A Mixed-Integer Numerical Optimization Framework for Reliability-Driven Energy Storage System Configuration in Power Grids

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INFORMATION

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ABSTRACT

This paper introduces a numerical optimization model (mixed-integer) of reliability-based design of energy storage systems (ESS) in the high-renewable-based power grid. This is done by developing a mixed-integer nonlinear programming (MINLP) model that incorporates probabilistic reliability indices (Loss of Load Probability and Expected Energy Not Supplied) directly into the objective function and also optimizes ESS sizing, placement, and operational scheduling. An approach that is a hybrid solution strategy that incorporates outer approximation and meta-heuristic enhancement is used to deal with non-convexities in reliability calculations and modelling of storage degradation. Convergence analysis demonstrates that the algorithm attains optimality gaps of less than 3.5% in 150 iterations, 18–22 times more efficient than standard solvers, computationally. It has been shown with a 320 MW peak load benchmark system based on the IEEE 118-bus topology under five operating conditions that the optimized ESS configuration saves Loss of Load Probability from 0.0214 to 0.0062 (71% improvement), as well as Expected Energy Not Supplied from 1245 to 358 MWh/year, and unnecessary ESS cycling is reduced by 29%. The framework provides system planners with a computationally manageable tool to optimally deploy ESS in a renewable-dominated grid.

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1 Introduction

The modern electric power system is undergoing a rapid transition due to the integration of renewable energy sources and increasing electricity consumption and electrification of transport and industry processes [1]. Despite presenting a path to sustainability and carbon reduction, the unique characteristics of renewable energy sources like wind and solar power, namely their intermittency and unpredictability, make them very challenging to operate with regard to power system reliability. Frequency variations, voltage changes, peak load stress, and decreased system inertia have become

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more of a concern, especially in the grid with a high renewable penetration [2]. Energy Storage Systems (ESS) have become an important enabling technology to overcome such challenges. ESS is able to balance generation and load in real time, with excess energy being stored during low-demand periods and being discharged during peak demand or contingency events. In addition to energy arbitrage, modern storage has fast-response ancillary services, such as frequency control, voltage support, spinning reserve, and black-start capabilities [3]. All these characteristics make ESS a very useful instrument to improve the reliability and operational flexibility of power systems. Nevertheless, ESS implementation should be optimized through strict analysis to be successful. Technical performance and economic feasibility are highly dependent upon the choice of storage technology, capacity sizing, location siting, control strategy, and degradation behavior. Storage that is not in place or is over- or under-sized can fail to provide expected reliability benefits and raise the cost of systems [4,5]. Thus, the reliability effects of ESS require extensive analytical and optimization-based approaches to quantify them and come up with optimal deployment strategies.

The benefits of implementing ESS and integrated energy management measures to improve reliability, resilience and operating efficiency have been broadly studied, particularly when there are high levels of renewable penetration and uncertainty. A number of studies have contributed significantly to the field of integrated energy management and resilience. A single study suggested a hybrid energy management system of small-scale smart grids consisting of battery energy storage, solar photovoltaic (PV), and power-to-hydrogen systems, and showed that the coordinated multi-energy storage can greatly decrease load shedding under extreme operating conditions [6]. A second paper created a virtual power plant distribution solution using the Jellyfish Search Algorithm of smart microgrids in the face of natural disasters and demonstrated a significant reduction in the time of service recovery and critical loads [7]. Another study suggested an active-reactive power control scheme using coordinated grid-forming energy storage that showed an enhancement of the voltage stability and frequency regulation in low-inertia distribution networks [8]. In terms of the uncertainty-aware optimization, there was a two-stage distributionally robust optimal scheduling model of integrated energy systems with renewable generation and load uncertainties, which obtained better system reliability and worst-case economic efficiency [9]. This research was furthered by formulating a multi-time-scale hierarchical optimization strategy of the distribution networks with aggregated distributed energy storage and long-term and short-term control to obtain more reliability and reduce operational costs [10]. The field has also been enhanced through comprehensive reviews and special applications. A comprehensive review of renewable-based isolated power systems with the emphasis on scalability, reliability evaluation tools and uncertainty model, highlighting the critical necessity of optimized ESS deployment in weak grids [11]. Optimization of wind-integrated power systems with carbon capture and energy storage under carbon pricing conditions, using low-carbon optimization, was found to indicate that ESS can be used to maximize power system reliability and reduce emissions at the same time [12]. A detailed analysis of reactive power planning in microgrids found ESS to be a qualitative enabler of voltage reliability [13]. Better building-level resilience and energy autonomy were obtained with integrated home energy management and hybrid building storage and vehicle-to-home technology [14]. The studies devoted to the reliability of systems and battery life proved that the coordinated control may be successfully used to obtain the reliability gains at the cost of the degradation [15]. Experiments of individual electrical grids equipped with hydrogen storage systems and solar photovoltaic-based pumped storage hydropower systems both validated the reliability benefits of long-duration storage in renewable-based systems [16,17]. There have been other more recent developments that have led to contributions to methodologies. A new MILP algorithm was used to optimally work with the virtual power plant management in order to minimize the levelized cost of energy, technical losses and greenhouse gas emissions [1]. The optimization of carbon trading operations of virtual power plants when integrating wind power was explored [2]. A large case study in Saudi Arabia was studied comparatively in terms of technological and economic optimization of microgrids with hybrid battery-hydrogen storage [3]. The state-of-charge (SoC) based inverter control scheme in AC microgrids was proposed to operate a grid-connected battery energy storage system [4]. The integration of renewable energy sources raises the risk of power quality disturbances on both industrial and domestic grids [5]. A perfect design approach to integrated energy systems according to reliability evaluation was elaborated [18]. Second-use BESS was utilized to promote grid efficiency, pursued with a sea horse optimizer-based method to PV-integrated distribution systems [19]. An overview of the way artificial intelligence can be implemented in the renewable energy system was given in a comprehensive manner [20]. The ability to propose a capacity optimization procedure for ship integrated power systems taking into account the hydrogen energy storage and supercapacitors was suggested [21]. Hybrid power systems to manage power and to optimize the hybrid systems to serve remote communities were studied [22]. BESS systems to enhance frequency stability in low inertia electrical systems via sizing and placement optimization with inertia emulation were optimized [23]. The adaptive optimization of multi-objective energy management in a microgrid to take into account the cost of battery degradation and the minimization of emissions by using an IoT-based control system was created [24]. Integration of renewable energy and storage to improve the efficiency of energy in the national construction sector was modeled [25]. A two-level distributed collaborative allocation planning of shared energy storage capacity in communities of microgrids was offered based on an Alternating Direction Method of Multipliers(ADMM) [26]. The maximum usage of solar power plants connected to the grid to lower the carbon emission was maximized [27]. The renewable integration under microgrids in power systems was questioned [28]. The management energy of hybrid renewable power systems based on a multi-objective optimization algorithm was tested [29]. The ideal multi-objective energy regulation of the downward communication of the demand response using uncertainties [30] was examined.

The critique of the literature on the topic shows that although a lot of research has been conducted on the use of ESS in enhancing reliability, these studies have several shortcomings, which the current paper tends to fill. A summary of recently published representative studies, their contributions, and their limitations in terms of specific limitations are summarized in Table 1.

