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MODEL ORDER REDUCTION OF SOLIDIFICATION PROBLEMS

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Solidification processes in the material processing industry are complex and need to be modelled numerically. The underlying multi-physics equations for fluid flow, heat transfer, and phase change must be coupled to one another [1]. Moreover, non-linearities (e.g. temperature dependent fraction of solid) within the described processes make for challenging numerical modelling and high computational costs, which is problematic when fast predictions are needed. To address these computational challenges, reduced order models have been developed [2], [3]. These models aim to approximate solutions efficiently at little or no expense of accuracy. Fast and reliable predictions open up new possibilities to study a numerical problem for varying parameters.

The question when a process can be modelled efficiently with a reduced basis is subject of open research. Advection driven problems are known to be difficult to model with a reduced basis because of a slow decay of the Kolmogorov n-width [4]. This paper investigates how this challenge transfers to the context of solidification problems. In solidification problems, the problem is not the advection per se, but rather a moving solidification front. This paper studies reduced spaces of 1D step functions that move in time, which can either be seen as advection of a quantity or as a moving solidification front. Furthermore, the reduced space of a solidification in 2D is compared with the reduced space of the classical flow around a cylinder. The results show that the smoothness of the moving step function is crucial for the decay of the singular values and thus the quality of a reduced space representation. The 2D case study shows that alloy solidification problems featuring a smooth interface with a solid-liquid mixture are of lower rank than problems with sharp interfaces.

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