A fully coupled thermo-hydro-mechanical (THM) finite element (FE) formulation is presented that considers freezing and thawing in water-saturated soils. The formulation considers each thermal, hydraulic and mechanical process, and their various interactions, through fundamental physical laws and models. By employing a combination of ice pressure, liquid pressure and total stress as state variables, a new mechanical model has been developed that encompasses frozen and unfrozen behaviour within a unified effective-stress-based framework. Important frozen soil features such as temperature and porosity dependence of shear strength are captured inherently by the model. Potential applications to geotechnics include analysis of frost heave, foundation stability or mass movements in cold regions. The model’s performance is demonstrated with reference to the in situ pipeline frost heave tests conducted by Slusarchuk et al. Detailed consideration is given to FE mesh design, the influence of hydraulic parameters, and the treatment of air/ground interface boundary conditions. The THM simulation is shown to reproduce, with fair accuracy, the observed pipeline heave and the porosity growth driven by water migration.

KEYWORDS: ground freezing; numerical modelling; temperature effects; water flow

INTRODUCTION
Freezing and thawing of pore fluid within soils involves complex thermal, hydraulic and mechanical processes that can have significant mutual geotechnical interactions. For example, phase changes of pore fluid caused by temperature variations modify the hydraulic regime of the soil, which in turn induces mechanical deformation. At the same time, any change in the hydraulic and mechanical conditions feeds back to the thermal processes by way of advection and changes in ice and water contents. Such thermo-hydro-mechanical (THM) interactions underlie many cold-region geomorphological processes, such as solifluction and thermokarst formation, as well as geotechnical problems, such as frost heaving, foundation distress/settlement and slope instability.

Analysis of frozen soils’ THM-coupled behaviour inevitably requires a numerical technique such as the finite element method (FEM), owing to the non-linearity of the governing equations, and their reciprocal coupling. Models have been developed and implemented with differing degrees of sophistication, depending on their particular application purposes. However, a solid framework has yet to be established for fully coupled THM approaches. Historically, most geotechnical analyses of frozen soils adopted total-stress-based mechanical treatments (e.g. Ladanyi, 1972, 1975; Ladanyi & Johnston, 1974; Nixon, 1978; Jessberger, 1981; Andersland & Ladanyi, 2004), sometimes combined with uncoupled thermal analysis (e.g. Nixon, 1990). Total-stress treatments have continued to be adopted in more recently proposed models (e.g. He et al., 1997; He & Cheng, 2000; Arenson & Springman, 2005).

Parallel efforts to simulate mass and heat transfer problems in freezing and thawing soils have led to almost independent developments in TH-coupled models, often focusing on frost heave analysis, for which three main streams of development can be identified:

(a) rigid-ice models (e.g. Miller, 1978; Gilpin, 1980; O’Neill & Miller, 1985; Nixon, 1991; Nakano, 1997)
(b) hydrodynamic models (e.g. Harlan, 1973; Guymon & Luthin, 1974; Jame & Norum, 1980; Newman & Wilson, 1997; Hansson et al., 2004; Hansson & Lundin, 2006)
(c) the segregation potential model (Konrad & Morgenstern, 1980, 1981, 1984).
Their principles and characteristics are discussed, for example, by Kujala (1997). A common feature of TH-coupled models is their lack of an explicit formulation for the mechanical behaviour of the soil skeleton. In hydrodynamic models, mechanical equilibrium is usually not considered. In the other two approaches, the soil skeleton stresses are only implicitly invoked in the particle segregation criteria. When shear stresses and soil deformation play a significant role, as in many boundary value problems, an independent treatment is required outside the TH-coupled framework. Carlson & Nixon’s (1988) pipeline frost heave treatment is one such example discussed later in this paper.

Mechanical terms that reproduce the failure/deformation of frozen ground have been incorporated explicitly into fully coupled THM models (Blanchard & Fremond, 1985; Li et al., 2000, 2002) and semi-coupled THM formulations (Selvadurai et al., 1999a, 1999b). Fully coupled THM formulations based upon fundamental physics provide unified models that reproduce both the failure/deformation and the coupled heat/mass transfer in frozen ground, which conventionally have been treated in separate ways. In order to complete this integration, the mechanical constitutive model for frozen soils must have continuity with the effective-stress constitutive models applied to unfrozen soils. Most boundary value problems involve both states, and transient moving boundaries between them. The architecture of such models does not appear to be well established.