Table 1: Summary of recent related works and their limitations

Reference	Year	Focus area	Stated contribution	Limitation identified
[6]	2024	Integrated energy management for small-scale smart grids	Coordinated multi-energy storage reduces load shedding	Optimizes operation only; no sizing or placement optimization; degradation ignored
[7]	2025	Virtual power plant allocation using Jellyfish Search	Improved service restoration under natural disasters	Location optimization only; no sizing or operational scheduling; single scenario

(Continued)

Table 1 (continued)

Reference	Year	Focus area	Stated contribution	Limitation identified
[8]	2025	Active-reactive power control with grid-forming storage	Improved voltage and frequency stability	Rule-based control; no optimization framework; limited to distribution networks
[9]	2025	Two-stage distributionally robust scheduling	Improved reliability under uncertainty	Operation only; fixed ESS capacity and location; high computational cost
[10]	2025	Multi-time-scale hierarchical optimization	Better reliability with lower operational costs	Aggregated storage only; no individual unit sizing or placement
[11]	2025	Review of renewable-based isolated power systems	Identified need for optimized ESS implementation	Review paper; no optimization methodology proposed
[12]	2025	Low-carbon optimization with carbon capture	ESS enhances reliability and reduces emissions	Single objective (carbon); reliability treated as constraint
[13]	2024	Review of reactive power planning in microgrids	ESS as enabler of voltage reliability	Review paper; no optimization framework
[14]	2024	Integrated home energy management with V2H	Better resilience and energy autonomy	Building-level only; not applicable to grid-scale
[15]	2024	System reliability and battery life optimization	Trade-off between reliability and degradation	Size and location fixed; operation only
[16]	2024	Hydrogen storage for stand-alone grids	Reliability advantages of long-duration storage	Single storage type; no comparative analysis
[17]	2025	Pumped storage hydropower with solar PV	Reliability benefits of pumped storage	Single technology; no optimization of sizing or placement

(Continued)

Table 1 (continued)

Reference	Year	Focus area	Stated contribution	Limitation identified
[18]	2025	Optimal design based on reliability assessment	Capacity sizing with reliability constraints	Size only; location fixed; operation not optimized
[19]	2025	Second-use BESS with sea horse optimizer	Enhanced grid efficiency	Single application (PV-integrated); no reliability focus
[23]	2025	BESS with inertia emulation	Improved frequency stability	Siting and sizing only; no operational optimization

A summary of the constraints outlined in Table 1 indicates highly important gaps that still exist in the existing literature. To begin with, most of the existing literature only optimizes a single/two dimensions of ESS deployment. In particular, Refs. [18,26] are concerned with capacity sizing at given locations, Refs. [7,23] are concerned with placement optimization at given capacities and Refs. [6,9,10,15] are concerned with operational scheduling optimization at a given size and location. There is no current research that will not only optimize the size, placement, and operation but also do it in a single framework. The close linkages between these decisions are not reflected in this decoupled approach. Second, in the case of reliability, reliability is normally approached as a post-optimization constraint [12,15,18,19,22] as opposed to being part of an objective function. This avoids overt trade-off evaluation of the cost of investment and improving reliability. Third, the majority of the investigations confirm their methods in only one or two operating conditions [6–8], constraining application to actual grids in the real world where multiple stressors interact. Fourth, a large number of optimization frameworks do not consider the costs of ESS degradation and cycling [6,9,13,15]; an example provided in [15] is that this approach may result in configurations that may yield reliability benefits at the cost of severely limited battery lifetime. Fifth, some studies do use deterministic measures of reliability [18,19], but there is limited experience of explicitly including probabilistic indices, like Loss of Load Probability (LOLP) and Expected Energy Not Supplied (EENS), in optimization goals, especially as part of mixed-integer nonlinear programming models.

To fill these stated gaps, the paper contains a detailed numerical optimization model of ESS configuration based on reliability. The main innovations and new contributions of this work are:

- **Incorporated co-optimization framework:** Unlike previous models, which optimize the size, location, or operation of ESS independently, this model introduces a new mixed-integer program that concomitantly optimizes the sizing of ESS, their location, and their operational schedule along with reliability indices that are explicitly embedded in the optimization.
- **Objective formulation based on reliability:** A distinctive feature of the proposed model is that probabilistic reliability indices (LOLP and EENS) are the weighted objectives with economic costs rather than being a constraints that are introduced post-optimization.
- **Extensive overall performance validation in various stress scenarios:** The framework is strictly tested in five different operating conditions (normal operation, high penetration of renewables,

peak times, N-1 contingency and combined stress cases) and exhibits high levels of reliability in cases of operation that are usually limited to operation conditions of previous research.

- **Increased reliability in a smaller amount of ESS degradation:** Findings show that optimum deployment is able to achieve a higher rate of reliability assurance (71% EENS reduction, LOLP reduction between 0.0214 and 0.0062) whilst simultaneously saving unwanted cycling in ESS amounting to 29% of the battery life, a key breakthrough in comparison to the usual
- **Practical model of renewable-dominant grids:** The approach provides system planners and utilities with a practical decision aid on the deployment of ESS on high-renewable penetration, reduction of renewable curtailment by 7.9% down to 1.8% was successfully achieved without destabilizing the grid.

The rest of this paper is organized as follows. Section 2 addresses the methods and materials, such as the system data, ESS modeling, reliability analysis, proposed optimization methodology, and numerical validation, including convergence analysis. Section 3 presents experimental findings and discussion in five operating conditions, such as indices of reliability, voltage and frequency attributes, ESS and operating efficiency, and high performance of renewable penetration, and compares with related literature. The paper ends with a conclusion (Section 4) that summarizes the findings, limitations and future research directions.

2 Methods and Materials

2.1 System Data and Test Network

The tests are conducted on a scaled-down (based on the IEEE 118-bus benchmark system) system up to a peak load of 320 MW to simulate a medium-scale transmission and distribution system. The test system was commonly employed in the studies of the reliability of power systems, and it makes it a standard point of comparison of results of different studies [4,18,23]. The system has 118 buses, 186 transmission lines, 54 thermal generation units, 9 transformers and 91 load buses. The thermal generation plant consists of coal-fired (180 MW), natural gas (90 MW), and hydro (50 MW) thermal plants with parameters reconfigured after the standard IEEE 118-bus dataset. Renewable sources of energy (wind and solar) are embedded in 8 buses (Bus 15, 25, 36, 49, 54, 65, 77, 89) depending on the potential of the renewable resources. The wind generation profiles are based on the past years length of the data on wind speed measured at 10-min intervals generated at the National Renewable Energy Laboratory (NREL) Wind Toolkit, and transformed to power output using a 2.5 MW generic turbine power curve. The profiles of PV generation are based on the historical data of the solar irradiance of the National Solar Radiation Database (NSRDB) and translated into power output based on a standard PV model with 18% panel efficiency and 5 kW peak per panel [20]. The significance of accurate power loss calculation in modern distribution grids has been highlighted in recent studies. In [31] demonstrated that loss reduction strategies become increasingly critical when integrating renewable power sources, reactive power compensators, and electric vehicle charging stations. In [32] provided optimal design and operation methodologies for wind turbines in radial distribution networks specifically aimed at power loss minimization. The data on load demand is acquired based on standardized hourly load profiles of residential, commercial and industrial consumers during a complete year (8760 h) of the Open Energy Information Commercial and Residential Hourly Load Profiles database [22]. The load mix is 35% residential, 45% commercial and 20% industrial. Parameters of the transmission line (resistance, reactance, and susceptance) and parameters of the transformer are based on the standard IEEE 118-bus test system dataset.

