

Dynamic Observability Method for Durability Assessment Considering Measurement Noise

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Abstract. *Due to the inevitable degradation of material properties in structures in daily use, such as stiffness degradation due to cracking in concrete elements, their durability will definitely be influenced, and their serviceability and safety could be in danger. Thus, understanding and identifying the change in the structural parameters provides new approaches to evaluate their durability. Structural system identification by dynamic observability method, which is using subsets of masses, natural frequencies and modal shapes, is a powerful tool to detect the change of structural parameters. Taking into account the presence of noise in the measurement data in real world structures, this method establishes the relative dynamic equation with the error separation items. The equation is solved by error minimization of an objective function combining the measured frequencies and mode shapes through the parameter MAC (Modal Assurance Criterion). Additionally, the algorithms and the steps are introduced based on the dynamic eigenvalue equation, which can fully demonstrate the performance of observability techniques. The present paper provides an example on how to successfully identify structural parameters. Its suitability for practical applications is demonstrated in a large frame structure. The result is a much more accurate identification of the parameters involved in the durability of the structure even in the case of noise-corrupted measurement signals*

Keywords: *Durability, Structural System Identification, Dynamic Observability Method, Frequencies, Mode Shapes.*

1 Introduction

Existing structures are inevitably exposed to the natural environment (wind, earthquake, temperature difference, or even some extreme climate) and human operation (traffic, impact, daily degradation by use). Thus, the material properties are degraded such as the bending stiffness, axial stiffness, and mass. In this case, the durability of the structures may be influenced, leading them structures into a dangerous condition instead of the originally safe state.

Regarding the durability of structures, understanding and identifying the change in the structural parameters is a great way to estimate the structure condition and safety. Structural system identification (SSI) has become a powerful tool to help engineer decision making during the life cycle of civil and infrastructures systems. SSI can be classified as the static and dynamic analysis, as well as deterministic and probabilistic methods. The former research classification depends on the type of excitation and the latter is related to the uncertainty of the parameters involved.

It is worth mentioning that the development of Observability method (OM) makes relevant contributions to deal with the remaining problem of SSI, and has been used in several engineering fields, such as hydraulics, electrical and power networks, or transportation. The

SSI by OM under static conditions was proposed to identify 2-D beam element models in its plane, dealing with the partial observability problem and achieving an accurate assessment. While it is much easier to dynamically excite a large structure than statically, especially in large scale structures, the dynamic data are more difficult to deal with than the static one. As a result, the standard observability method used with dynamic data has shown deficient in identifying degradation processes endangering durability.

Thus, this paper proposes a new constrained dynamic observability SSI methodology, which allows the identification of a subset of characteristics of a structure, related to its durability parameters such as stiffness and mass. Subsets of natural frequencies and modal shapes are used. Firstly, the description of dynamic constrained OM method is demonstrated step by step. Secondly, the verification of this method is revealed by one example considering the measurement noise. Lastly, the potential of dynamic constrained OM when applied to large structures is shown.

2 Methodology

The dynamic equation of motion of a system with no damping and no external applied forces is shown in equation (1), expressing what is the eigenvalue or characteristic value problem.

The equation can be expressed for a two-dimensional structure with N_N nodes, N_B boundary conditions and R vibration modes as:

$$[K]^{[(3N_N-N_B) \times (3N_N-N_B)]} \{\Phi_i\}^{[(3N_N-N_B) \times 1]} = \lambda_i \{M\}^{[(3N_N-N_B) \times (3N_N-N_B)]} \{\Phi_i\}^{[(3N_N-N_B) \times 1]} \quad (i = 1, 2, 3 \dots, R) \quad (1)$$

Where λ_i and Φ_i are the eigenvalues and mode shapes of i th mode, and the $[K]$ and $[M]$ represent the stiffness matrix and mass matrix. Φ_i includes the displacement in the x-direction (u_{ik}), y-direction (v_{ik}) and rotation (w_{ik}) at each node k for each vibration mode i .