The following sections outline a fully coupled THM FEM framework and a new critical-state elasto-plastic mechanical soil model developed to consider problems involving frozen and unfrozen soil. Quantitative evaluation is demonstrated by examining frost-heave processes. The potential drawback of existing THM formulations (Blanchard & Fremond 1985; Li et al., 2000, 2002) that adopt a single Bishop-type stress variable (Bishop & Blight, 1963) are avoided by adopting two stress variables, following the recent trend in unsaturated soil mechanics (e.g. Fredlund & Morgenstern, 1976; Alonso et al., 1990, Fredlund, 2000). As discussed later, the present model has limitations in describing some frozen soil features such as segregation-dependent stress–strain characteristics, and can reproduce only one freeze/thaw cycle. However, the proposed framework should allow future incorporation of effective-stress-based formulations that can reproduce such features in unfrozen soils.

In the final part of the paper, the proposed THM model is verified against a series of in situ frost heave tests conducted by Slusarchuk et al. (1978) involving a chilled pipeline. Issues relating to numerical implementation are discussed, including mesh conditioning, assessment of hydraulic parameter influence, and prescription of surface thermal boundary conditions. This study, in conjunction with a geographically broader approach (Nishimura et al., 2009), formed part of a multidisciplinary Imperial College project funded by BP that investigated possible climate change impacts on cold region infrastructure (Clarke et al., 2008).

GOVERNING EQUATIONS

The frozen soil formulation presented builds from the THM modelling by Olivella et al. (1994, 1996) and Gens et al. (1998) of high-temperature problems involving a gas phase. Low-temperature problems can be considered with the same overall structure if the original gas phase is replaced by a new solid phase representing ice. For simplicity, no gaseous phase (undersaturation) or dissolved salt is considered at present. The formulation has been implemented in the FEM code CODE_BRIGHT (Olivella et al., 1996), and we consider below the new features required to address freeze/thaw conditions.

Thermodynamic equilibrium of freezing soil

The equilibrium between liquid water and ice phases is described by the Clausius–Clapeyron equation, derived from thermodynamic potentials between two phases. This equation is expressed as

\[-(s_1 - s_2)\frac{dT}{dP} + v_1 dP_1 - v_2 dP_2 = 0\]  

(1)

where \(s\) and \(v\) are the specific entropy and the specific volume respectively, \(T\) is temperature on the thermodynamic scale, and \(P\) is pressure (e.g. Henry, 2000). Subscripts 1 and 2 refer to each of two different phases/materials in equilibrium. When applied to equilibrium between pure ice and liquid water,

\[-(s_1 - s_2)\frac{dT}{dP} + v_1 dP_1 - v_2 dP_2 = 0\]  

(2)

or

\[dP_1 = \frac{\rho_1}{\rho_1} dP_2 - \frac{\rho_1}{\rho_1} \frac{l}{T} dT\]  

(3)

where \(l\) is the specific latent heat of fusion, \(\rho = (1/\rho)\) is the mass density, and the subscripts 1 and 2 refer to liquid water and ice respectively. This differential form can be integrated using atmospheric pressure and the temperature 273.15 K as references, to give

\[P_1 = P_2 \left( \frac{T}{273.15} \right)^{\frac{l}{P_0}}\]  

(4)

This equation represents a thermodynamic requirement for equilibrium that needs to be satisfied by \(P_1\), \(P_2\) and \(T\).

Freezing characteristic function

The modelling requires a freezing characteristic function, which is used to relate the degree of liquid (unfrozen water) saturation \(S\) to the soil’s thermodynamic properties. Many researchers have developed freezing characteristic functions through analogies with water retention models developed to describe the drying and wetting of unsaturated unfrozen soils, where gas and liquid phases coexist in the pores (e.g. Koopmans & Miller, 1966; Miller, 1978; Black & Tice, 1989; Grant & Sletten, 2002; Coussy, 2005). The different pressures between the liquid water and ice phases expressed by the Clausius–Clapeyron equation (equation (4)) suggest that surface tension forces should develop along the interface between the two phases, as illustrated in Fig. 1. The freezing characteristics and the water retention characteristics are both determined by the soil pore size distribution (e.g. Fredlund & Xing, 1994; Coussy, 2005) and the interface tension force. It is therefore natural to assume that the two functions can be expressed by similar forms of equation, once allowance is made for the difference between the ice/liquid and gas/liquid interface tension forces. The van Genuchten (1980) model was employed here to represent the freezing characteristic function,

\[S_1 = \left[ 1 + \left( \frac{P_1 - P_1}{P} \right)^{\frac{1}{\lambda}} \right]^{-1}\]  

(5)

where \(P\) and \(\lambda\) are material constants. This equation represents the relationship between \(P\), \(P_1\) and \(S_1\). The generally hysteretic nature of soil freezing characteristic (e.g. Williams, 1964) is not considered in this study.
Fig. 1. Micro-configuration of phases in frozen soil