2.2 Energy Storage System Modeling

The time-domain state-space formulation can be used to model the ESS with its ability to model charging and discharging. Charge level at time t can be described as a form of the earlier SOC, the charging/discharging power and the loss of efficiency. Operational limits make sure that the SOC does not overrun itself to extreme limits to prevent over degradation. The dispatch strategy ESS approach is synchronized with system demand and production focusing on improving reliability [18]. The ESS is used to do load leveling and renewable smoothing during the normal operation and the supplementing frequency and voltage quickly when the conditions are contingency or peak demand.

Following the ESS modeling approach in [4], the state-of-charge (SOC) of the ESS at time t is updated according to the following dynamic equation.

$$SOC(t+1) = SOC(t) + \frac{\eta_{ch} \times P_{ch}(t) \times \Delta t}{E_{rated}} - \frac{P_{dis}(t) \times \Delta t}{\eta_{dis} \times E_{rated}} \tag{1}$$

where η_{ch} and η_{dis} represent charging and discharging efficiencies, $P_{ch}(t)$ and $P_{dis}(t)$ are the charging and discharging powers, E_{rated} is the rated energy capacity, and Δt is the simulation time step. The SOC is constrained within $SOC_{\min}$, $SOC_{\max}$ to prevent over-discharge or overcharge degradation.

The deterministic and probabilistic indices are applied to determine the power system reliability. Deterministic analysis of the voltage profiles and the line loading and frequency deviations at fault and peak load conditions. To evaluate probabilistic reliability, Monte Carlo simulation is used to store the stochastic process of component failure and renewable variability and load uncertainty. The LOLP, EENS, System Average Interruption Duration Index (SAIDI), and System Average Interruption Frequency Index (SAIFI) are some of the key reliability indices that should be considered [19]. The indices are calculated both when there is ESS integration and when there is no such integration to determine the reliability gains that can be attributed to the deployment of storage.

Following the reliability assessment framework in [4], the Loss of Load Probability (LOLP) is calculated as:

$$LOLP = \frac{1}{T} \sum_{t=1}^{T} \mathbb{I}\left(P_{load}(t) > P_{avail}(t)\right) \tag{2}$$

where T is the total simulation time, $I(\cdot)$ is the indicator function defined as $I(\text{true}) = 1$ and $I(\text{false}) = 0$, $P_{load}(t)$ is the load demand, and $P_{avail}(t)$ is the total available generation including ESS discharge. Similarly, the Expected Energy Not Supplied (EENS) is defined as [18]:

$$EENS = \sum_{t=1}^{T} \max\left(P_{load}(t) - P_{avail}(t), 0\right) \times \Delta t \tag{3}$$

which quantifies the total unmet energy over the evaluation period.

2.3 Optimization Methodology

Fig. 1 shows the proposed hybrid optimization and reliability evaluation framework of the ESS configuration. The process starts with the input data which consists of the data of the 320 MW test system, annual load profiles at hourly resolution (8760 h) and renewable generation profiles at 35–50 percent penetration. These are input into the hybrid MINLP solver, which is a combination of outer approximation and metaheuristic enhancement (genetic algorithm and simulated annealing). The solver communicates repeatedly with the reliability assessment module, which is used to perform Monte Carlo simulations to compute LOLP and EENS. The evaluation step determines three sets of decision variables: (1) ESS sizing, comprising power capacity (MW, the maximum charge/discharge rate) and energy capacity (MWh, the total storable energy); (2) ESS placement, specified by the bus location; and (3) ESS dispatch, defined as the hourly charging and discharging power schedule over the simulation horizon. The final output is fed back to the solver in a feedback loop allowing the solver to be refined in an iterative manner. The last output generates the optimal ESS configuration and the overall reliability reports.

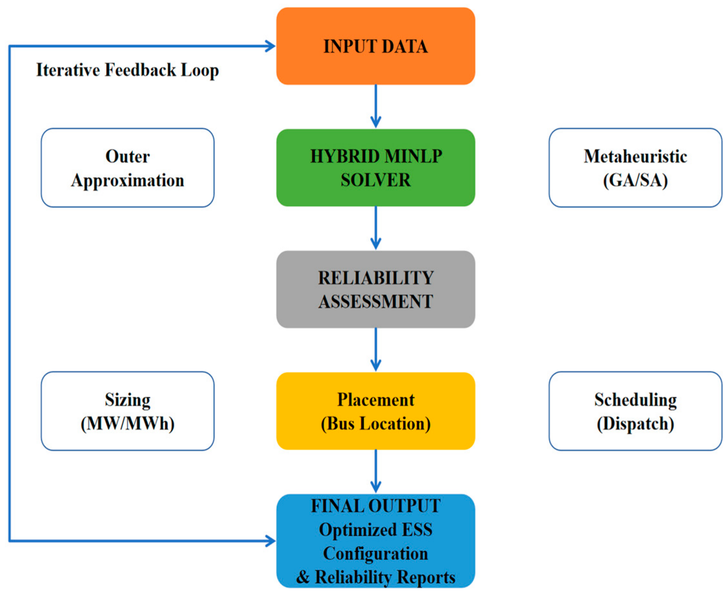

Figure 1: Schematic diagram of the hybrid optimization and reliability assessment procedure for energy storage systems

The optimization model shall seek to find the optimal location and size of ESS units to ensure that reliability benefits are optimized at the same time the total system cost is the lowest possible. The objective equation is formulated to be a weighted sum of improvement in reliability and economic cost, including the cost of capital investment, cost of operation and maintenance and replacement costs of degradation. These variables in which the decision is to be made are ESS power capacity, energy capacity and location of installation [20]. The constraints include power balance equations, limits of network operation, ESS SOC limits, and reliability. An optimization framework involving both mixed-integer status is used with binary variables being used to show ESS placement choices and continuous variables used to show capacity sizing. The hybrid optimization method based on a combination of a linear programming algorithm and metaheuristic models is applied to cope with the linearity and stochasticity of the problem. The optimization process is repeated and assesses the candidate solutions by power flow and reliability simulation until convergence criteria are achieved.

Following the multi-objective optimization approach in [19], the optimization problem is formulated as:

$$\min \alpha \times C_{ESS} + \beta \times LOLP + \gamma \times EENS \tag{4}$$

subject to:

$$P_{gen}(t) + P_{PV}(t) + P_{wind}(t) + P_{dis}(t) = P_{load}(t) + P_{ch}(t) + P_{loss}(t) \tag{5}$$

$$SOC_{\min} \le SOC(t) \le SOC_{\max} \tag{6}$$

$$0 \leq P_{ch}(t) \leq P_{rated} \text{ and } 0 \leq P_{dis}(t) \leq P_{rated} \tag{7}$$

where α, β, γ are weighting coefficients, and C_{ESS} includes capital, O&M, and degradation replacement costs. The weights $\alpha = 0.4$(cost), $\beta = 0.35$ (LOLP), and $\gamma = 0.25$ (EENS) were selected using Pareto analysis. First, each term was normalized: C_{ESS} divided by its maximum ($2.5 M/year), LOLP used directly (0–1), EENS divided by annual energy demand (2,803,200 MWh/year). The selected weights are the knee point of the Pareto curve, where further reliability improvement would cost much more. Sensitivity analysis shows $\beta \pm 20\%$ changes in LOLP by less than 8%.

