VIII International Conference on Particle-Based Methods
PARTICLES 2023
U. Perego, M. Cremonesi, A. Franci (Eds)

PARTICLE-BASED SEMI-RESOLVED COUPLING MODEL FOR THE SIMULATION OF INTERNAL EROSION IN SOIL STRUCTURES

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Abstract: Internal erosion, caused by seepage flow inside the soil, accelerates soil failure during a natural disaster. Numerical simulation can be an effective tool to quantitatively evaluate the relationship between internal erosion and the instability of the ground as a whole. Internal erosion and multiphase flow simulation of fluid and granular materials with a particle size distribution require coupling simulations that can represent the interaction between particles and pore water and the movement of particles. There are two main types of coupling models: "Resolved coupling model," which can calculate detailed flow and fluid forces, and "Unresolved coupling model," which is based on empirical drag and seepage flow models. Previous studies have indicated that both models should be judged appropriately based on the ratio of particle-fluid spatial resolution. However, applying a resolved coupling model to the vast number of soil particles that make up the ground is impractical from a computational cost perspective, and empirical unresolved coupling model has difficulty in representing localized failures such as internal erosion. Therefore, developing a new coupling model that satisfies both computational accuracy and efficiency is desirable. In this study, we applied ISPH (Incompressible Smoothed Particle Hydrodynamics) for fluid analysis and DEM (Discrete Element Method) for soil particles to develop a fluid-soil coupling simulation model that can directly represent the movement of soil particles during the internal erosion process. Through numerical experiments using a particle layer with the vertical upward flow, we understand the limitations of the conventional coupling model and propose a new hybrid type of semi-resolved coupling model that combines these two models appropriately.

Keywords. Internal erosion, SPH, DEM, Semi-resolved coupling

1 INTRODUCTION

Internal erosion, caused by seepage flow inside the soil, accelerates ground failures during a natural disaster such as a tsunami or heavy rainfall, leading to infrastructural failures. This phenomenon is caused by the movement and loss of fine soil particles due to the water channels (e.g. weak areas such as pores and cracks) in the ground, which induce loosening and collapse of the ground. Internal erosion is extremely difficult to observe and measure on a particle scale, and many unknowns remain. Numerical simulation, which enables microscopic analysis through virtual numerical experiments, could be an effective tool to quantitatively assess the relationship between internal erosion and the destabilisation of the entire soil structure. Numerical simulation of internal erosion requires coupling analysis that can represent even the movement of soil particles during erosion, based on appropriate modelling of the interaction between soil particles and pore water. The coupling methods can be broadly classified into 'the resolved coupling model', which enables detailed flow and fluid force calculations, and 'the unresolved coupling model', which is based on empirical and experimental drag and seepage flow models. Previous studies have provided indications that the use of both coupling models should be appropriately determined from the ratio of particle to fluid spatial resolution [1, 2]. That means that the ratio of the fluid resolution (e.g. CFD cell width) to the solid particle size must be greater than 3 for the unresolved coupling model and less than 1/10 for the resolved coupling model to ensure the accuracy of the fluid force estimation and modelling of the local complex fluid flow, respectively. However, the application of a resolved coupling model to the vast number of particles that make up the ground is impractical from a computational cost point of view, and the unresolved coupling model has difficulty in representing localised failure, such as internal erosion [3]. Therefore, it is desirable to develop a new coupling model that satisfies both computational accuracy and efficiency. In this study, the SPH (Smoothed Particle Hydrodynamics)[4, 5] is applied to the fluid analysis and the DEM (Discrete Element Method) [6] to the soil particles. This enables the development of a fluid-soil coupling simulator that can directly represent the movement of soil particles during the internal erosion process. Numerical experiments using a particle layer subjected to vertical upwelling are carried out to understand the application limits of the conventional coupling model. Also, a new hybrid type of resolved and unresolved coupling model is proposed.

Fig.1 shows a schematic diagram of the proposed semi-resolved coupling model, which is a hybrid type of resolved and unresolved model. In the proposed model, coarse particles, which are larger than the mean particle size, are considered to have a significant influence on the pore flow inside the ground, and the resolved coupling model is applied. On the other hand, for fine particles, their shape has less influence on the flow and they are assumed to behave as if they are being pushed along by the flow, so the unresolved coupling model is applied.