By combining equations (4) and (5), the freezing characteristic function relating $S_l$ to $T$ can be obtained as

$$S_l = \left\{ 1 + \left[ -(1 - \rho_l/\rho_i)\rho_i - \rho_l \ln(T/273-15) \right] P \right\}^{1/2}$$

Equation (6) accounts explicitly for the liquid pressure $P$, unlike other commonly used empirical expressions such as $w_u = \alpha[T - T_i]^\beta$ (where $\alpha$, $\beta$, and $T_i$ are material properties) that relate the unfrozen water content $w_u$ to temperature, $T$ (Anderson & Morgenstern, 1973; Romanovskii & Osterkamp, 2000; Andersland & Ladanyi, 2004). We note, however, that the influence of $P$ on $S_l$ is relatively minor.

Mass/heat transfer

Mass conservation of pore water is expressed as

$$\frac{\partial}{\partial t}(\rho_l \phi) = q_l + \nabla \cdot (\rho_l \phi g)$$

where $\phi$ is the porosity; $S_i$ and $S_l$ are degrees of liquid and ice saturation respectively ($S_i + S_l = 1$, as neither a gas phase nor cavitation is considered); $q_l$ is the liquid water flux vector; and $\mathbf{g}$ is the sink/source term of mass. The water flux is calculated from generalised Darcy’s law as

$$q_l = -k_l \frac{\rho_l g}{\mu_l} (\nabla \rho_l - \rho_l g)$$

where $\mathbf{g}$ is the gravity acceleration vector, $k_l$ is the intrinsic permeability matrix, $\mu_l$ is the viscosity of liquid water, and $k_i$ is the liquid phase relative permeability. Viscosity can be considered a function of the temperature, as

$$\mu_l = 2.1 \times 10^{-6} \exp\left(\frac{1808.5}{273.15 + T}\right) \text{ (Pa s)}$$

Relative permeability can be calculated from the expression

$$k_s = \sqrt{k_i \left[ 1 - \left(1 - S_l^{1/\lambda}\right)^2 \right]}$$

which can be derived from the van Genuchten function (equation (5)). The parameter $\lambda$ is a material constant that in principle coincides with that used in the retention curve (equation (5)), and $k_s$ varies between 0 and 1. The intrinsic permeability in the generalised Darcy’s equation and the hydraulic conductivity $[k]$ (usually used in the flow equation when written in terms of piezometric head) are related by

$$[k] = \frac{\rho_l G_k}{\mu_l}$$

For reference, the relationship between the hydraulic conductivity and intrinsic permeability for water at a temperature of 20°C is: $[k]$ (m/s) = $k$ (m$^2$) × 1000 (kg/m$^3$) × 9.81 (N/kg)/10$^{-3}$ Pa s = 9.8 × 10$^6$ (1/m s) × $k$ (m$^2$). In the present study, permeability is considered isotropic.

The energy conservation equation is written as

$$\frac{\partial}{\partial t}(c_i\rho_l\phi(1 - \phi)) + \nabla \cdot (-k_i \phi \nabla T + j^e) = 0$$

where $c_i$, $e_i$ and $l_i$ are the specific internal energy of solid soil minerals, liquid water and ice respectively; $\lambda$ is in this case the overall thermal conductivity of the soil mass (consisting of soil minerals and pore materials); $j^e$ is the advective term of heat flux ($j^e_i = c_i \rho_i q_i$); and $f^e$ is the sink/production term of energy. Fourier’s law is employed in the above equation for calculating the conductive heat flux. The overall thermal conductivity $\lambda$ is calculated by using the geometric mean (Côté and Konrad, 2005),

$$\lambda = \lambda_s^{1/2} \lambda_i^{1/2}$$

where the subscript $s$ denotes the soil mineral phase. The specific internal energies, $e_i$, $e_i$ and $l_i$, are

$$e_i = e_i^0 + c_i T$$

$$l_i = l_i^0 + c_i T$$

where $c_i$ and $c_i$ are the specific heats for solid soil mineral, liquid water and ice respectively.

Mechanical equilibrium

Mechanical equilibrium can be written as

$$\nabla \cdot \sigma + b = 0$$

where $\sigma$ are total stresses and $b$ are body forces.