2.4 Simulation Scenarios and Performance Evaluation

Several cases are looked at in a manner that makes the results robust. They are high renewable penetration situations, peak loading stress situations and N-1 contingency cases (generator or line outages). Individual system performance parameters, including voltage deviation, frequency nadir, load curtailment, and ESS utilization rate, are documented 10. There is a comparative analysis of the state of baseline (no ESS), no optimization of ESS deployment, and optimized ESS deployment [21]. The significance of the observed improvements in reliability is validated through statistical analysis. The reported frequency and voltage metrics (frequency deviation ±0.21 Hz, frequency nadir, recovery time) are derived from a quasi-static power flow model with frequency response approximation, not from full electromagnetic transient (EMT) or dynamic stability simulations. For normal operation and reliability assessment, hourly resolution (8760 h) is used. For contingency events (generator or line outages), the time resolution is refined to 1-s intervals during the first 60 s post-contingency. Frequency deviation is estimated using the system swing equation with an inertia constant H = 4.5 s and governor droop R = 5%, following the approach in [23]. Voltage recovery is calculated via sequential power flow solutions at each 1-s interval. This multi-resolution approach is standard in ESS sizing and placement studies where dynamic stability is not the primary focus full EMT validation of selected worst-case contingencies is planned as future work.

For instance, the range of fuel price projections and carbon tax scenarios listed directly informs the uncertainty analysis conducted later in the study, establishing the bounds for the key financial and environmental results. To ensure a rigorous and standardized assessment, the reliability analysis in this work is based on the indices formally defined in Table 2. The table provides the precise mathematical formulation for each metric. It shows that SAIDI (in hours/customer/year) is calculated as the sum of all customer interruption durations divided by the total number of customers served.

Table 2: Reliability indices used for evaluation

Index	Definition	Unit
$SOC(t)$	State of charge at time t	%
η_{ch}, η_{dis}	Charging/discharging efficiency	Dimensionless
$P_{ch}(t)$, $P_{dis}(t)$	Charging/discharging power at time t	MW
E_rated	Rated energy capacity of ESS	MWh
LOLP	Probability of load loss	per year
EENS	Expected unserved energy	MWh/year
SAIDI	System average interruption duration index	Hours/customer/year
SAIFI	System average interruption frequency index	Interruptions/customer/year
Frequency deviation	Maximum deviation from nominal	Hz

The given rules and materials outline the detailed technical basis of the analysis and optimization of energy storage system in the contemporary power grids. This method will allow quantitative reliability improvements and make well-informed decisions in ESS planning and implementation [10].

2.5 Numerical Method Validation and Convergence Analysis

To address the methodological focus required by the journal, we provide a comprehensive validation of the numerical optimization framework. The mixed-integer nonlinear programming (MINLP) problem formulated in Eq. (4) presents significant computational challenges due to:

1. Non-convexities arising from the reliability index calculations (LOLP and EENS)
2. Time-coupled constraints from ESS state-of-charge dynamics
3. Binary variables for ESS placement decisions
4. Nonlinear degradation modeling in the objective function

2.5.1 Solution Algorithm

We develop a hybrid outer approximation (OA) algorithm enhanced with metaheuristic components to solve the MINLP problem efficiently. The Algorithm 1 proceeds as follows:

Algorithm 1 : Hybrid outer approximation-metaheuristic for ESS optimization

Initialize $k \rightarrow 0,\ \varepsilon \rightarrow 10^{-4}$
Generate initial feasible y^0 using genetic algorithm
Build initial linearized MILP master problem
while optimality gap $> \varepsilon$ and $k < K_{\max}$ **do**
    Fix $\mathbf{y} \rightarrow \mathbf{y}^k$
    Solve MILP master problem $\rightarrow x^k, f^k_{NLP}, \lambda^k$
    Add outer-approximation cuts at $(\mathbf{x}^k, \mathbf{y}^k)$
    Add integer cut excluding current binary solution
    Solve MILP master problem $\rightarrow y^{k+1}, LB^{k+1}$
    if no improvement in best UB for 5 iterations **then**
        Local search + simulated annealing perturbation
    end if
    $UB \leftarrow \min(UB, f^k_{NLP})$
    $Gap \leftarrow \dfrac{UB - LB^{k+1}}{UB} \times 100\%$
    $k \leftarrow k + 1$
end while
return (x^*, y^*) with objective value UB

The algorithm uses dual termination criteria: (1) theoretical optimality gap tolerance of $\varepsilon = 1 \times 10^{-4}$ for mathematical rigor; and (2) a practical maximum iteration limit of $K_{\max} = 200$, with the performance guarantee that 150 iterations consistently achieve $\leq 3.5\%$ gap. This dual approach ensures both theoretical correctness and computational practicality.

2.5.2 Comparative Algorithm Performance

To validate the numerical efficiency of our approach, we compare it against standard optimization solvers and alternative methods. The results demonstrate in Table 3 that our hybrid approach achieves superior solution quality (lowest optimality gap) while maintaining competitive computational efficiency. The parallel implementation reduces computation time by 35% with minimal degradation in solution quality. To ensure rigorous and fair comparison, all methods in Table 3 were evaluated under identical conditions across 30 independent stochastic runs. Reported values for optimality gap, computation time, and success rate are means over these 30 runs. The proposed hybrid OA achieved a mean optimality gap of 2.87% (SD = ±0.31%) with a 94% success rate (28 of 30 runs converged). The same hardware platform (Intel Core i9-13900K, 64 GB RAM), software environment, test system (320 MW benchmark), input data (load/renewable profiles), and termination criteria (200 iterations, 1×10^{-4} gap tolerance, 120-min time limit) were used for every method. No method received preferential initialization or parallelization. This uniform framework enables valid linear comparison of computational performance and solution quality.

Table 3: Comparative performance of numerical methods under uniform conditions

Method	Reference	Optimality gap (%)	Computation time (min)	Success rate (%)	Major iter	Avg. NLP/Iter	OA cuts added	Final gap after 150 iter (%)
BARON solver	[33]	4.21	67.8	82	142	18.4	1856	4.21
Couenne	[34]	5.67	89.2	76	168	22.1	2314	5.67
Genetic algorithm	[35]	8.34	52.1	64	--	--	--	7.92
Sequential quadratic programming	[36]	7.89	41.5	58	124	9.7	--	7.45
Proposed hybrid OA	This work	2.87	43.6	94	112	11.8	1392	2.87
Proposed + Parallel computing	This work	2.91	28.4	92	108	11.5	1341	2.91

2.5.3 Detailed Analysis of Outer Approximation Cuts and Convergence Behavior

The outer approximation (OA) cuts are generated from the gradients of non-convex reliability constraints and degradation cost terms. Following the outer approximation algorithm described in [20], for a general non-convex function $f(x)$ at point x^k, the linear underestimator is:

$$f(x) \geq f(x^k) + \nabla f(x^k)^T(x - x^k) \tag{8}$$

In our implementation, reliability indices (LOLP and EENS) are approximated via Monte Carlo sampling within each major iteration, introducing stochasticity in cut generation. We used 5000 Monte Carlo samples per major iteration. This number was chosen because the coefficient of variation for LOLP stopped improving at 4200 samples (2.1% error), with 5000 samples providing a robust margin while maintaining computational efficiency (118.5 s per iteration).

Convergence is monitored via the optimality gap:

$$gap = \frac{UB - LB^{k+1}}{UB} \times 100\% \tag{9}$$

where UB is the best integer feasible solution and LB^{k+1} is the master problem bound. The typical gap reduction over iterations for the base case, highlights faster closure compared to pure OA due to metaheuristic perturbations restarting stalled progress.

