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KNOCKDOWN FACTORS FOR TWO KINDS OF STIFFENED CYLINDERS UNDER AXIAL COMPRESSION

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Abstract. Stiffened cylinder is widely used in launch vehicle structures because its stiffener can prevent buckling wave to spread in the skin. However, it is still sensitive to shape imperfection more or less. Knockdown factors of stiffened cylinders have been researched all the time. However, comparison among different grid types is seldom. In this paper, both orthogrid stiffened cylinder and isogrid stiffened cylinder under axial compression are studied. Linear eigenvalue method and nonlinear implicit method are adopted to calculate the bearing capacities of the cylinders with ABAQUS. The sinusoidal function is used to model the imperfection. Given different imperfection parameters, the sensitive of bearing capacities to the shape imperfection is researched for two kinds of cylinders. Axial compression experiments are also implemented for the stiffened cylinders, and the results agree with numerical result. Knockdown factor of orthogrid cylinder is about 0.5~0.7. Meanwhile, it proves that when the stiffener is strong enough, the effects of shape imperfection on load capacity can be ignored for orthotropic grid, and its knockdown factor is nearly 1.

1 INTRODUCTION

Most primary structures in launch vehicle are thin-walled cylinders, such as tanks and interstage skirts. In 1940s and 1950s, unstiffened cylinder is first applied in aerospace engineering. However, many experiments revealed that its axial compression bearing capacity was very sensitive to manufacturing imperfection. When the radius-to-thickness rises to 1000, knockdown factors of bearing capacity could even be less than 0.2.

In order to describe the sensitivity of thin-walled cylinders, knockdown factor is defined as the ratio of actual bearing capacity to theoretical bearing capacity. Many scientists gave the lower bound curves of scatted experiment data as knockdown factor. The most famous one is the report NASA SP-8007, which has been used as for many years[1].

$$\gamma = 1 - 0.901(1 - e^{-\Phi}) \tag{1}$$

$$\Phi = \frac{1}{29.8}\left[\frac{R}{\sqrt[4]{\dfrac{D_x D_y}{E_x E_y}}}\right]^{1/2} \tag{2}$$

Many experiments have proved that NASA SP-8007 may be too conservative[2,3]. In recent years, many countries and corporations restarted to study this problem based on advanced measurement and algorithm, such as NASA, ESA and China. Wagner et al. [3] defined the local buckling load as the ultimate load of thin-walled cylinders and obtained the knockdown factors as:

$$\gamma = \Omega \cdot (R/t)^{-\eta} \tag{3}$$

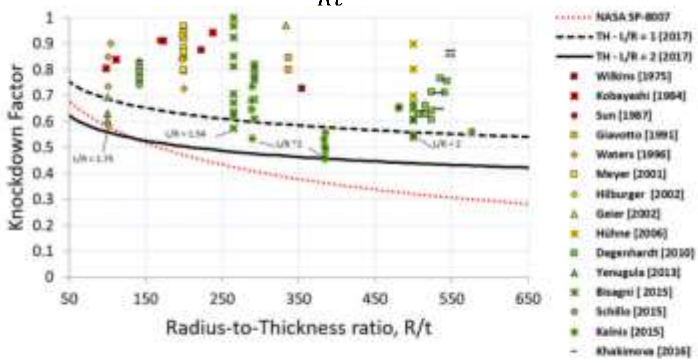
$$\Omega \approx -0.0196 \cdot \left(\frac{L}{R}\right)^2 - 0.0635 \cdot \left(\frac{L}{R}\right) + 1.3212 \tag{4}$$

$$\eta \approx -0.013 \cdot \left(\frac{L}{R}\right)^2 + 0.061 \cdot \left(\frac{L}{R}\right) + 0.08 \tag{5}$$

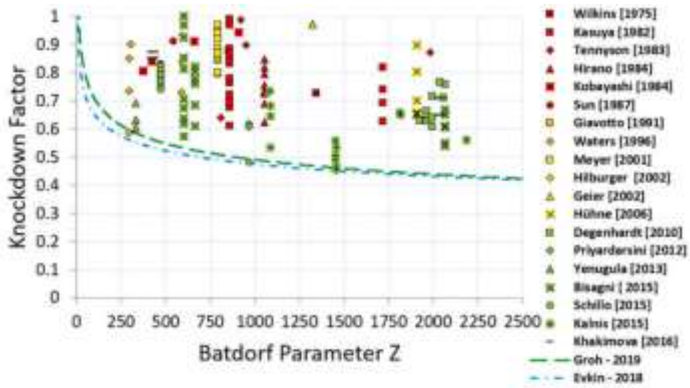
Some researchers think that L/R is also important to determine the bearing capacity of cylinders. Therefore, they considered both R/t and L/R into the knockdown factors. For example, Evkin[4] proposed the following fitted equation according to large amount of experiments:

$$\gamma = 1.23 \cdot Z^{-0.138}, \quad 50 \le Z \le 7000 \tag{6}$$

$$Z = \frac{L^2 \cdot \sqrt{1-\upsilon^2}}{Rt} \tag{7}$$



a



b

Figure 1: Knockdown factors of axially compressed cylinders for different R/t ratios (a) and different Batdorf Parameter Z (b)[5]