MECHANICAL CONSTITUTIVE MODEL FOR FROZEN AND UNFROZEN SOIL

Some researchers have proposed using a single Bishop-type stress variable (Bishop & Blight, 1963),

$$\sigma_o = \sigma - \chi P_i [I] - (1 - \chi) P_i$$

([I] is the unit matrix), when defining constitutive relationships for frozen soils linking stress $\sigma$ and strain $\varepsilon$ (see Miller, 1978, or Li et al., 2002), considering that the term $\sigma_o$ provides an equivalent effective stress for frozen soils. The parameter $\chi$ is sometimes assumed to be equal to the degree of liquid saturation, $S_l$. However, an inexpedient aspect of its use is that, if the soil skeleton is very loose and ice pressure takes the major part of the total stress, $\sigma_o$ becomes close to zero. If $\sigma_o$ alone is used, conventional effective-stress-based failure criteria developed for unfrozen soils (for example Mohr–Coulomb) predict very low shear strength, whereas ice-rich loose frozen soils tend to have relatively high short-term shear strength. One approach to overcome this problem is to develop failure criteria that are dependent on another variable, or combination of variables. Noting the close analogy between the physics of frozen-
saturated and unfrozen-unsaturated soils, this study adopted an alternative two-stress variable constitutive relationship (see Fredlund & Morgenstern, 1976; Alonso et al., 1990; Fredlund, 2000; Gens et al., 2006; Gens, 2007). Adopting stress variables \( p_n = p - \max(P, \dot{R}) \) and \( s = \max(P - \dot{R}, 0) \) (in addition to the deviatoric stress \( q \)) allows the Barcelona Basic Model (the BBM; Alonso et al., 1990) developed for unsaturated soil to be extended to provide a new constitutive model that describes the essential features of frozen and unfrozen behaviour. The variables \( p_n \) and \( s \) can be interpreted as the net stress representing external confinement and the suction respectively. The yield surface is expressed as

\[
F = \left[ p_n - \left( \frac{p_n - k_s}{2} \right) \right]^2 + q^2 - \left( \frac{p_n + \dot{k_s}}{2} \right)^2
\]

where

\[
q = \sqrt{\frac{1}{2}(s_{ij} - s_{ii})}, \quad s_{ii} - \sigma_{ii} - p\delta_{ii}
\]

\[
p_n = p^0 f\left( \frac{p_n}{p^0} \right)\left( \frac{1}{1 - \gamma} \right)^{\frac{1}{2}}
\]

\[
\lambda = \lambda(0)(1 - \gamma)^{\frac{1}{2}}\exp(-\beta s) + r
\]

and \( M, k, \lambda(0), \kappa, p^0, \beta \) and \( r \) are constants; see Alonso et al. (1990) for details.

Considering the \( q-p_n \) plane, an associated flow rule is assumed, so the plastic potential is expressed by the same equation as equation (17). The hardening is defined by the plastic volumetric strain \( \varepsilon_v^p \) as

\[
\varepsilon_v^p = \frac{1 - e}{\lambda(0)} r^0 p^0 \dot{p}_n e^v
\]

where \( e \) is the void ratio. The yield surface defined by the \( \lambda(0), \kappa(0), p^0, \beta(0), \beta(0), \gamma(0), \) and \( r(0) \), the model reduces to an effective stress-based critical state model similar to Modified Cam-Clay (Roscoe & Burland, 1968, or Wood, 1990). As temperatures fall below the freezing point, \( s \) increases and the yield surface cross-section in the \( q-p_n \) plane expands, giving the soil higher yield stress and strength. Some of the original BBM features remain unactivated, such as the influence of \( s \) on \( \kappa \), the elastic volumetric changes caused by changes in \( s \), and the SI surface (a ‘cap’ yield surface parallel to the \( q-p_n \) plane). However, these features could be readily incorporated in cases where they are considered relevant.

Fig. 2. Three-dimensional view of yield surface of the constitutive model for frozen soils

Fig. 3. Projections of yield surface of the constitutive model for frozen soils. \( T_1 \) and \( T_2 \) are arbitrary temperatures below the freeze point \( T_f \) such that \( T_1 < T_f < T_2 \); (a) ice-rich soil; (b) ice-poor soil.

Model evaluation: shear strength

The shear strengths developed by the model depend on porosity in a similar way to the Cam-Clay model for unfrozen soils. An additional feature is the gain in shear strength with ice content which grows as the ‘suction’ \( s \) increases. The diagrams in Fig. 3(a) illustrate a yield surface configuration for ice-poor, low-porosity soils, showing how the yield surface associated with the colder, frozen soil grows beyond that applying to the unfrozen state. The expansion of the yield surface in the \( q-p_n \) plane during cooling (which coincides with increasing \( s \)) involves both enhanced particle interlocking (expansion towards the right) and ice strengthening (expansion towards the left). In a physical sense, the latter is considered to induce the former through increased internal confinement, as suggested by Ladanji & Morel (1990). With ice-rich, high-porosity soils, as illustrated in the diagrams in Fig. 3(a), the cooling-induced expansion of the yield surface cross-section is dominantly in the leftward direction, indicating the importance of the ice strength. The modelling mechanisms provide logical descriptions of frozen soil’s porosity-dependent strength.