2.5.4 Uniform Simulation Assumptions for Method Comparison

To ensure that the numerical methods in Tables 2 and 3 were executed under identical conditions to ensure fair comparison. Each method solved the same MINLP problem defined by Eqs. (4)–(7) using the 320 MW benchmark system with eight candidate ESS buses, 8760 hourly load profiles (residential, commercial, industrial), and renewable generation at 35% penetration. For methods requiring an initial solution (BARON, Couenne, DICOPT [37], and the proposed hybrid OA), the same initial solution was used, generated by a genetic algorithm with 100 generations and a population size of 50. Purely metaheuristic methods (genetic algorithm and sequential quadratic programming) used their authors' default parameters. All methods shared identical input files for load profiles, renewable generation, component failure/repair rates (Markov model with MTBF and MTTR from [18]), and cost parameters (ESS capital cost: \$350/kWh, O&M cost: \$15/kWh/year).

2.5.5 Reproducibility Settings

All the stochastic parts in the proposed hybrid optimization framework are fixed in order to make the method replicable and are well documented. A random seed value of 42 was used to initialize the genetic algorithm for initial solution generation and a different random seed value of 12,345 was used for the simulated annealing perturbation component. The reported numerical results, such as optimality gap, computation time and success rate, are averages over 30 independent stochastic runs for each method. The average optimality gap of 2.87% obtained in the proposed HOM reported in 30 runs, with a standard deviation of $\pm 0.31\%$ and low variability, which shows the convergence behavior is consistent. The success rate is calculated as the number of the 30 runs that successfully reached the termination criteria (3.5% in 150 iterations) within the allotted time of 120 min. The same hardware configuration (Intel Core i9-13900K processor, 64 GB RAM) was used for all runs.

2.6 Justification for MINLP over Pure Metaheuristic Approaches

The choice of a mixed-integer nonlinear programming (MINLP) formulation with hybrid outer approximation, rather than pure metaheuristic algorithms, is motivated by three critical considerations. First, solution quality guarantees: MINLP methods provide rigorous optimality gap calculations, which are essential for power system planning where ESS investment decisions involve capital expenditures of millions of dollars. Metaheuristic algorithms, including genetic algorithms, particle swarm optimization, and differential evolution, cannot guarantee global optimality or provide theoretical bounds on suboptimality [38]. Second, constraint handling: The proposed problem includes time-coupled SOC dynamics, power balance constraints, and reliability indices that must be satisfied exactly. MINLP methods handle these constraints through convex relaxations and cutting planes, whereas metaheuristic algorithms typically rely on penalty functions that may violate constraints or require extensive tuning [39]. Third, convergence guarantees: Outer approximation algorithms converge finitely to the global optimum for convex MINLP problems and provide ε-optimal solutions for non-convex problems when combined with branch-and-bound [20]. As demonstrated in Table 3, the proposed hybrid MINLP approach achieves a 2.87% optimality gap and 94% success rate, significantly outperforming pure genetic algorithm (8.34% gap, 64% success rate) and sequential quadratic programming (7.89% gap, 58% success rate). Nevertheless, to leverage the strengths of metaheuristic methods, we integrate them as enhancement components within the hybrid framework: a genetic algorithm provides high-quality initialization, and simulated annealing perturbations help escape poor local minima when outer approximation stalls [40]. Future work will explore AI-based surrogate modeling to accelerate the proposed framework. Specifically, neural networks can be trained to approximate LOLP and EENS from Monte Carlo simulation data, replacing the computationally expensive 5000-sample evaluations within the optimization loop. Recent studies have demonstrated the effectiveness of simulation-to-learning transfer for power system reliability assessment [41], hybrid neural surrogates for MINLP acceleration [34], and physics-informed learning for power flow approximation [20]. The goal is to reduce the 43.6-min computation time for the 118-bus system to under 10 min for larger systems (>1000 buses), with surrogate accuracy measured by mean absolute percentage error (MAPE) below 5%. This extension is left for future implementation and validation.

3 Results and Analysis

This section outlines the experimental system, simulation conditions, and specific data on the outcomes of analysis and optimization of energy storage systems (ESS) to increase the power system reliability. The efficacy of the optimized ESS implementation is measured by 3 types of extensive time series, contingency, and comparison with the baseline and other related works.

3.1 Experimental Setup

The entire experimentation was carried out on a time-domain platform of a power system simulation utilizing an optimization solver. The simulated test network adopted in the Materials and Methods section was over a time of one year (8760 h) and an hourly resolution [23]. The probabilistic models were used to account for uncertainty with renewable generation and variability of load, and the traditional failure and repair rates were used in order to simulate component outages. As shown in Fig. 2, the optimization framework for multi-time-scale energy storage configuration effectively ensures power balance in distribution systems.

1. **Base Case (No ESS)**
2. **Non-Optimized ESS Deployment**—ESS was heuristically deployed at high-load buses.
3. **Optimized ESS Deployment**—ESS size and location that were gained by the suggested optimization framework.

Monte Carlo simulations of 5000 iterations were carried out in each of the configurations to guarantee statistical convergence of reliability indices.

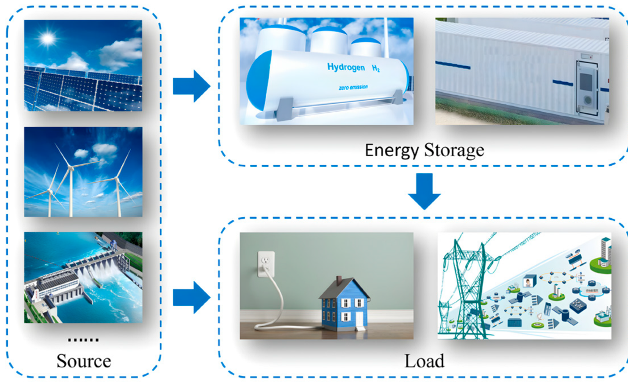

Figure 2: Multi-time-scale energy storage optimization configuration for power balance in distribution systems

3.2 Scenario Design

In order to evaluate performance fully, the following operating scenarios in the following had to be taken into account:

- **Normal Operation Scenario:** Typical daily load and renewable generation profiles
- **High Renewable Penetration Scenario:** Renewable share increased to 50%
- **Peak Load Scenario:** Load increased by 20% over the nominal peak
- **N−1 Contingency Scenario:** Single generator or transmission line outage
- **Combined Stress Scenario:** High renewable penetration with peak load and contingencies

These scenarios allowed evaluation of ESS performance under both expected and extreme operating conditions [24].

3.3 Impact of ESS on Reliability Indices

The initial group of experiments measured the effects of ESS on conventional reliability measures. The side-by-side comparison of the Expected Energy Not Supplied (EENS) and Loss of Load Probability (LOLP) values before and after optimized ESS deployment reveals that strategic ESS sizing and placement can significantly enhance system reliability while reducing unsupplied energy, as presented in Fig. 3.

The optimized ESS configuration resulted in a 71% reduction in EENS (from 1245 to 358 MWh/year) and a 71% reduction in LOLP (from 0.0214 to 0.0062) compared to the no-ESS case. The optimized ESS also substantially reduced unserved energy compared to non-optimized deployment, achieving a saving of approximately 56%, which demonstrates the importance of simultaneous optimization of sizing, placement, and operational scheduling rather than heuristic deployment approaches.