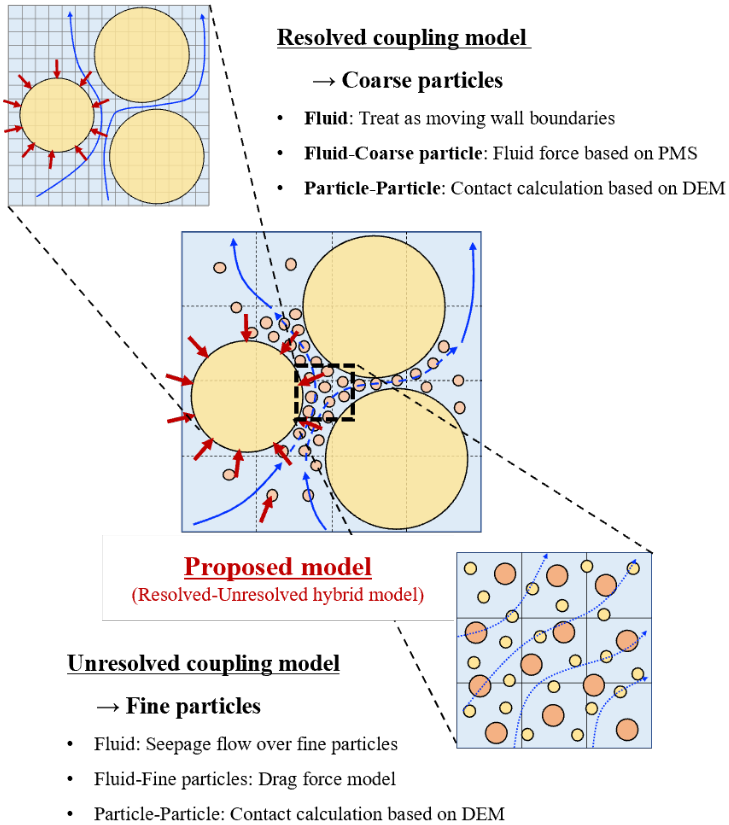


Figure 1: Schematic diagram of the proposed hybrid coupling model

2 SPH FOR FLUID SIMULATION

In the case of fluid-soil multiphase flow such as internal erosion, it is necessary to treat seamlessly the seepage flow inside the soil and the free-surface flow outside the soil. The following Darcy-Brinkman type unified governing equations, which unify the Navier-Stokes equation for free surface flow and the extended Darcy law for porous flow, are used in this study. This is then discretized in the framework of an ISPH (Incompressible SPH) for multiphase flow calculation[3, 7].

$$\nabla \cdot \boldsymbol{v}_D = 0 \tag{1}$$

$$\frac{C_r(\varepsilon_f)}{\varepsilon_f}\frac{D\boldsymbol{v}_D}{Dt} = -\frac{1}{\rho_f}\nabla p + \boldsymbol{g} + \nu_E(\varepsilon_f)\,\nabla^2\boldsymbol{v}_D + \boldsymbol{F}_r^*(\boldsymbol{v}_D, \varepsilon_f) \tag{2}$$

where t, p, ρ, ν, $\boldsymbol{g}$ are time, pressure, density, velocity and gravitational acceleration respectively, and the subscript f indicates the physical properties of the fluid. The effective kinematic viscosity $\nu_E(\varepsilon_f) = (\nu_f + \nu_T)/2$ is a function of the porosity ε_f is given by the kinematic viscosity of the fluid ν_f and the eddy viscosity ν_T based on the Smagorinsky eddy viscosity model. The Darcy flow velocity is defined by $\boldsymbol{v}_D = \varepsilon_f \boldsymbol{v}_f$, and the apparent fluid density inside the porous medium is given by $\hat{\rho_f} = \varepsilon_f \rho_f$. $C_r(\varepsilon_f)$ is the virtual mass coefficient. In this study, $\boldsymbol{F}_r^*$, which is important when coupled with DEM, is the resistance force that the fluid receives from the solid.