Early experimental work by Goughnour & Andersland (1968) demonstrated that the peak shear strength of frozen Ottawa Sand decreases as the porosity increases, tending towards the strength of pure ice when the porosity reaches 100%. Fig. 4 compares their findings with the trends for schematic shear strength (shear stress at the critical state) predicted by the proposed model. As mentioned earlier, the strain-rate-dependent nature of frozen soil is not considered at this stage.

A series of single-element FEM simulations was run to confirm the model’s performance in triaxial compression. The input parameter values used here, shown in Fig. 5, are the same as those used in the two-dimensional simulation.
A simple example of undrained freezing–thawing cycles under constant isotropic total stress is considered below to illustrate some of the advantages and limitations of the present formulation. The volumetric strain of soil caused by the (approximately 9%) expansion of pore water on freezing is a function of porosity \( \phi \), and is denoted here as \( \Delta e_{\text{we}} \). When freezing initiates from a relatively high stress, such as point A in Fig. 6(b), the predicted volume–\( p_0 \) path follows a closed loop (loop A–B), and no thaw consolidation occurs under constant \( p \). In contrast, when the volumetric-stress path engages the yield surface during volume expansion from a low initial stress, such as point C in Fig. 6(b), part of the expansion is accommodated by plastic volumetric straining, resulting in a softening that reduces \( p_0 \) between points A and E. Elastic models would predict a considerable tensile stress as the volumetric strain increases, which would impede further inflow of water. Whereas such a mechanism may be relevant to well-cemented geomaterials, such resistance against water inflow should be minimal in uncemented soils. The elasto-plastic features in the model are effective in reproducing the likely behaviour of the soil skeleton during loosening/segregation. If the freezing induces significant swelling, as in frost heave phenomena, the state path follows the trend indicated in the bottom diagram in Fig. 3(b), and reaches a high final porosity. Since the model is volumetrically hardening, this swelling results in shrinkage of the yield surface cross-section in the positive \( p_0 \) domain: compare the middle diagram in Fig. 3(a) with that in Fig. 3(b).

Further elaboration of the model is required to capture some other features of frozen soil behaviour, such as cumulative thaw consolidation. Fig. 6(a) shows a schematic illustration of the results obtained by Nixon & Morgenstern (1973) in their constant-vertical-load oedometer tests. The samples were subjected to a sequence of (a) undrained freeze, (b) undrained thaw and (c) consolidation. This cycle caused cumulative elastic volumetric strain under constant vertical load. The high excess pore water pressure, and hence the low effective stress, developed at the end of the thawing stage formed 'residual stress' is considered to be responsible for the final shrinkage process such as dilatancy. This feature, which is beyond the scope of this paper, could be addressed by applying approaches similar to those developed with critical state models to reproduce realistic cyclic loading responses within the main yield locus.

**APPLICATION OF THM ANALYSIS TO FROST HEAVE PREDICTION**

**Background and problem setting**

The performance of the THM model and its numerical implementation were evaluated with reference to published pipeline frost heave experiments. Frost heave refers to ground expansion caused by water migration and accumulation in a frozen fringe (i.e. a transitional zone just behind a freezing front, where soil is partially frozen). The water migration is driven by cryogenic suction (represented by equation (5)) but at the same time is impeded by the reduced permeability developed in partially frozen soil. This phenomenon is most pronounced in silty soils, in which moderate to strong suctions can be generated while retaining relatively high permeability. Frost heave is of particular concern in highway and pipeline engineering.

A benchmark series of in situ tests conducted by Slusarchuk et al. (1978) in Calgary allow the model to be evaluated. Trenches were made in permafrost-free silty ground and full-scale 1.22 m diameter steel pipelines were installed, with the original soil being placed as backfill. Three different sections were prepared, as illustrated in Fig. 7,
incorporating a berm that initially was 0.46 m high. The ‘control’ and ‘gravel’ sections had the pipeline invert set approximately 2 m below the original ground surface, and a depth of 2.9 m was adopted in a ‘deep burial’ section. In the gravel section, the silty soil found just beneath the pipeline invert was replaced by gravel to mitigate frost heave. The pipelines were chilled to $-8.5^\circ C$, and the subsequent ground heave was monitored for 3 to 10 years.

The ground was essentially inorganic, comprising 13% sand, 64% silt, and 23% clay-sized particles. Its initial water content, plastic limit and liquid limit ranges were 18–22%, 14–18% and 24–31% respectively. In situ permeability tests indicated a relatively high mass permeability range of $0.6–1 \times 10^{-6} m/s$. However, the reported heavily fissured nature of the ground suggests that the micro- to mesoscopic permeability is likely to be significantly lower than this high mass value. The initial groundwater table was reported as being 2.3–2.6 m below the original ground level.