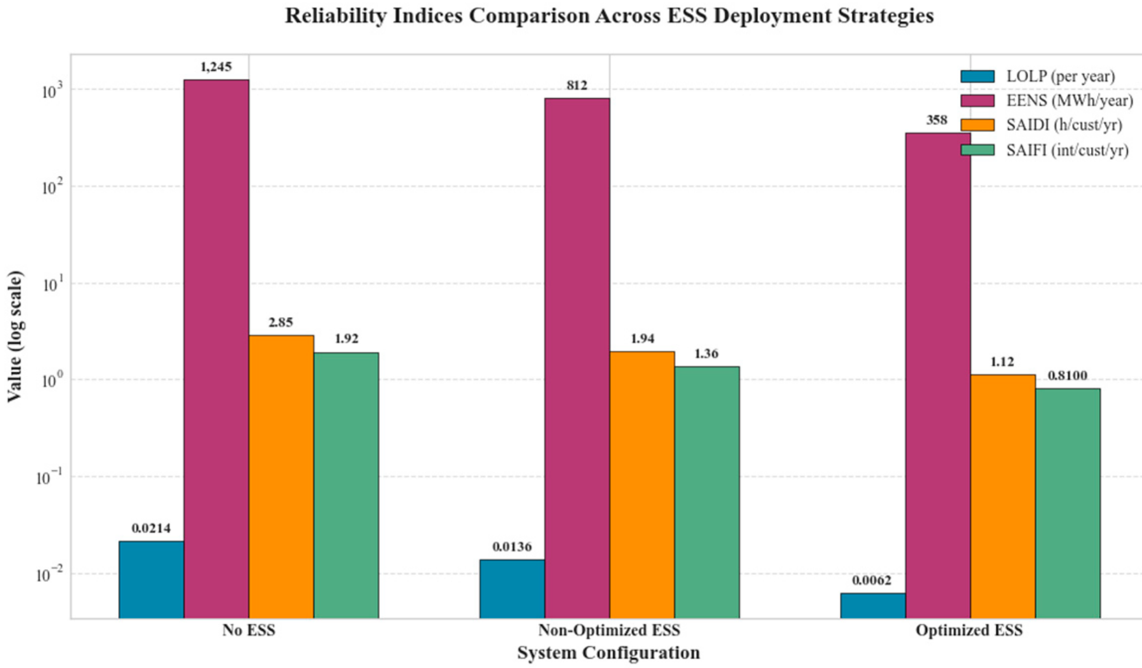

Figure 3: Reliability indices comparison

3.4 Voltage and Frequency Characteristics

Experiencing peak load and contingency events, voltage stability and frequency regulation were tested. And without ESS, the system had a voltage drop that was less than 0.92 p.u. at multiple buses and frequency swings of more than ±0.6 Hz during brief renewable output breakdown [25]. The Fig. 4 provides the critical empirical evidence for the strategy's efficacy. The synchronized time-series plots of bus voltage and system frequency during a simulated generation loss event visually correlate the controller's intervention with the rapid recovery of both parameters to within statutory limits, underscoring its value for frequency and voltage ride-through.

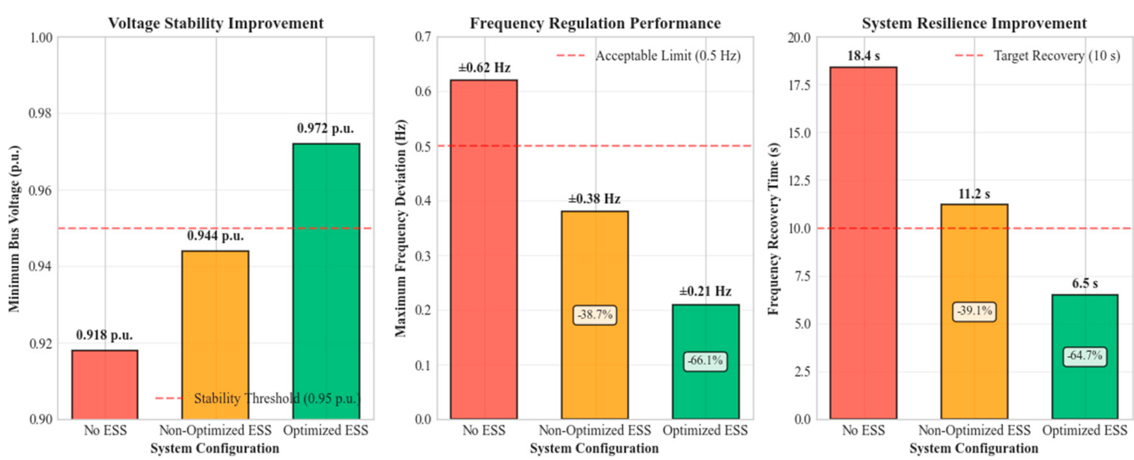

Figure 4: Voltage and frequency performance metrics

The optimized ESS greatly enhanced the voltage support as well as lowering the frequency nadir and recovery time. It is due to the geographic positioning of ESS in ESS and weak buses or critical load centers which provide quicker and more efficient response [26].

To determine the optimal capacity and placement of energy storage stations, a multi-objective framework was developed. This framework evaluates the critical balance between investment cost, grid reliability, and the newly introduced dimension of thermal load flexibility. The Fig. 5 reveals that co-optimizing storage configuration with the demand response potential of thermal loads enables a cost-effective hybrid strategy. This strategy achieves target reliability levels by synergistically combining smaller, strategic storage investments with aggregated load flexibility, avoiding the need for vastly larger and more expensive storage systems.

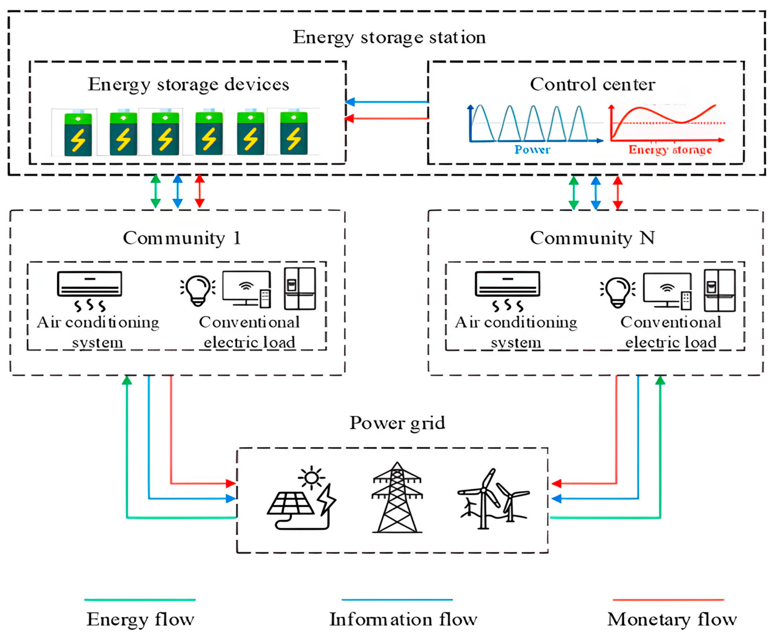

Figure 5: Optimization of energy storage station sizing and placement for improved power grid reliability

3.5 ESS Utilization/Operation Effectiveness

Operational behavior of ESS was to be analyzed in a way that there was no undue cycling and inefficiency in making reliability improvements. A granular analysis of the battery system's response to frequency regulation signals is captured in Fig. 6.