The arrangement of equation (1) is done in a similar way to the static observability method, all the unknowns of the system are set up to one column vector and the known variables are joined into a vector $\{D\}$. In this way, the previous equation might be written as:

$$[B_i] \{z_i\} = \begin{bmatrix} K_{i,0}^{*(3N_N-N_B) \times rx} & -M_{i,0}^{*(3N_N-N_B) \times mx} \end{bmatrix} \begin{Bmatrix} \Phi_{Ki,0}^{*rx \times 1} \\ \Phi_{Mi,0}^{*mx \times 1} \end{Bmatrix} \\ = \begin{Bmatrix} M_{i,1}^{*(3N_N-N_B) \times nx} \Phi_{Mi,1}^{*nx \times 1} - K_{i,1}^{*(3N_N-N_B) \times sx} \Phi_{Ki,1}^{*sx \times 1} \end{Bmatrix} = \{D_i\} \quad (i = 1, 2, 3 \dots, R) \quad (2)$$

Where z_i represents all the unknowns, B_i and D_i are the corresponding coefficient matrix and augmented matrix of i th mode.

Equation (2) is the description in i th mode. When multiple modes are considered together, the equation will be built by combining information of several modes. For example, the first R modal information is given by $[B]\{z\} = \{D\}$ shown as follows:

$$[B]\{z\} = \begin{bmatrix} B_1 & 0 & 0 & 0 \\ 0 & B_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & B_R \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ \vdots \\ z_R \end{Bmatrix} = \begin{bmatrix} D_1 & 0 & 0 & 0 \\ 0 & D_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & D_R \end{bmatrix} = \{D\} \quad (3)$$

Expression in which $[B]$ is a matrix of constant coefficients, $\{D\}$ is a fully known vector and $\{z_i\}$ contains the full set of unknown variables. This system contains coupled and single variables. Thus, the identified coupled variables (*e.g.*, $EI_j w_{ik}$ that exist in Eq.(2)) are referred as observed variables. In order to uncouple the observed variables, *e.g.*, $EI_j w_{ik} = EI_j * w_{ik}$, the dynamic COM (constrained observability method) is here proposed based on a similar procedure as in the static case. However, in this case, the objective function is defined as:

$$E = \sum_{i=1}^R (\Delta f_i)^2 + \sum_{i=1}^R \left(1 - \frac{[\phi_{mi}^T \bar{\phi}_{mj}]^2}{(\phi_{mi}^T \phi_{mi})(\bar{\phi}_{mj}^T \bar{\phi}_{mj})} \right) \quad (4)$$

The modal assurance criterion (MAC) is used in Eq. (4), which consists of computing the so-called MAC values as a measure for the correspondence between the calculated mode shape ϕ_{mi} , obtained from the inverse analysis using the estimated stiffnesses and areas, and the measured shape $\bar{\phi}_{mj}$ as shown in Equation (5). Besides, Δf_i are the differences between the measured and estimated frequencies.

$$\text{MAC}(\phi_{mi}, \bar{\phi}_{mj}) = \frac{[\phi_{mi}^T \bar{\phi}_{mj}]^2}{(\phi_{mi}^T \phi_{mi})(\bar{\phi}_{mj}^T \bar{\phi}_{mj})} \quad (5)$$