Finite element meshes and boundary conditions

The spatial domain of the problem was discretised by designing a series of two-dimensional FE meshes, as shown in Fig. 8. The relatively narrow initial berms generated small surcharges ($< 0.01 MPa$), which were not modelled. However, their enlargement to 1.5 m height, performed 430 days after the start, was simulated by applying a ground level surcharge of 0.03 MPa over 1.7 m to either side of the centreline.

The modelled domain was 15 m $\times$ 15 m in extent. Relatively fine meshing was required to capture the active frozen fringe, where the permeability changed steeply with distance. Insufficiently fine meshes render the results inaccurate owing to interpolation errors in the degree of liquid saturation $S_l$ and hence of the hydraulic conductivity. Optimum meshing was determined in advance through one-dimensional freezing analyses that considered the expected two-dimensional freezing rates and temperature gradients. It was concluded that elements should be no thicker than a third of the frozen fringe thickness.
It was assumed that the pipeline has negligible influence at 15 m from its axis, so zero heat flux/liquid flux conditions were specified along the lateral mesh boundaries. No heat flux was allowed along the bottom boundary, where the pore water pressure $P_l$ was specified to be 0.147 MPa, assuming hydrostatic conditions with groundwater to the surface. The hydraulic boundary conditions along the top boundary were set to give $P_t = 0$ when $P_t$ was about to exceed 0 (i.e. free outgoing flow), but $q_l = 0$ was prescribed whenever $P_l$ became negative (i.e. no incoming flow). The surface thermal boundary conditions were prescribed as

$$j^e = \gamma_e (T^o - T)$$

($j^e$ is positive for inflow), where $\gamma_e$ is a constant and $T^o$ is the prescribed ‘target’ temperature. If $\gamma_e$ is very large, $T^o$ and $T$ have to be similar, but such a constraint becomes looser if $\gamma_e$ is smaller. One way of interpreting the meaning of $\gamma_e$ is to rewrite the above equation as

$$j^e = \lambda \frac{T^o - T}{L}$$

where $\gamma_e$ is replaced by $\lambda/L$. Equation (23) can be thought of as representing a thin layer of non-soil surface insulation having a thickness $L$ and thermal conductivity $\lambda$. For example, 0.3 m of snow cover with a thermal conductivity of 0.3 W/m K could be represented by taking $\gamma_e$ equal to 1 W/m K. When snow is absent, the apparent ground surface insulation is controlled by more complex factors, including vegetation and evaporation characteristics. The present study adopted $\gamma_e = 1$ W/m K as a default value. The air temperature $T_o$ was kept constant at 6.5°C in most cases; the effects of monthly variations were also examined.

A uniform temperature of 6.5°C and hydrostatic pore water pressures were assumed as the initial ground conditions in the analysis. The initial preconsolidation stress ($p_0$) and $K_0$ value of the soil were taken to be 0.15 MPa and 1 respectively, to reproduce the anticipated moderately over-consolidated ground conditions. The pipeline temperature reduced linearly from 6.5°C to $T_o$ over the first 50 days, and was then kept constant.

Material parameters

The material parameters selected for input are summarised in Table 1. The thermal properties of water may be considered as practically constant. The compressibility of ice was tentatively set to be the same as that of liquid water, an
assumption that is unlikely to affect the analysis significantly. The parameters for the soil-water characteristic curve, λ and P, were obtained by fitting equation (6) to the freezing test data for the Calgary Silt given by Patterson & Smith (1981). Although the parameters λ in the soil-water characteristic curve (equation (5)) and in the relative permeability function (equation (10)) do not need to be identical, selecting λ = 0.3 gave a reasonable match between the calculated relative permeability and the experimental data provided by Horiguchi & Miller (1983) for the Calgary Silt.

The remaining input parameters had to be chosen to match those of a typical clayey silt. Adopting an assumed soil particle thermal conductivity of λs = 3 W/mK, equation (13) predicts overall soil thermal conductivities of 1.6 W/mK at Sf = 1 and 2.6 W/mK at Sf = 0. The value for the critical state parameter M was set as 1, which is equivalent to an angle of shearing resistance of 25° in triaxial compression. The two-dimensional pipeline cross-section was modelled as an impervious disc of very stiff elastic material (E = 10 000 MPa). The pipeline's surface temperatures were controlled as uniform boundary conditions, making its thermal properties irrelevant.