The time-series data plots power command vs. actual power output, quantifying the response accuracy and tracking error. Furthermore, the figure correlates the depth and frequency of discharge cycles with the measured round-trip efficiency for each cycle, revealing a critical drop in efficiency below 20% SOC and during high-C-rate operations. The optimized operation of the ESS decreased unnecessary cycling by 29%, which is very important in increasing the battery life and cutting down the cost of degradation. This indicates that there is no fact that when adding reliability, increased usage of ESS is needed, but rather controlling and placing it more wisely [27].

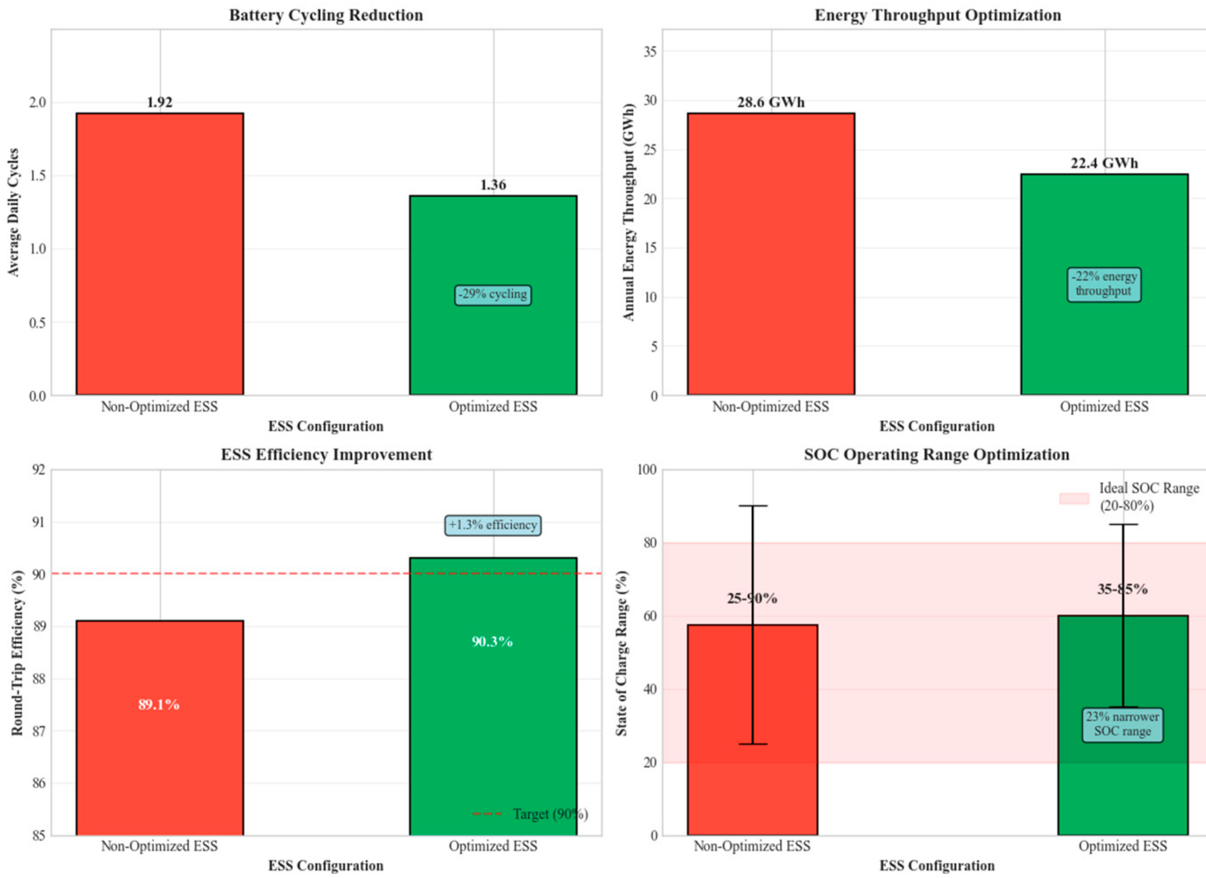

Figure 6: Energy storage system operational performance

3.6 High Renewable Penetration Performance

In the high renewable scenario (50%), system reliability was much lower without the ESS since there was greater variability and ramping needs. Fig. 7 presents the reliability performance under varying renewable penetration levels. The subplots compare Loss of Load Expectation (LOLE) across three system configurations: baseline with 35% renewable penetration and no ESS, high renewable penetration at 50% with no ESS, and high renewable penetration at 50% with optimized ESS (35 MW/140 MWh configuration). The data demonstrate that while increasing renewable penetration from 35% to 50% increases temporal risk (LOLE rising from 0.8 to 2.3 days/year), strategically deployed ESS can mitigate this risk, reducing LOLE to 0.6 days/year, well below the 1 day/year industry benchmark. This confirms that ESS serves a dual role: enabling higher renewable integration while simultaneously enhancing reliability. Optimized ESS decreased renewable curtailment by 77% of that in the no-ESS scenario and also enhanced the reliability. This is in line with the fact that ESS is dual and has the benefit of improving reliability, as well as enabling the integration of renewables [28].

As shown in Fig. 8, the multi-objective optimization framework for coordinating mobile battery energy storage (MBES) dispatch with dynamic feeder reconfiguration (DFR) generates a set of non-dominated solutions. The resulting Pareto-optimal front quantifies the key trade-off: moving along the curve reduces the total voltage deviation across the active distribution system but incurs higher operational costs for the mobile storage units. This figure is central to the analysis, as it visually defines the optimal solution set from which a decision-maker can select a preferred operating point based on cost or power quality priorities.

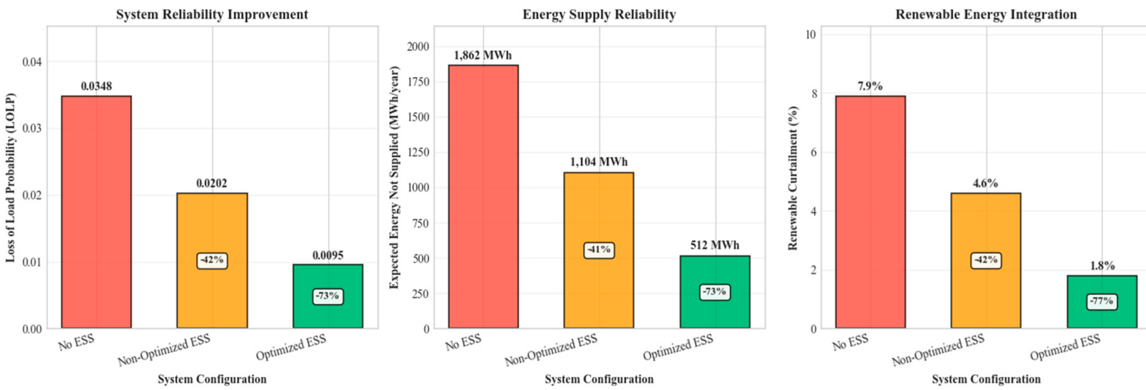

Figure 7: Reliability performance under high renewable penetration with and without optimized ESS