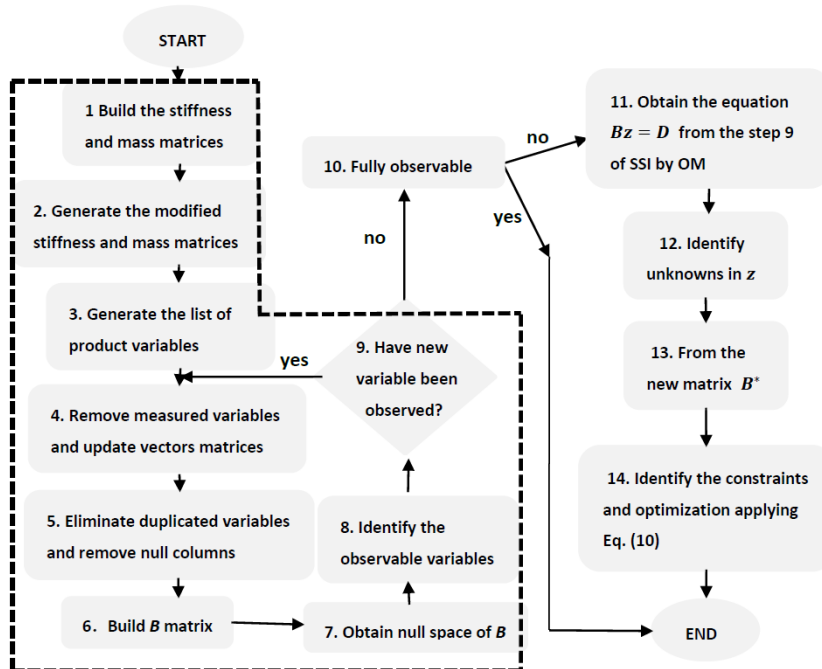


Figure 1. Flow-chart of algorithm of dynamic observability method.

The solution is obtained by minimizing equation (4) with the imposed constraints of the form: $EA_j u_{ik} = EA_j * u_{ik}$, $EI_j v_{ik} = EI_j * v_{ik}$, $EI_j w_{ik} = EI_j * w_{ik}$ present in equation (3), which is the so-called Constrained Observability Method .

The specific steps about the method are shown in Figure.1, which is developed on the base of the static COM method.

3 Example Analysis

3.1 Reinforced Concrete Beam

The dynamic constrained observability method is verified in a reinforced concrete beam. This beam is shown in Figure 2, with a length of 6m and equally divided into 30 elements, 10 parts, whose divisions can be seen minutely in Figure 2. The free vibration test of the beam was conducted by Ellen Simon, with the aim of obtaining its dynamic information, frequencies and mode shapes. The structure stiffnesses are estimated in the following way: $EI_i = x_i EI_{int}$, $i = 1 \sim 10$, $EI_{int} = 7.23 \times 10^6 \frac{N}{m^2}$, $x_i = [1.15, 1.04, 0.86, 0.90, 0.80, 0.54, 0.51, 0.57, 0.72, 0.86]$ by using the experimental data after some controlled damage was produced in the beam.

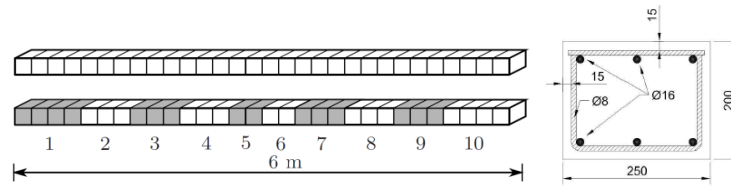


Figure 2. Set-up of static loading.

There is one way in using the estimated stiffnesses of EI_i as the input to do the direct analysis of the first four frequencies and mode shapes since the real experimental data is difficult to get from paper. Therefore, 100 samples of frequencies are generated for this set with an error of 2% while 100 samples of mode-shapes are obtained with an error of 10%, because frequency accuracy obtained from dynamic tests is normally higher than for mode. One sample for a given measurement set is the collection of the associated measurements generated by Eq. (6).

$$\tilde{\delta} = \delta(1 + E_{level} \cdot \xi) \quad (\tilde{f} \text{ yielded in the same way}) \quad (6)$$

Here, the measurements $\tilde{\delta}$ are simulated by adding proportional errors to the theoretical values, δ are obtained by direct analysis. E_{level} is the error level in the measurement and ξ is a random number following a normal distribution with mean zero and standard deviation 0.51 to guarantee the 95% confidence interval. The samples of \tilde{f} are yielded in the same way.