Finite element analysis is the most popular method to study imperfection sensitivity. In order to acquire knockdown factors, shape imperfections need to be introduced into numerical model. Up to now, there are several methods to establish the imperfections. For example, buckling mode shape can be firstly calculated by eigenvalue analysis and then be introduced into finite element model. Mode shape corresponding to the first eigenvalue is usually used. However, the stiffness would be largely weakened. Some people proposed single perturbation load approach (SPLA)[6] and multiple perturbation load approach (MPLA)[7]. Hao[8] combined MPLA with an optimization framework, and then proposed the worst multiple perturbation load approach (WMPLA). The WMPLA has been successfully validated by stiffened cylinder experiments[9].

Although the imperfection of cylinders has been frequently researched, there are seldom comparison between different stiffener modes. There are also few axial compression experiments for large scale cylinder. In this paper, orthogrid stiffened cylinder and isogrid stiffened cylinder are compared and researched through numerical and experimental methods.

2 STIFFENED CYLINDERS AND IMPERFECTION MODE IN NUMERICAL MODEL

Based on finite element methods, three kinds of cylinders are carefully researched and compared, including unstiffened cylinder, orthogrid cylinder and isogrid cylinder. They have the same radius R = 1500mm, the same height H = 1500mm and the same skin thickness t_s = 2.5mm. The height of stiffener h is adjusted to research the influence brought by mass balance coefficient m_b. All the sizes in finite element model are listed in table 1.

Table 1: The sizes for two kinds of stiffened cylinder

		No stiffener	Orthotropic stiffener				Isogrid stiffener			
Distance of vertical stiffener	b_{s1}	-	147				196			
Distance of horizontal stiffener	b_{s2}	-	300				-			
Thickness of stiffener	b_w	-	5				3.3			
Height of stiffener	h	-	2.5	5	10	20	2.5	5	10	20
Mass balance between stiffener and skin	m_b	0	0.05	0.1	0.2	0.4	0.05	0.1	0.2	0.4

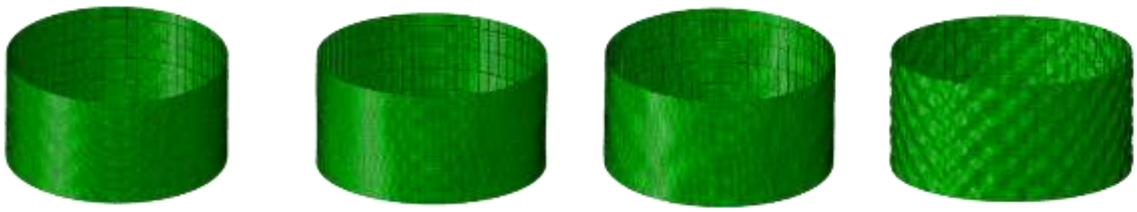
In order to simplify the process of introducing imperfection, a new bi-directional sinusoidal function is used to describe the imperfection.

$$w(x,\theta) = a \cdot sin\left(\frac{2\pi m x}{H}\right) \cdot sin(n\theta) \tag{7}$$

where, a is the imperfection amplitude; m and n is the wave length number along vertical direction and circumferential direction. In order to ensure that the wave length along vertical and circumferential directions is the same, n is always 6 times m. Using Python script in ABAQUS, the function can be easily applied to modify the coordinates of nodes in numerical model. Seven kinds of arrangements of three parameters are listed in Table 2, and some are explicitly given in Figure 2.

Table 2: Imperfection parameters adopted in FEM for cylinders

Variables		Unit	Case ID						
			1	2	3	4	5	6	7
Imperfection amplitude	a	mm	2.5	5	10				20
Wave number along vertical direction	m	-	4			1	2	3	4
Wave number along circumferential direction	n	-	24			6	12	18	24



a Case1 b Case4 c Case5 d Case7

Figure 2: Several imperfection modes for isogrid stiffened cylinders

3 INFLUENCE BY IMPERFECTIONS FOR THREE KINDS OF CYLINDERS

3.1 Mechanical behaviors of three kinds of cylinders

Firstly, unstiffened cylinder is analyzed. The axial bearing capacity of the perfect cylinder can be easily deduced to be 1602kN with the following equation:

