|_{r_i}$ is a key issue and it :ongly affects the distribution of element sizes in the mesh. This definition can based on different mesh optimality criteria and some of these are presented a later section.

3 Element refinement parameter

We can define now a single *element refinement parameter*, combining the tisfaction of the global and local conditions (a) and (b) of previous section as

$$\xi_i = \bar{\xi}_i \xi_g = \frac{\|e\| \; \|e\|_i}{\eta \|U\| \; \|e\|_{r_i}} \tag{10}$$

The element refinement parameter ξ_i was first introduced in ref[1] and since en it has been used by many authors as the basis for deciding the new element es in a general AMR strategy [1-9]. However, ξ_i can be also interpreted from 3) as the result of trying to satisfy both the global and local error conditions a successive manner. Eq.(10) provides all the terms involved in this combined ocess and these could play individually a very different role as explained in xt section.

MESH OPTIMALITY CRITERIA AND AMR PROCEDURES

1 Mesh optimality criterion based on the equal-distribution of the global error

A very popular mesh optimality criterion for structural analysis is based on e so called equal-distribution of the error, i.e. a mesh is defined as optimal if e global error is equally distributed over all the elements [1-10]. On the basis this assumption we can define the required error for each element as the ratio tween the global error and the total number of elements in the mesh. Thus ting that only the square of the error norm is additive (see eq.(5)) we have

$$\|e\|_{r_i} = \frac{\|e\|}{\sqrt{n}} \tag{11}$$

Combining (9) and (11) yields the expression of the local error parameter as

$$\bar{\xi}_i = \frac{\|e\|_i}{\|e\| n^{-1/2}} \tag{12}$$

The element refinement parameter is now obtained viz. eq.(10) as

$$\xi_i = \bar{\xi}_i \xi_g = \frac{\|e\|_i}{\eta \|U\| n^{-1/2}} \tag{13}$$

The parameter ξ_i can now be readily interpreted as the ratio between the element error and the distributed value of the permissible error over the mesh.

Expression (13) is in fact identical to that used in ref[1]. However, the multiplicative form $\xi_i = \bar{\xi}_i \xi_g$ allows to derive the correct AMR strategy. Thus by noting that the convergence rates of the element and global error norms are

$$\|e\|_i \simeq O(h_i^m)\Omega_i^{1/2} \simeq O(h_i^{m+\frac{d}{2}}) \tag{14a}$$

$$\|e\| \simeq O(h^m) \tag{14b}$$

where h_i and h are the existing element size and the average size of all the elements in the mesh, respectively, m is the degree of the shape function polynomials ($m = 1$ for linear elements, $m = 2$ for quadratic elements, etc.), and d is the number of dimensions of the problem ($d = 1, 2, 3$ for 1D, 2D and 3D problems, respectively) we can deduce that the new element size $\bar{h}_i$ can be obtained in terms of the existing size using the expression

$$\boxed{\bar{h}_i = \frac{h_i}{\xi}} \tag{15}$$

with

$$\boxed{\xi = \bar{\xi}_i^{\frac{2}{2m+d}} \xi_g^{1/m}} \tag{16}$$

The expression of the element size parameter ξ as given by (16) takes into account the different convergence rates of the element and global error norms.

Zienkiewicz and Zhu, followed by others [1-10], use a simpler expression for ξ based directly on the element refinement parameter ξ_i as

$$\xi = (c\xi_i)^{1/m'} \tag{17}$$

where c is a relaxation factor and the new exponent m' is taken as m except for elements adjacent to singularities where $m = \lambda$ is used (λ being the singularity strength).

The authors have found that the computation of ξ as given by (17) with the very common choice of $c = 1$ and $m' = m$ [1-10], leads to a non consistent mesh refinement [16]. This is shown by an oscillatory re and de-refinement of the same mesh zones in the AMR process. This problem dissappears if the correct expression (16) for ξ used (see first example and [16]).

.2 A mesh optimality criterion based on the equal-distribution of the specific error

An alternative criterion is to assume that a mesh is optimal if the square of ıe error per unit area (or volume) is the same over the whole mesh. It is clear ıen that in the optimal mesh

$$\frac{\|e\|_i}{\Omega_i^{1/2}} = \frac{\|e\|}{\Omega^{1/2}} \tag{18}$$

'bviously in (18) Ω_i and Ω denote the element and total area (or volume) ɜspectively.