$$\boldsymbol{F}_r^*(\boldsymbol{v}_r, \varepsilon_f) = \begin{cases} -a^*(\varepsilon_f)\,\varepsilon_f\,\boldsymbol{v}_r - b^*(\varepsilon_f)\,\varepsilon_f^2\,|\boldsymbol{v}_r|\,\boldsymbol{v}_r & (\varepsilon_f < 0.80) \\ -c^*(\varepsilon_f)\,|\boldsymbol{v}_r|\,\boldsymbol{v}_r & (\varepsilon_f \geq 0.80) \end{cases} \tag{3}$$

$$a^*(\varepsilon_f) = \alpha \frac{\nu_f\,(1-\varepsilon_f)^2}{\varepsilon_f^3\,d_s^2} \tag{4}$$

$$b^*(\varepsilon_f) = \beta \frac{1-\varepsilon_f}{\varepsilon_f^3\,d_s} \tag{5}$$

$$c^*(\varepsilon_f) = -\frac{3}{4}\,C_d\,\frac{1-\varepsilon_f}{\varepsilon_f^{2.7}\,d_s} \tag{6}$$

$$C_d = \begin{cases} \dfrac{24\,(1+0.15\,Re^{0.687})}{Re} & (Re \leq 1000) \\ 0.44 & (Re > 1000) \end{cases} \tag{7}$$

$$Re = \frac{\hat{\rho_f}\,d_s\,\varepsilon_f\,|\boldsymbol{v}_r|}{\mu_f}\,, \tag{8}$$

where $\boldsymbol{v}_r = \boldsymbol{v}_f - \boldsymbol{v}_s$ is the relative velocity of the fluid with respect to the soil particle, and a, b, c are resistance coefficients which vary with porosity ε_f. The fluid-soil unresolved interaction force is given by Eq.(3) is the resistive force term of Akbari and Namin [7] in seepage condition with low porosity ($\varepsilon_f < 0.80$) and the drag force proposed by Wen and Yu [8] in the floating condition with high porosity ($\varepsilon_f \geq 0.80$). In this way, the flow conditions from the seepage flow in the soil to the floating flow of soil particles are corresponded. Note that in this study, the above unified equations are solved by the stabilised ISPH[9]. Various enhanced method, such as SPH(2)-based differential operator [10] and some particle shifting schemes[11, 12], are introduced in our fluid simulation.

3 DEM FOR SOIL SIMULATION

The behaviour of soil particles is represented by DEM in this study. In the numerical experiments on internal erosion with a gap-graded particle size distribution in the next chapter, small particles that have little influence on the fluid flow are represented by spherical DEM particles and a unresolved coupling is implemented. On the other hand, coarse particles, which have a strong influence on the fluid flow, are represented by a Cluster DEM, in which multiple particles are rigidly bonded, and a resolved coupling is applied.

3.1 Spherical DEM for fine particles

Fine particles are represented by the ordinary DEM [6], and their behaviour is represented by solving the equations of motion below for each particle and calculation time step.

$$m_s \frac{d\boldsymbol{v}_s}{dt} = m_s \boldsymbol{g} - \nabla p V_s + \boldsymbol{f}_d + \sum \boldsymbol{f}_c \tag{9}$$

$$i_m \frac{d\boldsymbol{\omega}_s}{dt} = \sum (\boldsymbol{m}_c - \boldsymbol{m}_r) \tag{10}$$

where m_s, V_s, $-\nabla p V_s$, $\boldsymbol{f}_d$ and $\boldsymbol{f}_c$ are the mass, volume, buoyancy, drag and contact force between DEM particles respectively. For the contact forces, the spring-dashpot model used in typical DEM is employed to represent contact and friction. The drag force $\boldsymbol{f}_d$, which is the interaction force with the fluid, acts as a reaction to the resistance force of the fluid particles in Eq.3. i_m, $\boldsymbol{\omega}_s$, $\boldsymbol{m}_c$ in the equations of rotational motion represent the moment of inertia of the spherical particles, the angular velocity and the torque due to the contact force (tangential component). The rolling friction $\boldsymbol{m}_r$ is taken into account to represent the rolling resistance of the unevenly shaped soil particles by means of pure spherical particles.