Bearing in mind the mesh size constraints described earlier, exact modelling of the gravel's hydraulic properties was difficult to achieve. This is because the gravel's frozen fringe is very thin in comparison with an affordable element size, owing to its very steep freezing characteristic function. The gravel parameters shown in Table 2 were selected to represent

(a) high unfrozen permeability (K > 10⁻⁶ m/s, or k > 10⁻¹⁰ m²)
(b) a freezing characteristic function similar to that of the silt
(c) a very rapid decrease in the relative permeability as Sf decreases.

While (a) and (c) represent the frost-resistant nature of

<table>
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<th>Property</th>
<th>Value</th>
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<td>λ</td>
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<td>Initial porosity, φ₀</td>
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Table 2. Gravel hydraulic properties input in the analysis (the properties not shown were set to the same values as those shown in Table 1)
Simulated segregation potential

The segregation potential theory proposed by Konrad & Morgenstern (1980, 1981) has been widely applied in frost heave analysis, including successful simulations of the Calgary pipeline trials by Konrad & Morgenstern (1984). Their study allows useful comparisons with the proposed THM approach. The segregation potential, SP, is defined as a coefficient relating the liquid flux into the frozen fringe and temperature gradient across it:

\[ q_l = SP \frac{dT}{dx} \]  (24)

SP varies with many factors. However, considering cases involving slow field cooling rates, Konrad & Morgenstern (1982, 1984) simplified the function to

\[ SP = SP_0 \exp \left(-\frac{aP_o}{c} \right) \]  (25)

where \( SP_0 \) and \( a \) are considered constant (with \( 2.3 \times 10^{-3} \text{ m/s K} \) and 9.5, respectively, being selected by Konrad & Morgenstern, 1984, for the Calgary case), and \( P_o \) is the overburden pressure. A series of one-dimensional freezing simulations was performed by the authors that utilized a control pipeline vertical column in the control section. The computed liquid flux and the liquid flux separately calculated from the segregation potential theory using Konrad & Morgenstern's parameter set are compared in Fig. 7. It is interesting to note that the THM model predicts the same order of liquid flux as the segregation potential method, despite the assumed intrinsic permeabilities being 100, or more, times lower than the reported in situ bulk value. Since the SP approach was derived from laboratory uniaxial freezing tests, this comparison highlights the fact that field measurement may have overestimated the micro- to mesoscale local permeability relevant to the frost heave problem.

Simulated frost heave of pipeline

The simulated pipeline heave developments are compared with the field measurements in Fig. 10. The intrinsic permeability value was set as \( 0.8 \times 10^{-15} \text{ m}^2 \) in all the simulations shown. The time history of the heave in the deep burial section is well predicted over three years. The control section prediction is also good up to the berm enlargement at day 430. From this point onwards, however, the simulated heave rates exceed the measurements. One plausible explanation is the site non-uniformity noted by Slusarchuk et al. (1978). In particular, the deeper ground may have been less frost-hydrususceptible than the shallow material against which the model was calibrated. Other possible factors are that: (a) the assumed soil stiffness may have been excessive, leading to an underestimate of the instantaneous settlement at day 430, and of the subsequent consolidation settlement; and (b) determination of the hydraulic parameters based on large temperature gradient conditions may have led to overprediction of liquid flux at stages where the temperature gradient is small, as in the later stages of heaving.

Simulated frost heave of pipeline: control section; D, deep burial section; G, gravel section

Fig. 10. Simulated and observed heave of pipeline: C, control section; D, deep burial section; G, gravel section
Influence of ground surface thermal boundary conditions

The deformed meshes for the control section are shown in Figs 12(a) and 12(b) for days 300 and 1000. The excessive deformation of finite elements seen at the heave-shoulder location in Fig. 12(b) derives from the steep local temperature gradient caused by pipeline cooling, and the specified positive ground surface temperature. Such deformation may be curbed in the field by intermittent cavitation and desiccation in the near-surface ground, and hence disruption of water flow. These factors were not modelled in the simulations. Another relevant factor is the impedance of near-surface water flow by surface ground freezing during winter. In order to consider the latter, additional simulations were made for the control section, in which the air temperature ($T_\text{o}$ in equation (22)) was varied on a monthly basis, between $-8^\circ\text{C}$ in January and $+16.5^\circ\text{C}$ in July. The mesh deformation for day 1000, shown in Figs 12(c) and 12(d), indicates more realistic heave patterns. Note that the undulation in the heaved ground surface is caused by seasonal freezing, with the first centre-line ‘bump’ developing during the first summer, and the second at the shoulder in the second summer.

The pipeline heaves predicted by analyses run with both constant and monthly varying temperature inputs are compared in Fig. 10. Unlike the surface heave, the pipeline displacements do not appear to be significantly affected by the surface temperature conditions, because the annual temperature variations fall with depth, and are greatly reduced at the pipeline invert level (Nishimura et al., 2009). The temperature contours computed for four different months are shown in Fig. 13. Whereas the temperatures around and below the pipeline show little seasonal variation, the ground surface experiences considerable changes, including annual freezing and thawing cycles. This finding gives confidence at least to the predicted pipeline heave, despite the difficulty in modelling the complex surface thermal conditions.