$$F_{cr} = 3.77Et^2 \tag{7}$$

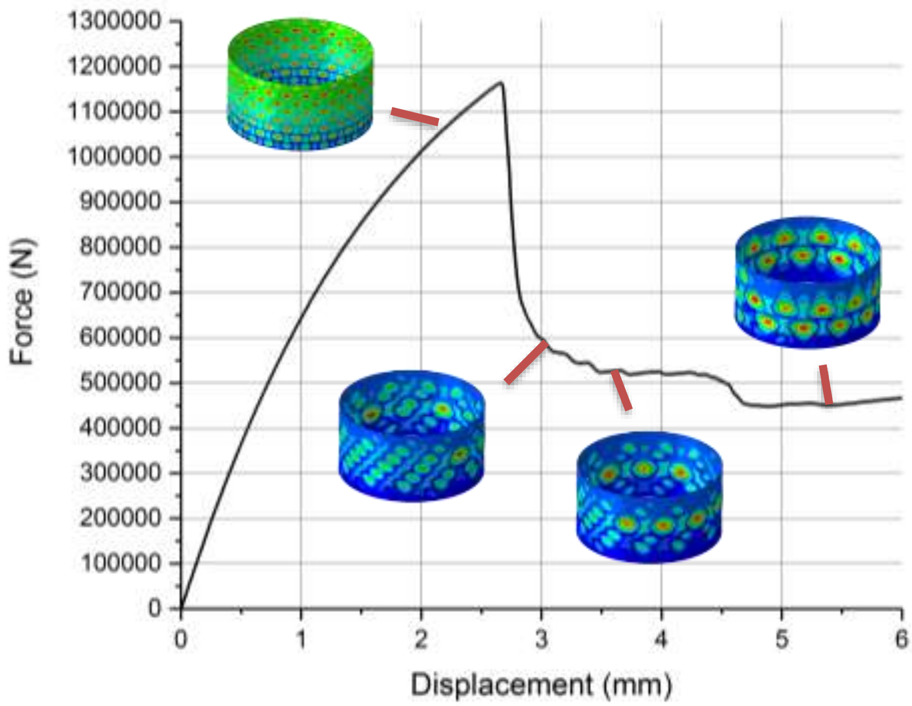


Figure 3: Force-displacement curves for unstiffened cylinder with imperfection amplitude a = 2.5mm

The force-displacement curves for different imperfections are almost the same, and four stages are included. Taking the curve for imperfection amplitude *a* = 2.5mm as example (Figure 3), there are some local bending deformation in pre-buckling stage, and they appear in the skins between the adjacent stiffeners. The bearing capacity is 1165kN, and knockdown factor is 0.73. Stage 2, 3 and 4 are post-buckling stages. In stage 2 (*u* = 2.6mm ~ 3.1mm), the load suddenly drops from 1170kN to 570kN, and the deformation changes into shear band. In Stage 3, shear band retains, and meanwhile a wave appears in the middle of the cylinder. In Stage 4, the load drops again, and two waves appear along the axial direction of the cylinder.

Figure 4 gives the force-displacement curve for orthogrid stiffened cylinder with parameters *h* = 5mm, *a* = 2.5mm and *m* = 4. For this stiffened cylinder, there are also some local bending deformation in pre-buckling stage, but they are weaker than that in unstiffened cylinder. Three Post-buckling stages is almost the same as that in unstiffened cylinder. However, More load suddenly drops between stage 2 and stage 3 because of the stiffeners. Much more energy is released during the transition of deformation modes.