3.2 Cluster-DEM for coarse particles

Cluster-DEM is used for the coarse particles that strongly influence the fluid flow in the gap-graded soil seepage problem. Cluster-DEM directly represents the solid shape by rigidly connecting spherical particles, contact forces and torques are evaluated at each contact point, and the rigid body motion is tracked by solving the equations of motion for the rigid body center of gravity.

$$M \frac{d\boldsymbol{V}}{dt} = M\boldsymbol{g} + \boldsymbol{F}_f + \boldsymbol{F}_c \tag{11}$$

$$\frac{d(\boldsymbol{I}\boldsymbol{\Omega})}{dt} = \boldsymbol{M}_f + \boldsymbol{M}_c \tag{12}$$

where M, $\boldsymbol{I}$, $\boldsymbol{V}$ and $\boldsymbol{\Omega}$ denote the mass, tensor of inertia, velocity and angular velocity of the rigid body respectively. $\boldsymbol{F}_c$, $\boldsymbol{M}_c$, $\boldsymbol{F}_f$, $\boldsymbol{M}_f$ represent the contact force with the soil particles acting on the rigid body, the torque due to contact, the fluid force acting from the fluid particles and the torque due to fluid force. Interaction forces between the coarse particles and the fluid are calculated based on the PMS(Passively Moving Solid) model [13].

4 NUMERICAL EXPERIMENT OF INTERNAL EROSION IN SOIL

4.1 Simulation conditions and model

We focused on confirming the capability of the proposed method to represent the features of internal erosion in soil. 2D numerical experiments are carried out in which internal erosion is caused by the inlet flow at a constant velocity from the bottom of a particle layer with a gap-graded particle size distribution. The simulation model is shown in Fig.2 and the computational conditions in Table 1. In order to confirm the difference in the internal erosion process depending on the inlet velocity $\boldsymbol{v}_{in}$, the fluid particles at the bottom of the soil layer are forced to flow vertically upward at $\boldsymbol{v}_{in} = 0.5$ and 2.0[cm/s]. A circulation cycle was prepared to allow the upward fluid particles to return to the bottom from the outside of the two walls placed inside the tank and to flow back into the interior of the soil. As simulation cases, we compared 1:applying an unresolved coupling model and 2:applying the proposed hybrid-type of semi-resolved coupling model to the whole model.