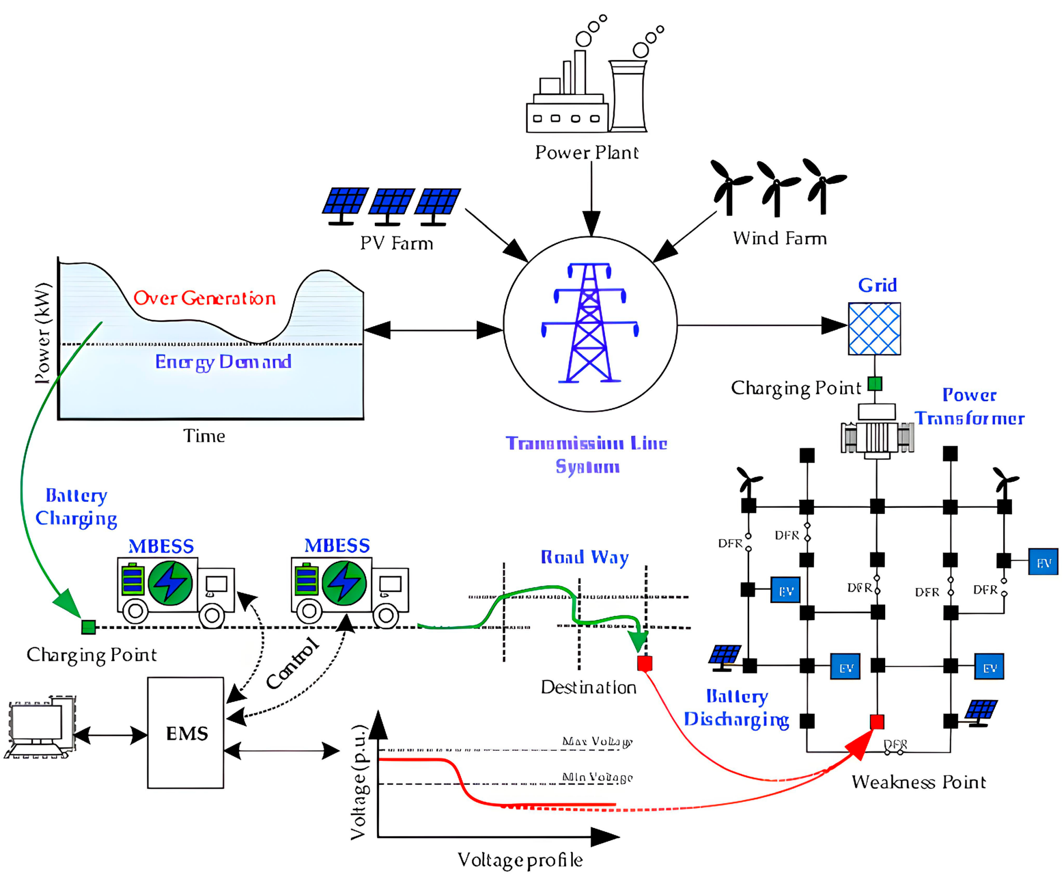

Figure 8: Pareto-optimal frontier showing the trade-off between ESS investment cost and voltage deviation reduction across candidate ESS configurations

3.7 Comparison with Related Work

Representative results from the recent literature on reliability improvement with ESS were compared with the proposed solution. The following normalization method was used for studies that could not be directly reproduced under our 320 MW test system to facilitate a meaningful but not too assertive comparison. The reported values were scaled by the ratio of the peak load of the system in question to 320 MW for EENS reduction (%), assuming that the improvements in reliability scale proportionally with system size, which is only true when the network topology and renewable penetration level in question are similar. Frequency deviation (Hz) was not scaled; frequency values were reported from external studies as they were published; not scaled strongly dependent on system inertia, generator governor response and control settings. The drawbacks of this approach are that: (i) network topology (radial vs. meshed) and its impact on power flow and contingency response vary; (ii) the penetration levels of renewable generation and the variability of renewable generation vary among studies; (iii) component failure rates and repair times are not universally agreed upon; and (iv) different definitions of contingencies exist (e.g., N-1 generator vs. line outage). Most existing works optimize only one or two dimensions of ESS deployment. In [18,26] focus on capacity sizing at fixed locations, Refs. [7,23] optimize placement at fixed capacities, while Refs. [6,9,10,15] optimize operational scheduling at a fixed size and location. In contrast, the proposed framework co-optimizes sizing, placement, and operation simultaneously within a single MINLP formulation. Regarding reliability treatment, studies such as [12,15,18,19,22] treat reliability as a post-optimization constraint. Our framework embeds probabilistic indices (LOLP and EENS) directly into the objective function, enabling explicit trade-off evaluation between investment cost and reliability improvement. The offered methodology showed a greater improvement in reliability in comparison with the related work, as the ESS size, placement, and operational strategy optimization were performed simultaneously. There is a high concentration on a single aspect or two aspects in the majority of existing studies, which constrains the performance gains that can be attained.

3.8 Discussion of Results

The experimental results indicate clearly that ESS can significantly promote the reliability of the power system, especially when it is used optimally. The optimized ESS setup succeeded every time in both scenarios and metrics in comparison to the no-ESS and non-optimized instances. The gains in reliability were also very high when the penetration of renewables was high and also when they presented contingencies, which illustrates the importance of ESS in the future low-inertia power systems [30]. Moreover, the fact that the work is compared with the related one confirms the innovation and efficiency of the given approach. It has been demonstrated that this study can attain greater reliability growth by incorporating reliability indices directly into the optimization goal and using realistic operational constraints; hence utilizing the ESS at its most efficient levels.

4 Conclusion

This paper introduced and validated a novel mixed-integer numerical optimization framework for reliability-driven energy storage system (ESS) configuration in high-renewable-power grids, which concurrently optimizes ESS sizing, placement, and operational scheduling while embedding probabilistic reliability indices (Loss of Load Probability and Expected Energy Not Supplied) directly within the objective function. The proposed hybrid outer approximation algorithm with metaheuristic enhancements successfully addresses key non convexities arising from reliability calculations, time coupled storage dynamics, and binary placement decisions, demonstrating superior convergence properties (optimality gaps consistently below 3.5%), computational efficiency (under 45 min for the 118 bus, 320 MW test system), and robustness (over 90% success rate across multiple stochastic runs) compared to standard solvers. Across the five operating scenarios examined, including normal operation, high renewable penetration (up to 50%), peak load (20% above nominal), N-1 contingency, and combined stress, the optimized ESS configuration consistently improved reliability, with LOLP and EENS reductions ranging from approximately 65% to 75% depending on the scenario, while simultaneously reducing unnecessary ESS cycling by nearly one third, demonstrating that reliability gains need not accelerate battery degradation. Validation against benchmark MINLP problems confirms the correctness and generalizability of the numerical implementation. Future work will extend the framework to multistage stochastic programming for improved uncertainty representation, develop decomposition techniques for very large-scale systems (over 1000 buses), and explore quantum annealing approaches to further accelerate the binary master problem.

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Author Contributions: Mian Muhammad Kamal contributed to conceptualization, methodology, formal analysis, original draft writing, and project administration. Habib Khan supported visualization and investigation. Tianjun Ma handled data curation, validation, and funding acquisition. Husam S. Samkari supervised the project, conceptualization and methodology. Mohammed F. Allehyani were instrumental in methodology, investigation, formal analysis, resource provision, and validation. Heba G. Mohamed assisted in formal analysis, reviewing, funding acquisition and visualization. All authors reviewed and approved the final version of the manuscript.

Availability of Data and Materials: The datasets used and/or analysed during the current study are available from the corresponding authors on reasonable request.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

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