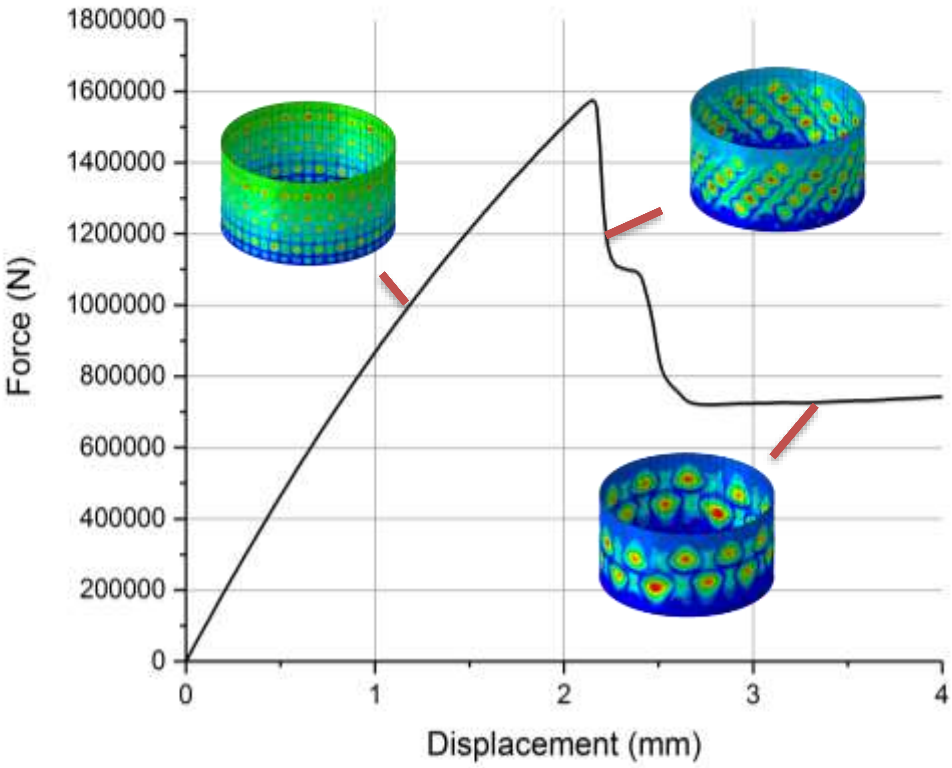


Figure 4: Force-displacement curve for orthogrid stiffened cylinder with parameters *h* = 5mm, *a* = 2.5mm and *m* = 4

Figure 5 lists the force-displacement curves for orthogrid stiffened cylinder with different imperfections. When *m* is 4, its stiffness becomes weaker with increasing imperfection amplitude. Bearing capacity generally becomes weaker for the larger amplitude. However, when the amplitude becomes from 10mm to 20mm, it is getting a little larger. When the imperfection amplitude is 10 mm, the cylinder stiffness is almost the same in case that m = 1 and no imperfection, and becomes weaker for m = 2 ~ 4. However, all the bearing capacities are almost the same, and m = 2 is the worst case. Moreover, for all the cases, lower bounded loads are almost the same, varying from 110kN to 140kN.

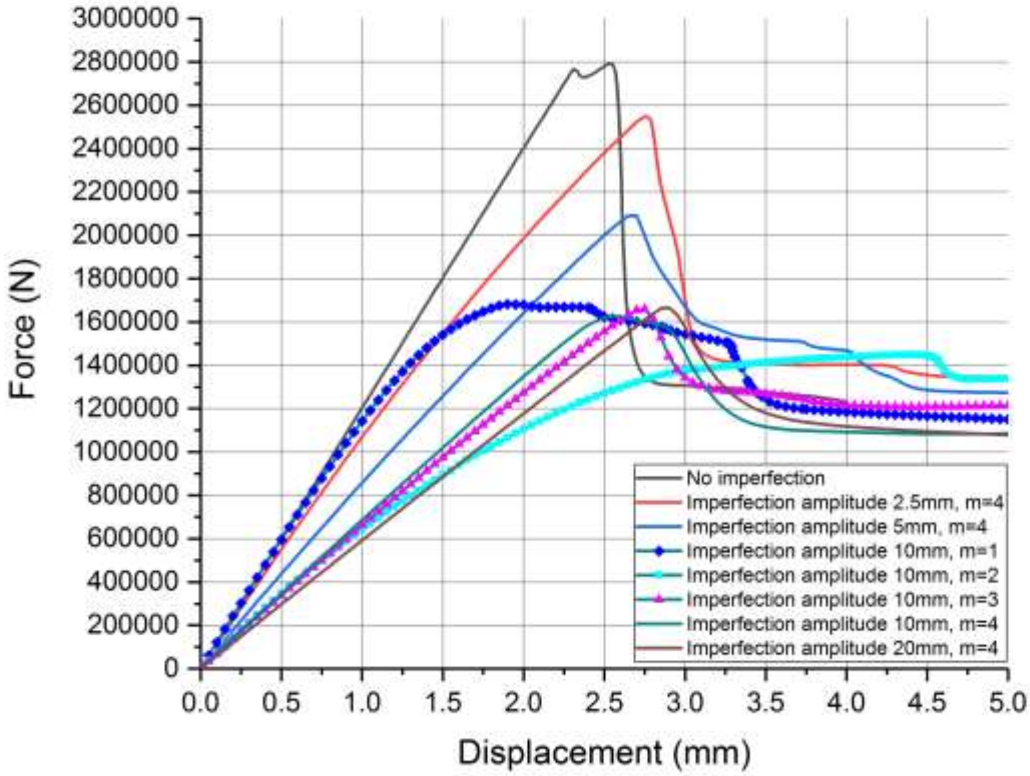


Figure 5: Force-displacement curves for orthogrid stiffened cylinder with different imperfection amplitudes

Figure 6 gives the force-displacement curve for isogrid stiffened cylinder with parameters h = 10mm, a = 10mm and m = 4. For this stiffened cylinder, there are also some local bending deformation in pre-buckling stage. However, only two post-buckling stages can be observed. One is the transition stage from 1690kN to 890kN, the other is another local bending deformation.