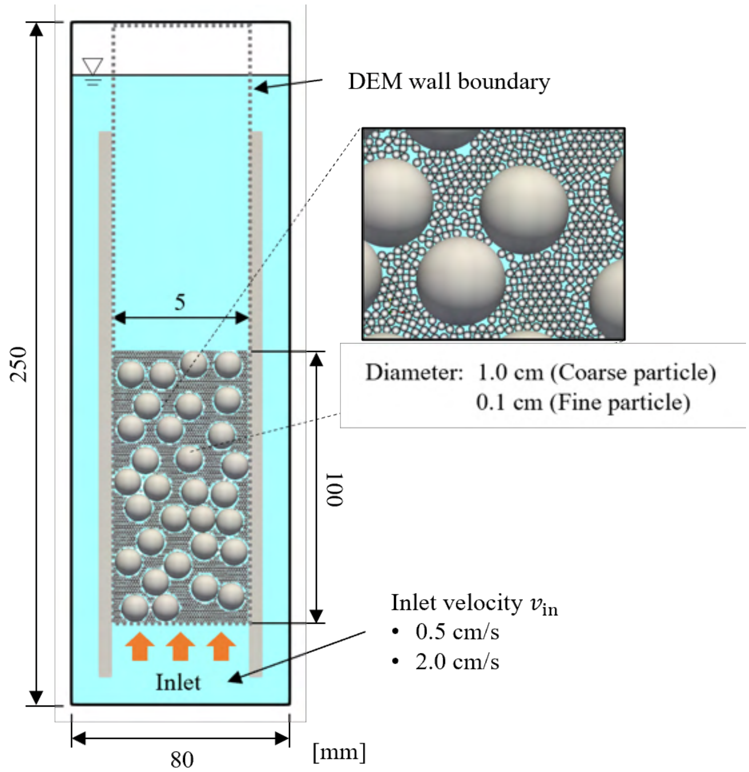


Figure 2: Analysis model for numerical experiment of internal erosion

Table 1: Computational conditions for numerical experiment

Fluid (ISPH)			
particle resolusion	d_x	0.10	[cm]
density	ρ_f	1.00	[g/cm^3]
kinematic viscosity	ν_f	0.01	[cm^2/s]
time increment for SPH	Δt_{SPH}	10^{-4}	[s]
Fine particles (Spherical-DEM)			
particle diameter	d_s	0.10	[cm]
density	ρ_s	2.60	[g/cm^3]
restitution coefficient	e	0.80	[-]
friction coefficient	μ_s	0.30	[-]
stiffness	k	25000	[N/m]
time increment for DEM	Δt_{DEM}	10^{-6}	[s]
Coarse particles (Cluster-DEM)			
component particle diameter	d_s	0.10	[cm]

4.2 Results of numerical experiments

4.2.1 Unresolved coupling simulation

Fig.3 shows the results of the simulation applying unresolved coupling to the whole area. The figures show an overlay of the particle distribution and the fluid velocity distribution, as well as a visualisation of the trajectories of randomly extracted Lagrangian fluid particles.

(a) For inflow velocity = 0.5 cm/s, at 10.0 s there is curling up of fine particles near the surface of the particle layer, but no change in the internal structure and no internal erosion occurs.

(b) Even at =2.0 cm/s, small particles can be seen floating from the surface, but the characteristics of internal erosion, where fracture propagates through the gaps between coarse particles (i.e. flow paths that potentially become water channels), can not be observed. The trajectories of the Lagrangian fluid particles shown in the figure indicate an almost vertically upward straight flow for all inlet velocities. This is due to the use of a unresolved coupling model that calculates the average seepage flow according to the Darcy law. Therefore, pore flows that concentrate in water channels between coarse particles and induce internal erosion are hard to represent in conventional unresolved coupling models.