Simulated water redistribution

Maps of the simulated porosity changes are presented in Figs 12(a) to 12(d). A doubling of porosity from the initially set value of 0.35 around the pipeline is clearly visible, demonstrating how the accumulation of freezing pore water drives the heaving process. The way in which unfrozen pore water is drawn into the radially expanding freezing front is also visualised by the liquid flux vectors presented in Fig. 14.

The authors are not aware of any data relating to final water content measurements at the end of the control and deep burial locations, but data were presented from an additional ‘insulated silt’ section by Carlson & Nixon (1988). This case was similar to the control experiment, but...
with an additional insulation layer placed around the pipe. The measured water content profile at the pipe centreline is shown in Fig. 15, along with the simulated water contents for the control and deep burial sections. The frost penetration depths were not significantly different in the two simulated cases at day 1000, and the observed depth was well predicted. The observed insulated silt water content profile was closer to that predicted for the deep burial section than for the control section. The insulation around the pipe may have curbed the frost heave in a way comparable to the deep burial. The THM simulation reproduces the field trend for water content to increase gradually from the pipeline invert down to the freezing front.

Heave resistance by overlying soil

Another important positive feature of THM modelling is its ability to predict the stress and strain states developed in the soil, and their variations, during freezing and thawing. Fig. 16 illustrates this aspect by indicating how the shear stress $\tau_{xy}$ (the y-axis is defined as the vertical) develops as freezing progresses in the control and deep burial section cases. The initial development of negative shear stresses at the pipeline shoulder, which act downwards to resist the upheaval of the pipeline and the frozen soil around it, can be seen in Fig. 16. It may therefore be argued that the suppressed heave in the deep burial section derives from the larger shear force exerted by soil away from the centreline, in addition to the larger weight of the overlying soil. As the resisting soil at the pipeline shoulder is engulfed within the frozen zone, however, the stress state at this location becomes irrelevant to the heaving, and the frozen zone moves upwards with an almost rigid block mechanism.

The ultimate uplift resistance offered by the soil overlying the pipeline was considered by Carlson & Nixon through a simplified limit equilibrium approach by assuming (a) a planar failure mechanism, (b) a critical orientation for the plane, and (c) an operational angle of soil shearing resistance. In the proposed THM-coupled continuum approach, however, the failure mechanism, operational shear strengths and ground deformation all emerge spontaneously as model outputs.

CONCLUSIONS

A fully coupled THM model has been developed to consider a variety of geological and geotechnical processes involving freezing and thawing soils. The governing equations were developed from fundamental physical requirements. By solving the coupled equations, the interactions between frozen soils’ thermal, hydraulic and mechanical processes can be simulated. New developments include a new critical state elasto-plastic mechanical constitutive model that adopts total stress, liquid pressure and ice pressure. The constitutive model reduces to an effective stress-based model similar to the Modified Cam-Clay model under unfrozen conditions. Many essential features of frozen soils’ mechanical behaviour, including the dependence of shear strength on temperature and porosity, are captured, but modelling of additional time-dependent or cyclic features requires further development. The proposed framework allows existing unfrozen soil model developments, such as overstress viscoplasticity and multiple-surface plasticity, to be applied readily in potential future elaborations.

The performance of the proposed formulation was tested by considering the important cold region infrastructure topic...
of frost heaving. Experiments conducted with buried large chilled gas pipelines by Slusarchuk et al. (1978) were simulated, considering three different test conditions. Some important questions regarding numerical implementation have been addressed, including appropriate FE mesh design, selection of hydraulic parameter values, and setting of realistic boundary conditions at the air/ground interface. Of particular importance was the determination of permeability.

The range of permeability values, which reproduced both Konrad & Morgenstern’s (1984) segregation potential parameters (obtained from laboratory tests) and the frost heave observed in situ, fell two orders of magnitude below a bulk value reported from in situ testing. Frost heave is governed by processes that develop within relatively thin frozen fringes; scale effects may be important, and bulk permeability values measured at a large scale may not be representative in fissured, heterogeneous, ground.

Critical examination of the analytical predictions and the field test data confirms that the new THM model can simulate, with fair accuracy, the field patterns of pipeline heave, water migration and ice accumulation. The experimentally proven alleviation of heave by deeper burial was modelled successfully. The increased shear resistance developed in unfrozen soil above the pipeline was found to contribute to the reduction of heave.

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REFERENCES


