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Robust discretizations for poroelastic problems: engineering and mathematical points of views in a presenation in pairs

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Porous materials are characterized by complex interactions between the deformation of a solid skeleton and the resulting flow of an interstitial fluid. As applications of such materials range from soil mechanics up to the description of biological tissue, accurate and robust solution strategies are highly desirable. Difficulties arise, as model constants vary over wide ranges and might lead to locking, when nearly incompressible solids are considered, or pressure oscillations for low permeablilities.

Progresses in the development of such solution strategies can only be made by merging the knowledge of mathematicians and engineers. In order to continue the tradition of fruitful interactions between applied mathematics and computational engineering this talk **(type 1, presentation in pairs)** takes different viewpoints on the robustness of discretizations for poroelastic problems. The engineering part of this talk will highlight the effects of the theoretical framework on the robustness. Nowadays, poroelasticity is commonly described either by the theory of Biot [1], Mixture Theory (MT) [2] or the Theory of Porous Media (TPM) [3]. While Biot derived his theory from a phenomenological viewpoint, MT and TPM are based on the homogenization of mixtures alongside with a thermodynamically consistent modeling of arising interaction terms. The mathematical part of this talk will show that the accurate approximation of the total stress tensor as well as the Darcy velocity is crucial to obtain reliability and robustness. However, standard methods do not lead to H(div)-conforming stresses and fluxes and surface or interface traction forces cannot be evaluated. We will demonstrates how to recover stress and flux quantities, following [4,5], and compare these results with direct approximations of the dual quantities.

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