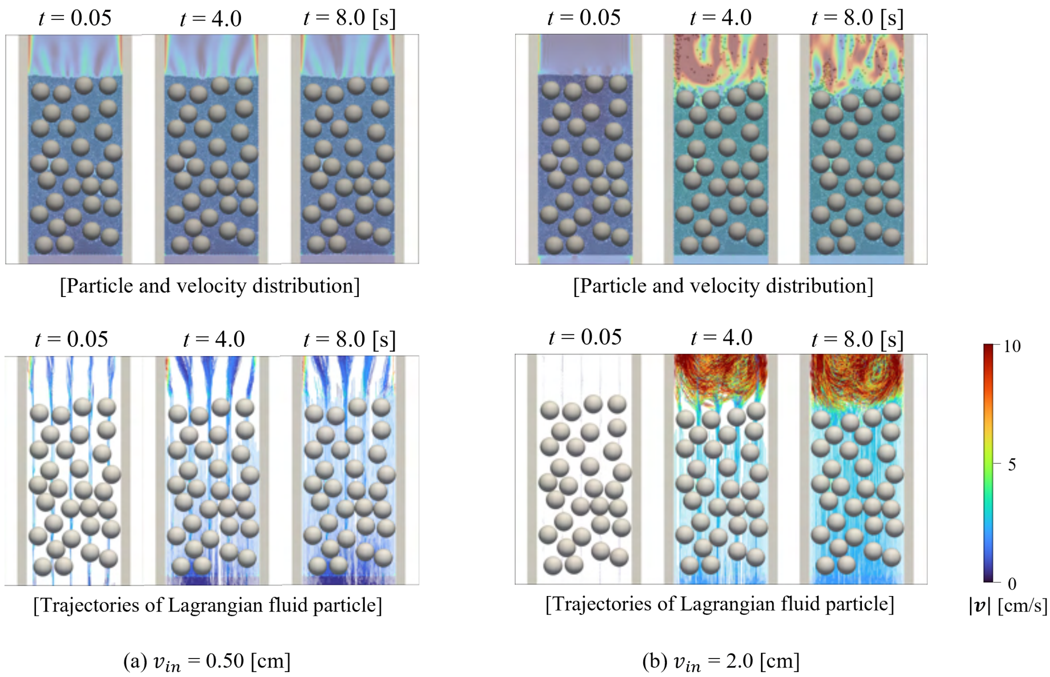


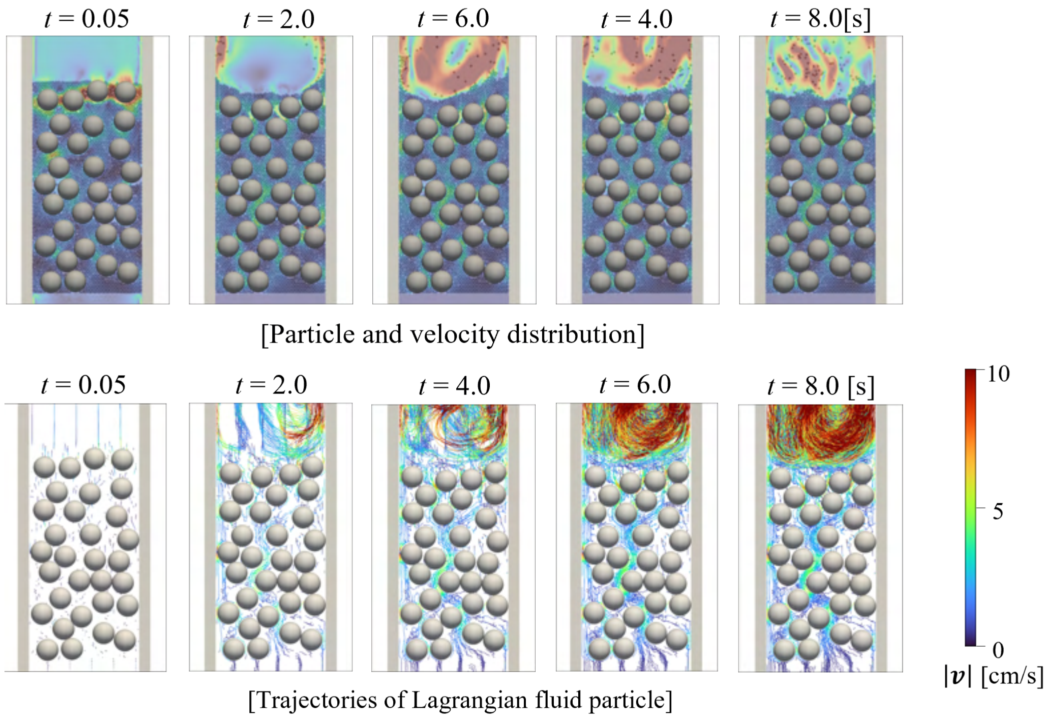
Figure 3: Numerical result of unresolved coupling simulation

4.2.2 Proposed hybrid-type semi-resolved coupling simulation

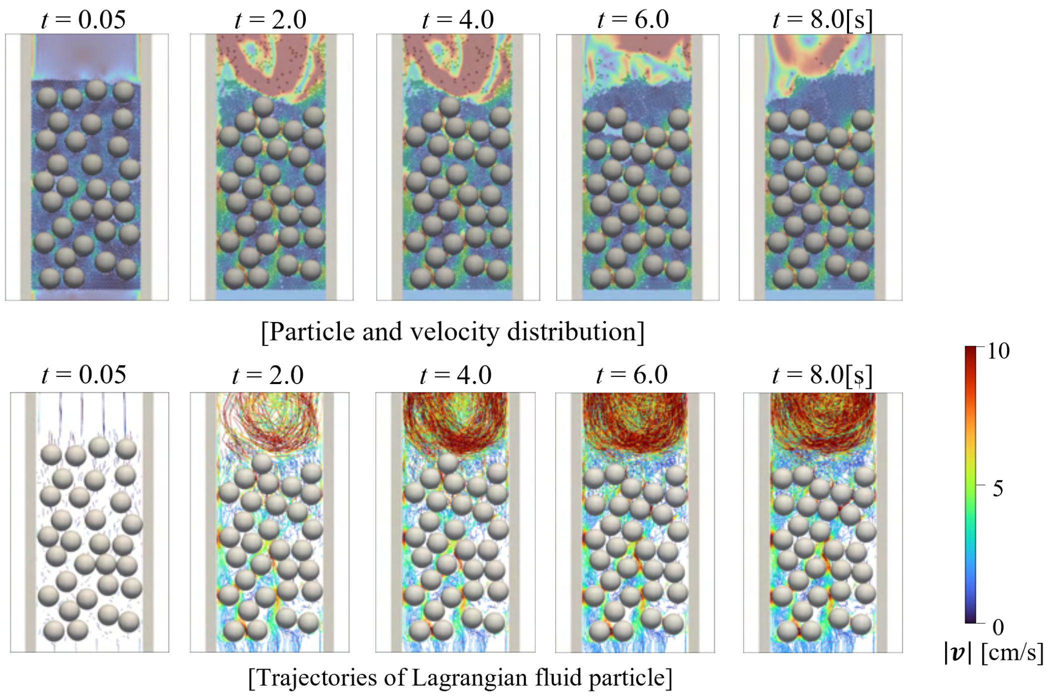
Fig.4 shows the results of the simulation applying the proposed semi-resolved coupling model, which is a hybrid type of the resolved and unresolved models.