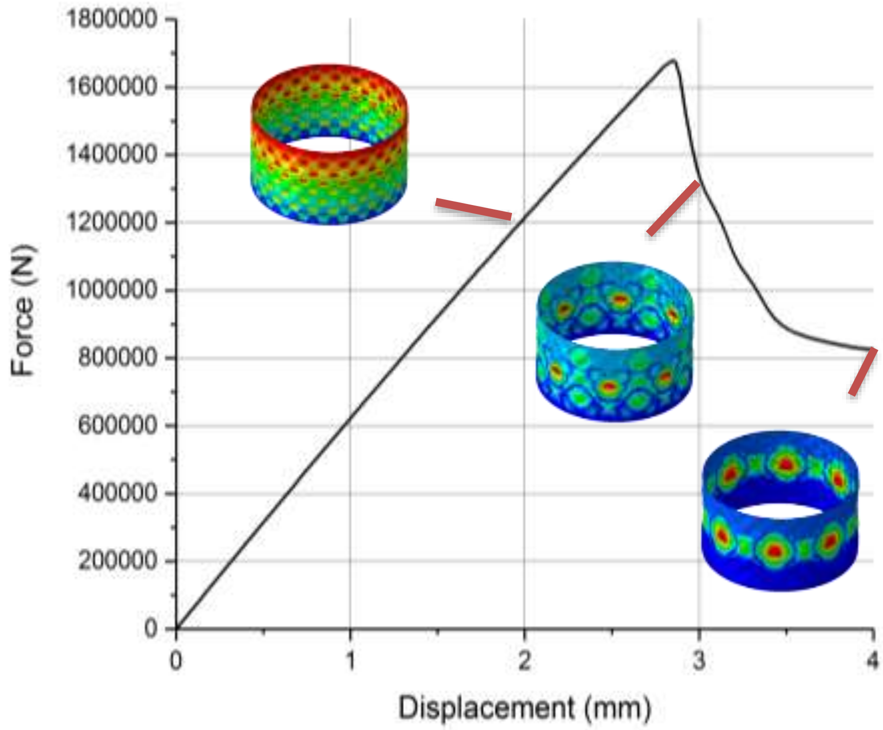
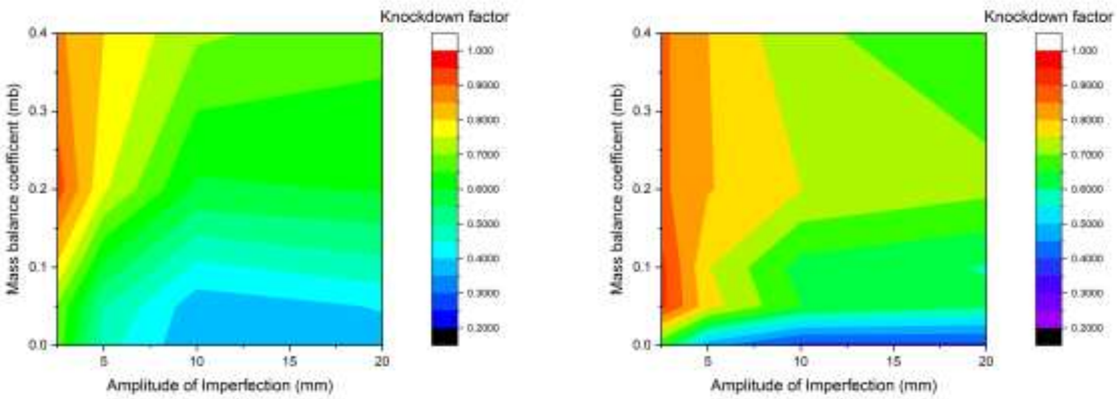


Figure 6: Force-displacement curves for isogrid stiffened cylinder with parameters h = 10mm, a = 10mm and m = 4