Comparing (8) and (18) yields the expression of the required error norm for ıch element as

$$\|e\|_{r_i} = \|e\|(\frac{\Omega_i}{\Omega})^{1/2} \tag{19}$$

The element error parameter $\bar{\xi}_i$ is obtained now using (9) and (19) as

$$\bar{\xi}_i = \frac{\|e\|_i}{\Omega_i^{1/2}}\left[\frac{\|e\|}{\Omega^{1/2}}\right]^{-1} = \frac{\|e\|_i}{\|e\|}\left(\frac{\Omega}{\Omega_i}\right)^{1/2} \tag{20}$$

The element refinement parameter is obtained from (9), (10) and (20) as

$$\xi_i = \bar{\xi}_i\xi_g = \frac{\|e\|_i}{\eta\|U\|}\left(\frac{\Omega}{\Omega_i}\right)^{1/2} \tag{21}$$

Note that eqs.(13) and (21) coincide if $\frac{\Omega}{\Omega_i} = n$ (i.e. all elements are equal size). This is however not the case for unstructured meshes which results in ıite different mesh distributions for each mesh optimality criterion as shown the examples.

Moreover, the way we have defined now the element error eliminates its ɜpendance with the element area. Therefore, the new convergence rate of the ement error can be deduced from (14a) as

$$\frac{\|e\|_i}{\Omega_i^{1/2}} \simeq O(h_i^m) \tag{22}$$

Note the coincidence of the convergence rates of the element and global ror norms (eqs.(14b) and (22)).

The new element size is obtained from (15) with ξ given now by

$$\xi = (\bar{\xi}_i\xi_g)^{1/m} = (\xi_i)^{1/m} \tag{23}$$

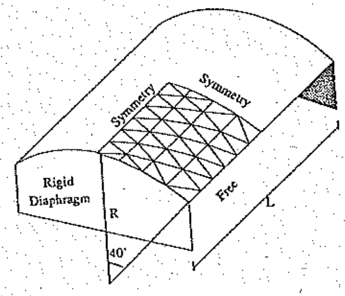
The new expression for the element refinement parameter coincides with that given by (17) with $c = 1$ and $m' = m$. Note, however, that (17) was introduced in the context the mesh optimality criterion based on the equal-distribution of the global error. The different forms obtained here clarify the correct expression to be used for each mesh optimality criterion chosen.

To our knowledge the criterion of equal-distribution of the specific error was introduced by Bugeda [17] and it has been used in the context of optimum structural design problems by Bugeda and Oliver [18] and for AMR analysis of plates and shells by Oñate, Castro and Kreiner [16]. This criterion can also be shown to be equivalent to that of equal error in stresses mentioned in [1].

4. EXAMPLES

4.1 Analysis of a cylindrical shell roof

The first example is the analysis of the classical Scordelis cylindrical roof under self weight shown in Figure 1 together with the initial mesh of 48 triangular facet shell elements used. The element formulation combines the 12 d.o.f. triangular Reissner-Mindlin plate element based on a linear interpolation for the lateral deflection, an incomplete quadratic interpolation for the rotations and an assumed linear shear strain field developed in [11–15] with a simple linear field for the in-plane displacements. The values of m and d in eq.(16) are $m = 1$ and $d = 2$. A value of the permissible global error $\eta = 10\%$ has been chosen.



$L = 600.0$
$R = 300.0$
thickness $= 3.0$
$E = 3.0 \times 10^6$
$\nu = 0.0$
B.C. : in diaphragm $u = 0,\ w = 0$
Load : Self weight, $q = 0.625$

Initial global error parameter $\xi_g = 6.2295$

Figure 1. Cylindrical shell roof under self weight. Symmetric quadrant used in the analysis and initial mesh of 48 triangular assumed shear strain facet shell elements [11–15].

Figure 2 shows the sequence of refined meshes obtained with the three AMR strategies studied in the paper. First column (strategy A) shows the results obtained using the criterion of equal-distribution of the global error over all the elements and the inconsistent value of ξ defining the new element sizes as given by eq.(17) with $c = 1$ and $m' = m = 1$. Note the oscillations in the AMR process clearly shown by the alternative re and de-refinements of the same mesh zones.

