

# Filtering Spurious Eigenmodes in Electromagnetic Cavities Discretized by Energy-Orthogonal Twenty-Nodes Hexahedral Finite Elements

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An electromagnetic cavity as discussed in this contribution will be a source-free region enclosed by a perfect conductor and filled with a linear dielectric in which modes of free oscillation can exist at an infinite number of discrete frequencies. These standing-wave fields and natural frequencies are solutions of the eigenproblem associated with the double-curl operator. If the electric field is the variable solved then the magnetic field will be called the induced field, and vice versa. As is well known, if the operator double-curl is discretized with nodal finite elements then eigenspectrum computed will be severely polluted with spurious eigenmodes, the spurious eigenvalues having the same order of magnitude as the physical eigenvalues and not easily distinguished from them [1].

A standing-wave field can be represented by the superposition of plane harmonic waves travelling in several directions. For these fundamental wave solutions, by a dispersion analysis, the behavior of the energy density of the induced field is researched in unbounded domains discretized by regular meshes of twenty-nodes hexahedral finite elements formulated in energy-orthogonal form [2]. In this formulation the element stiffness matrix is split into basic and higher order components, which are respectively related to the mean and deviatoric components of the element induced field. This decomposition is applied to the element energy of the induced field and holds for the finite element assemblage. The first physical branch of the dispersion equations is selected for the analysis. The main results are summarized. Given the mesh, in the limit of long wavelength, although the energy density does not vanish, its higher order component does vanish. Similarly, given the wavelength, as the solution converges on account of mesh refinement, the energy density is increasingly dominated by its basic component. Nevertheless, the higher order energy density vanishes as a cancellation of the component associated with the vertex nodes and the one associated with the mid-side nodes, which do not vanish but are equal and opposite in sign. A sign characteristic is also satisfied by the vertex and mid-side components of the basic energy density. The convergence values of these energy components do not depend on the direction of wave propagation, direction of polarization, and mesh parameters. The relationship between the density of the mesh, the higher order energy density, and the energy error is researched for high precision discretization.

For cavities discretized by twenty-nodes hexahedral finite elements is proposed to accept only those eigenmodes for which the basic energy component of the induced field prevails over the higher order energy component. Additionally, the behavior of the vertex and mid-side energy components must be in accordance with the fundamental behavior deduced for these energy components by the dispersion analysis carried out for plane harmonic waves. The numerical research reveals that applying these energy constraints the spurious eigenmodes can be efficiently identified.

## REFERENCES

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