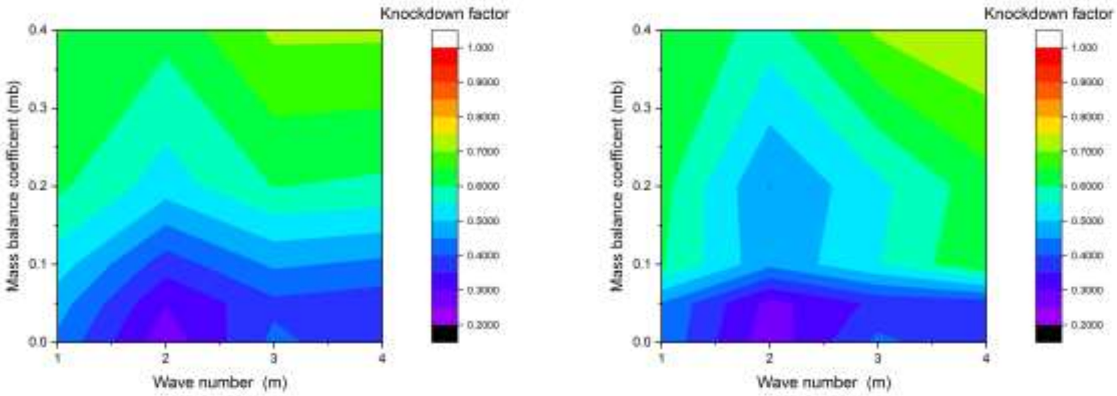
4.2 Comparison of knockdown factors for two kinds of stiffened cylinder

Figure 7 and 8 plot the knockdown factor contours with different imperfection amplitude a and wave length number m and mass balance coefficient m_b. When the imperfection amplitude is lower than 5mm, the bearing capacity is much sensitive to m_b. Meanwhile, in Figure 7, red area for isogrid cylinder is larger than that for orthogrid cylinder, which means that isogrid cylinder has a little higher knockdown factor than orthogrid cylinder. m = 2 is the worst wave length number for both kinds of cylinders, and this is possibly because the length-radius-radio is suitable to produce this wave number.



a Orthogrid stiffened cylinder b Isogrid stiffened cylinder

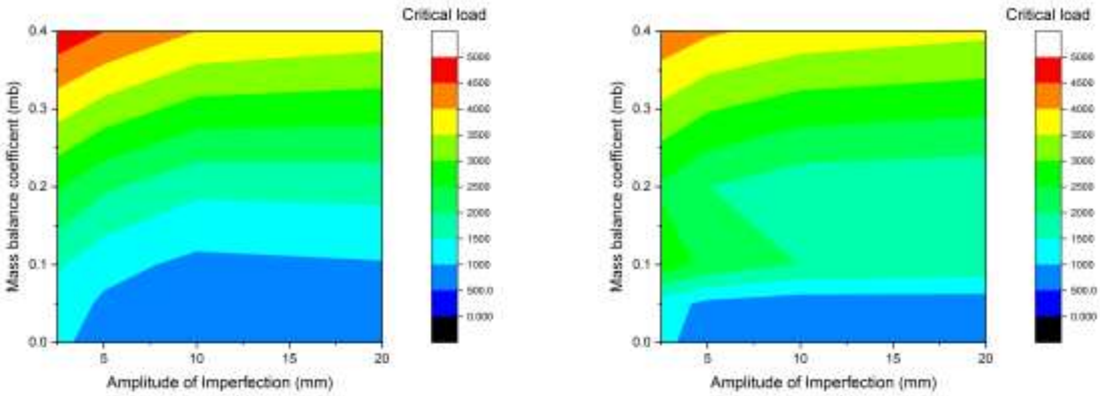
Figure 7: Knockdown factor .vs. imperfection amplitude for two kinds of cylinders



a Orthogrid stiffened cylinder b Isogrid stiffened cylinder

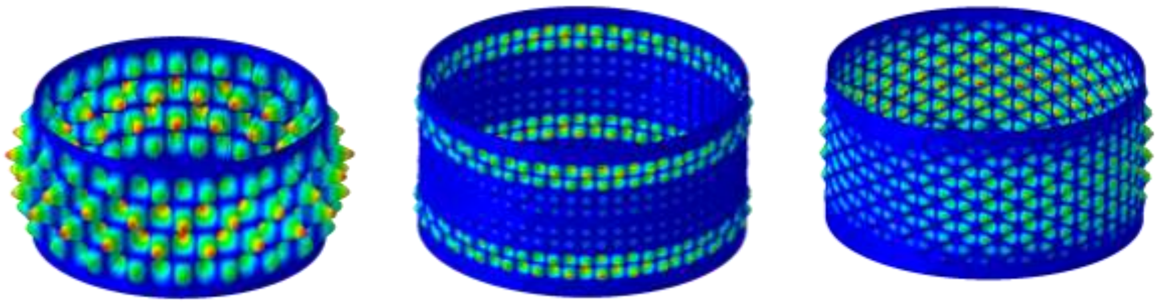
Figure 8: Mass balance coefficient mb .vs. wave number m for two kinds of cylinders