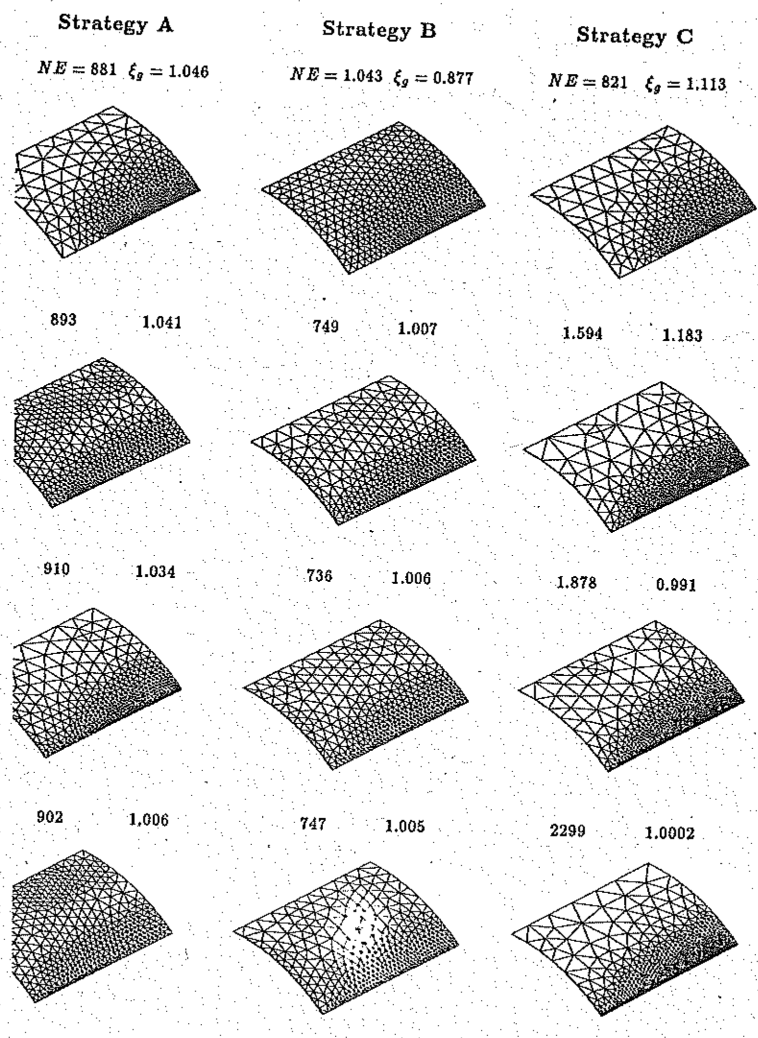


Figure 2. Symmetric quadrant of a cylindrical shell roof under self weight. Sequence of meshes obtained with AMR strategies based on: (A) Equal distribution of global error and inconsistent definition of parameter ξ (viz eq.(17)); (B) Idem with ξ consistently defined by eq.(16); and (C) Equal distribution of specific error. For each mesh is shown the number of elements (NE) and the value of the global error parameter ξ_g.

Results labelled as strategy B in Figure 2 have been obtained with the same mesh optimality criterion, but using now the correct expression for ξ as given by eq.(16). Note that the refinement oscillations dissapear and the AMR process converges in a consistent manner.

Results for strategy C have been obtained with the mesh optimality criterion based on the equal distribution of the specific error, with the element size parameter ξ as given by eq.(23). It can be clearly seen that: (a) The AMR process converges without oscillations, and (b) This AMR strategy concentrates more and smaller elements in the vicinity of the free edge (where the error is greater due to the higher membrane stress gradients), whereas in the rest of the mesh bigger elements than in the previous case are allowed. The prize to be paid is the increase in the total number of elements with respect to strategies A and B for the same global accuracy as shown in Figure 2.

Further details of this example and of the general AMR strategy for plates and shells can be found in [16].

4.2 Analysis of a cylindrical shell with a circular perforation under uniform traction

Figure 3 shows the geometry, material properties of the shell and the initial mesh used. The analysis has been performed using the same facet shell element as in the previous example. A value of the permissible global error $\eta = 10\%$ has been taken.

Figure 3 also shows the sequence of refined meshes obtained with the criterion of equal-distribution of the global error and the correct value for the element size parameter ξ as given by eq.(16) (Strategy B), and also with the criterion of equal distribution of the specific error with ξ given by eq.(23) (Strategy C).