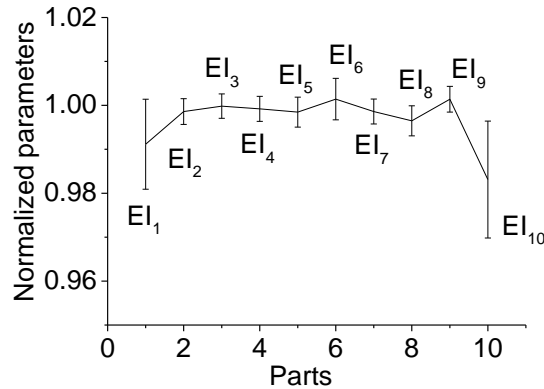


Figure 3. Mean and standard deviation of stiffness.

Figure 3 shows the estimated mean values of the 100 samples and their standard deviation for the estimated stiffness along the beam. It is shown how the errors in the mean values are less than 2% and the biggest standard deviations less than 1.5%, which is a reasonable range for parameter evaluation. However, it can be obviously observed that the standard deviations of EI_1 and EI_{10} are higher than other parameters, which may be due to the error accumulation from middle to the ends of the beam in the computational solution. Also the area of EI_1 and EI_{10} provides little information to the solution algorithm because of their free ends. This case can effectively explain the operability of the dynamic observability method to obtain the actual condition of the structure or its elements in relation to their durability.

3.2 Example of 13-Story Building

In order to show the possible applications and potential of the proposed methodology, a more complex example is presented in this section. The 13-story building shown in Figure 4 is taken under study. This structure was considered in Reference [16].

This frame is modeled using a total of 226 nodes and 273 elements and it is composed of a set of 8 different sections described in Figure 4, and the parameters are the same as in [16]. Therefore, the size of the system of equations is 678×678 . The study will be performed for the first two vibration modes, which can provide relevant modal information. This structure is analyzed from a local perspective, using the constrained dynamic observability technique to only a part of the unknowns as shown in figure 4.

In fact, there are occasions in which there is interest of obtaining only the properties of one or a few members of the structure to check, for example, if it is locally damaged. This leads to a more economically efficient application of the technique. The normal procedure is to measure some of the displacements around this element or elements in order to activate its matrix components.

For this example, an assumption is made that the bending stiffnesses EI_1 and EI_2 of the structure are unknown and that the mechanical properties of the floor slab of the 12th floor are different from the theoretical one, because this part of the structure is damaged. Its modal frequency (only the first frequency is required) and mass are assumed to be known. Therefore, the properties to be observed are the bending stiffnesses of the elements between nodes 27 and 41.