Figure 9 compares critical loads of two kinds of stiffened cylinder. Although isogrid stiffened cylinder is less sensitive to manufacturing imperfection, it has lower critical load. The reason is simple. The skin area between adjacent stiffeners is larger in isogrid cylinder, resulting in more easily skin buckling. For the same stiffener height h = 10mm, linear eigenvalue algorithm proves that skin buckling doesn't appear in orthogrid cylinder but in isogrid cylinder, as shown in Figure 10. Skin buckling can decrease the critical load of perfect isogrid cylinder, and of course increase knockdown factor of imperfect isogrid cylinder in the same time.



a Orthogrid stiffened cylinder b Isogrid stiffened cylinder

Figure 9: Mass balance coefficient mb .vs. imperfection amplitude for two kinds of cylinders

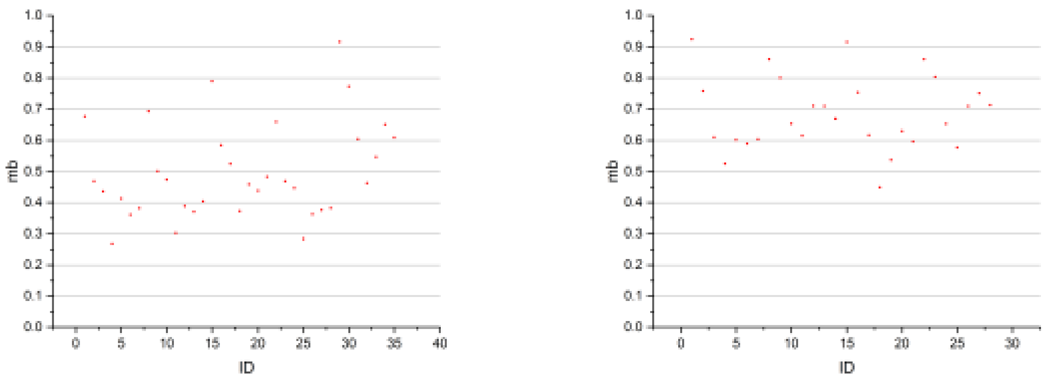


a Orthogrid stiffened cylinder with h=10mm b Orthogrid stiffened cylinder with h=20mm c Isogrid stiffened cylinder witht h=10mm

Figure 10: Buckling modes corresponding to first eigenvalue for different cylinders

4.2 Suggested criterion to choose knockdown factor

Knockdown factor is an important parameter in weak-stiffened cylinder. It should be used with "factor of safety" (FOS) in aerospace engineering. The latter is also to describe the error between failure load and allowable load caused by the errors of material, structure sizes and load. It is not necessary to choose too conservative knockdown factor. A new criterion is suggested in this part. The knockdown factors can be recorded based on different imperfection, and their statistics result can be used to determine suitable factor. When the cylinder is near to unstiffened, i.e., when $m_b \leqslant 0.1$, the suggested knockdown factor is 0.3. When m_b is larger than 0.2, the suggested knockdown factor is 0.55.



a mb≤0.1 b mb>0.2

Figure 11: Knockdown factors for all the simulation cases

4 AXIAL COMPRESSION EXPERIMENTS FOR GRID-STIFFENED CYLINDERS

In order to obtained actual knockdown factor, axial compression experiments of two stiffened cylinders are operated. One is an isogrid stiffened cylinder with m_b = 0.16, another is an orthogrid stiffened cylinder with m_b =0.75.

For the isogrid stiffened cylinder, two numerical methods are applied, including linear eigenvalue method and nonlinear implicit method. In the latter method, different imperfection amplitudes are considered. There is well agreement on failure modes between simulation and experiment. When the imperfection a = 1.5mm is introduced, the critical load is very closed to that in experiment. However, the knockdown factor can be up to 0.75. This value is much higher than the suggested value 0.55 in Part 4.2, but is much near to the average value 0.6~0.7 in Figure 11b.