Table I shows some characteristic results for each solution like the number of elements, the global error parameter ξ_g, the average value of the local error parameter $(\bar{\xi}_i^2)_a$ and its mean deviation $(\bar{\xi}_i^2)_\sigma$ over each mesh for the two AMR strategies used. From the numbers shown in this table we deduce

- Both AMR strategies converge to the global permissible error chosen
- Both AMR processes converge to an "optimal mesh", characterized by the appriate values $(\bar{\xi}_i^2)_a = 1.0$ and $(\bar{\xi}_i^2)_\sigma = 0.0$. However, the number of elements and its distribution in each mesh is very different for the two AMR strategies and again that based on the equal distribution of specific error (strategy C) tends to concentrate more and smaller elements in the vicinity of the central hole where stress gradients are higher (see Figure 3).

5. CONCLUDING REMARKS

We have shown that the correct evaluation of the rate of convergence of the different error norms involved in the AMR strategy is essential to avoid oscillations in the refinement process. Also, the mesh optimality criteria based on global and specific error distributions are conceptually very different. Thus, whereas the former leads to meshes with smaller number of elements, the second captures

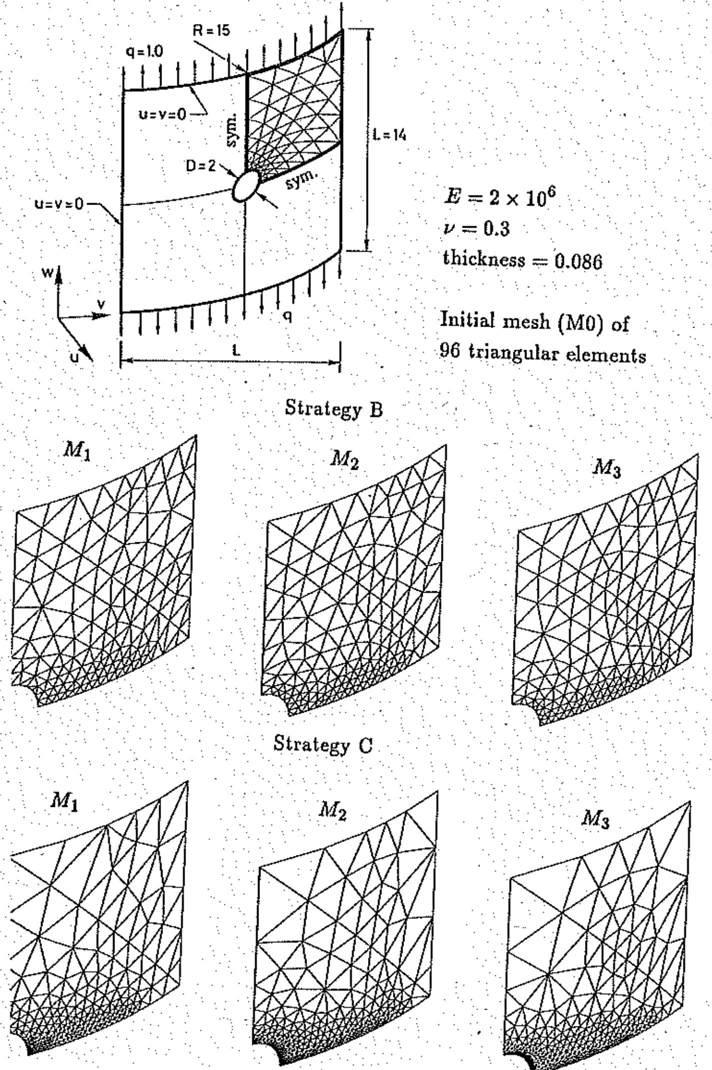


Figure 3. Analysis of a cylindrical shell under uniform traction. Sequence of meshes obtained with AMR strategies based on: Equal distribution of global error (Strategy B) and Equal distribution of specific error (Strategy C).

	Strategy B				Strategy C			
	NE	ξ_g	$(\bar{\xi}_i^2)_a$	$(\bar{\xi}_i^2)_\sigma$	NE	ξ_g	$(\bar{\xi}_i^2)_a$	$(\bar{\xi}_i^2)_\sigma$
M0	96	1.776	1.000	1.413	96	1.776	1.728	8.668
M1	234	1.078	1.000	0.613	393	0.993	1.545	3.432
M2	262	0.959	1.000	0.518	593	0.991	1.166	0.684
M3	266	0.968	1.000	0.384	729	0.979	1.050	0.381

TABLE I. Symmetric quadrant of a tractioned cylindrical shell. Some statistical results of the AMR processes based on the equal distribution of the global error (Strategy B) and the specific error (Strategy C).

better the effect of high stress gradients. Further research should allow to balance the possibilities of these two criteria for use in practical structural applications.

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