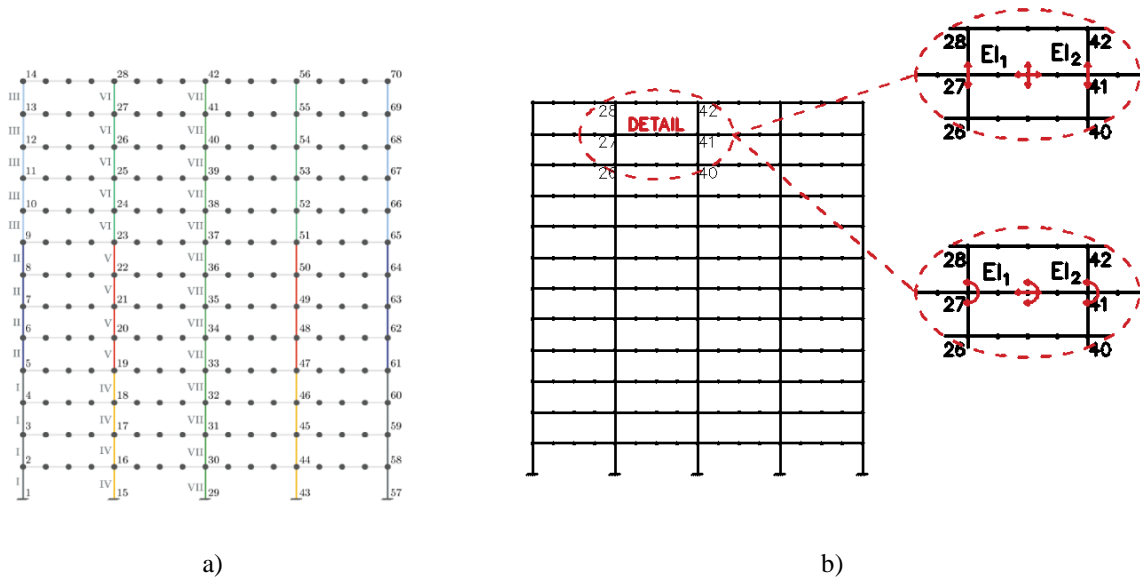


Figure 4. a) Members with different characteristics are represented with different colors.
 b) Sets of measurements used in the local analysis.

Two sets of measurements are proposed. The first one includes the vertical displacements in three nodes and the horizontal displacement in one node, which can be seen in the A detail in Figure 4(b) (4 measurements+ 1 frequency). The second one includes rotations in three nodes and horizontal displacements in one node, as shown in the B detail in Figure 4(b) (4 measurements+ 1 frequency).

After performing the analysis, the stiffnesses EI_1 and EI_2 can be identified by Constrained Observability assuming 2% and 5% errors in the frequencies and mode shapes respectively. The 100 samples are gained by Eq. (6). The estimated mean values and standard deviations of the 100 samples under the two measurement sets are illustrated in figure 5.

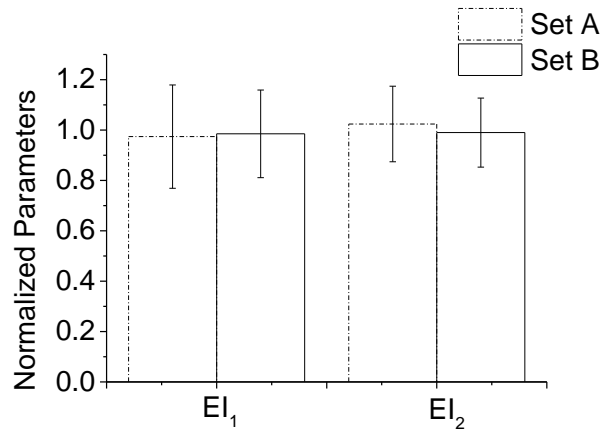


Figure 5. Mean and standard deviation of stiffness EI_1 and EI_2 .

From figure 5, the maximum standard deviations are 2.0% and 1.7% for set A and set B

respectively, and the mean of EI_1 and EI_2 under both sets are very close to the true value, with a 2.3% error. No matter the set of A or B, the estimated value of EI_2 is a bit better than for EI_1 because of the slightly smaller standard deviations, which may be due to the stronger boundary conditions of node 41 than node 27. It has to be emphasized that these bending stiffnesses can be observed without knowing the surrounding ones, which shows the potential of the method when applied to large structures where only a small number of measurements can be obtained. The potential application of constrained dynamic observability method to large scale structures is then fully illustrated.

4 Conclusions

The main objective of this paper is the application of constrained observability techniques for parametric estimation of structures using dynamic information such as frequencies and mode-shapes. The final objective is to show the applicability of the method in efficiently identify the possible damages existing in the structure that may affect its durability (mainly cracking affecting the bending stiffness). A new algorithm is introduced based on the dynamic eigenvalue equation. The merit of the dynamic constrained observability analysis is demonstrated in the analysis of a RC beam and a large frame structure.

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