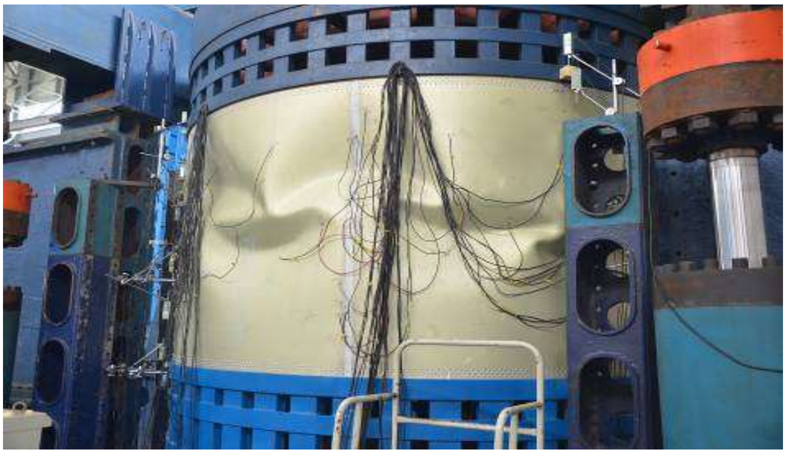
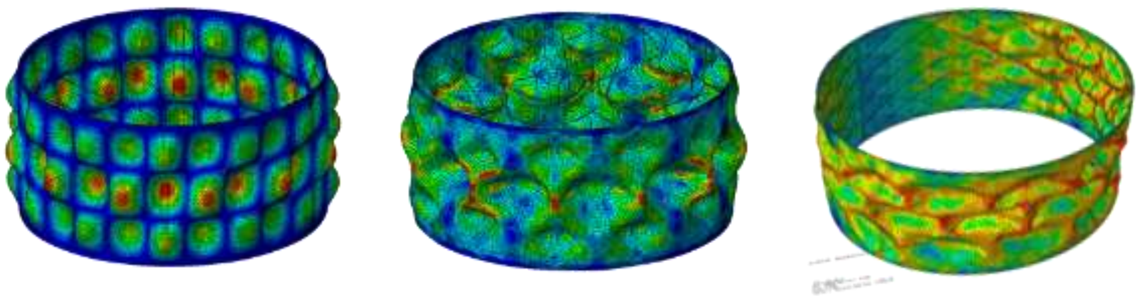


Figure 12: Failure mode in compression experiment of orthogrid stiffened cylinder



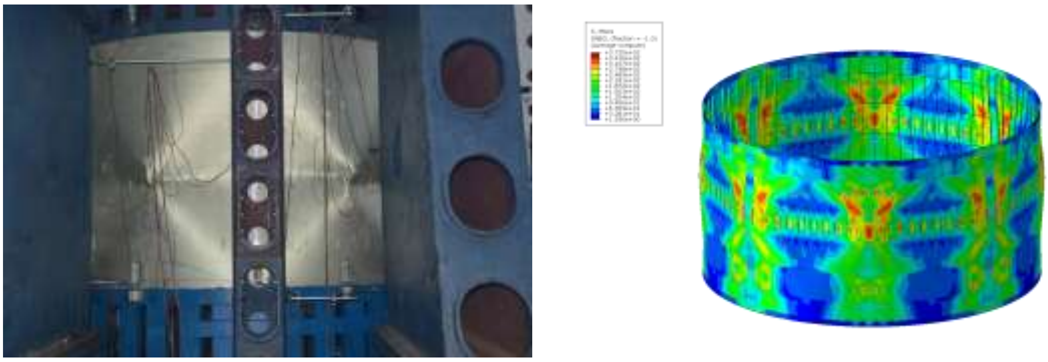
a Eigenvalue   b Nonilinear, a = 0   c Nonlinear, a = 1.5mm

Figure 13: Failure mode in simulation of orthogrid stiffened cylinder

Table 3: Example of the construction of one table

Method	Imperfection amplitude /mm	Critical load /kN: Experiment	Critical load /kN: Simulation	Error
Eigenvalue	-	4293	5720	25.0%
Nonlinear	0		5745	25.3%
	1		4654	7.7%
	1.5		4119	-4.2%

For the orthogrid stiffened cylinder, the knockdown factor is up to 0.92, which is much closed to 1. It means that the cylinder is not sensitive to manufacturing imperfection anymore. For the stiffened cylinders with higher m_b, the imperfections can be ignored.



a Experiment   b Simulation

Figure 14: Page layout

Table 4: Example of the construction of one table

Method	Critical load /kN		Error
	Experiment	Simulation	
Nonlinear	6835	6320	8.1%

5 CONCLUSIONS

In order to investigate the influence of manufacturing imperfection on axially compressed cylinder, three kinds of cylinders are compared using finite element methods, including unstiffened cylinder, orthogrid stiffened cylinder and isogrid stiffened cylinder. The bi-directional sinusoidal imperfection is introduced, and different imperfection parameters are introduced. Two compression experiments are also compared. Some useful conclusions can be obtained:

- Generally, the bearing capacity of the cylinder becomes weaker with larger imperfection. But sometimes, it can get a little larger.
- $m = 2$ is the most dangerous imperfection wave number for the cylinder with $R = H =$ 1500mm.
- Lower bounded loads are almost the same for all the imperfections.
- Compared with orthogrid cylinders, isogrid cylinders have a little higher knockdown factor, but have a little lower critical load.
- For $m_b \leqslant 0.1$, the knockdown factor is suggested as 0.3; for $m_b \geqslant 0.2$, the knockdown factor is suggested as 0.55.
- Axial compression experiment proves that the weak-stiffened cylinder is sensitive to manufacturing imperfection, and the influence of imperfection almost can be ignored when m_b exceeds 0.75.

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