(a) Even at an inlet velocity = 0.5 cm/s, where no erosion occurred in the previous unresolved coupling model, the proposed model shows the characteristics of internal erosion, where small particles erupt from between the coarse particles near the surface layer. The semi-resolved coupling can represent fluid behaviour bypassing the coarse particles, and the increased flow velocity and drag force on the fine particles in the pore space can be considered to represent localised failures.

(b) At an inlet velocity = 2.0 cm/s, fine particles are pushed upwards not only near the surface but also inside the particle layer, changing the internal structure of the soil skeleton. This deformation of the soil skeleton propagated through the water channel and caused sufficient failure to erupt the fine particles. This localised fracture and its propagation could only be reproduced if the high velocities (i.e. more than 20 times higher than the inlet flow velocity at the moment) occurring between the coarse particles could be represented. This cannot be reproduced by a unresolved coupling model that calculates a uniform seepage flow according to the Darcy law. We expect that internal erosion can be simulated only by applying a semi-resolved coupling model, which treats coarse particles as moving wall boundaries



(a) v_{in} = 0.50 [cm]



(b) v_{in} = 0.50 [cm]

Figure 4: Numerical result of hybrid-type semi-resolved coupling simulation

5 CONCLUSIONS

We proposed a semi-resolved coupling model in the framework of the particle method (ISPH-DEM), which is a hybrid type of resolved and unresolved models that can represent the characteristics of internal erosion in soil structures. The proposed method is a hybrid type which applies a resolved coupling model to coarse particles that form the soil skeleton and strongly influence the internal flow, and a unresolved coupling model to the fine particles whose shape effects are almost negligible and the surrounding flow is regarded as seepage flow. Therefore, it has the advantages of both conventional methods: the detailed flow reproducibility of the resolved coupling model and the computational efficiency of the unresolved coupling model. In this paper, we focused on confirming the performance of the proposed method in representing internal erosion, and carried out numerical experiments in which the fluid flows into a particle layer with a gap-graded particle size distribution. When applying the unresolved coupling model, the characteristics of internal erosion could not be represented because the local flow velocity distribution, which is the starting point of erosion, could not be reproduced. On the other hand, the proposed semi-resolved coupling model can represent the non-uniform velocity distribution because it can represent the flow bypassing the coarse particles. This enabled representing a series of internal erosion processes in which fine particles in pores with locally high velocities are affected by an increased drag force, which changes the whole structure through movement and loss of the fine particles. In this model, the computationally expensive resolved coupling model is applied only to coarse particles, an efficient unresolved coupling model is used for other fine particles to obtain the seepage flow and drag force. As a result of applying the proposed model, it is considered possible to reproduce the detailed internal erosion.

The proposed hybrid model requires the user to use a resolved/non-resolved coupling model according to the grain size. Therefore, when representing internal erosion in soils with a general grain size distribution, it is expected that the results will differ depending on the user's settings. To solve this problem, it is desirable to develop an extended unresolved type of semi-resolved coupling model, as in Wang et al.[14], which can be applied without restriction to the particle-fluid resolution ratio. If this extended unresolved type of semi-resolved coupling model can be developed within the framework of the particle method, it is expected that numerical simulation of internal erosion will be possible even for general soils.

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