

DEEP LEARNING BASED SURROGATE MODELLING OF WAVE PROPAGATION AND DAMAGE DETECTION IN CRACKED ROD

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Abstract. *Guided wave-based Structural Health Monitoring (SHM) tools utilize the guided wave responses to interrogate damage in structures. This research demonstrates the use of various objective functions in single (mono) objective and multi-objective genetic algorithms for damage identification in isotropic 1D structures. The time domain spectral element method and a deep-learning-based surrogate is utilized for simulating wave propagation in an isotropic cracked rod. The genetic algorithms employ results (“numerical experiment”) obtained from the spectral element model and the deep-learning-based surrogate to determine the optimized crack locations and crack depths as output parameters. The obtained optimized parameters from genetic algorithms are compared in terms of errors for various objective functions.*

1 INTRODUCTION

Engineering structures such as buildings, bridges, offshore platforms, dams, and other civil infrastructures may experience damage during their operational life due to natural or human actions. Even a minor flaw in a structure can lead to increased maintenance costs and significant failure. Therefore the early diagnosis of structural damage is vital and helpful from the perspective of productivity and safety [1]. Structural health monitoring (SHM), as the name suggests, monitors the system’s structural health. It monitors the structure’s performance and reliability in-service through sensing and can inform users of signs of deterioration. Guided waves are an important tool for SHM of engineering structures as these are highly sensitive to discontinuities in their propagation path, while they further have the potential to propagate across long distances.

For structural damage detection or identification, one of the optimization methods, genetic algorithms, has been studied widely. Traditional gradient-based algorithms perform substantially worse in terms of global optimization than these approaches. In contrast to standard algorithms, genetic algorithms may calculate the values of objective functions without requiring the objective function to be continuous, and

they don't require gradient information. So, the searching process makes it not only avoids falling into local minima but also more efficient and effective.

A single-objective optimization problem occurs when the system has just one item to optimize. However, actual engineering systems are sometimes thought to be so complex that one object is insufficient; instead, two or even more objects are required. As a result, genetic algorithms' so-called multi-objective function incorporates the benefits of several dynamic factors [2].

This paper presents a method for damage identification in an isotropic cracked rod using single-objective and multi-objective genetic algorithms (GAs). In these approaches, the structural dynamic responses (signals) of the system generated by a deep-learning-based surrogate models are used for ultra-fast computation of the structural dynamic responses necessary to build objective functions. And, these objective functions are compared with "numerical experiment" signals (obtained by time domain spectral element method).

2 CRACKED ROD SPECTRAL ELEMENT

Several numerical methods have been developed for solving wave propagation problems, such as the finite difference method (FDM), or the broadly exploited the finite element method (FEM). In this study, the spectral element method (SEM), in the time domain, is utilized for modeling wave propagation in an isotropic cracked rod [3]. A spectral rod finite element with a transverse open and non-propagating crack is presented in Fig. 1. The element has six non-uniformly distributed nodes and one degree of

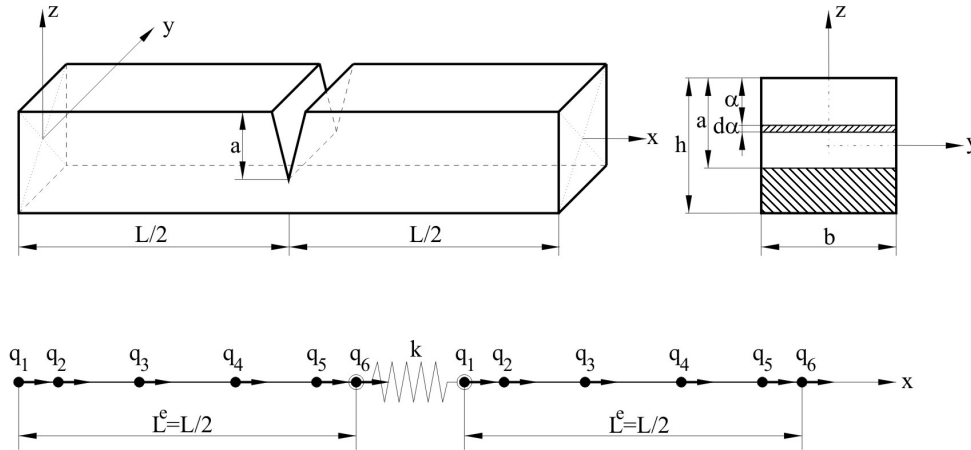


Figure 1: Spectral rod element. (based on Ref. [3]).

freedom per node corresponding to the longitudinal displacement. The nodal locations correspond to Gauss-Lobatto-Legendre integration points. The crack is represented by a spring of stiffness k . Spectral rod finite elements which include a crack can be formed by connecting two spectral finite elements with nodes separated by a spring. The flexibility/stiffness of the spring representing a transverse, non-growing crack is calculated by the laws of fracture mechanics. In these wave propagation simulations, a rod of isotropic material of density 7860 kg/m^3 and geometric length 2 m , cross-section height 0.02 m , and width 0.02 m is considered. An excitation signal in the form of sine with five cycles, modulated by the Hanning window, is applied.

3 DEEP-LEARNING-BASED SURROGATE MODEL

To predict the wave propagation signals in a cracked rod, a deep-learning-based framework is used [4]. In this framework, a full wavefield was predicted. But in our work, the framework is constructed and trained to predict the wave propagation signals sensed at both ends of the rod. To achieve this, the encoder and decoder of the autoencoder parts were trained jointly. So, the decoder part can then be individually used to reconstruct/predict the solutions from latent space. Next, a feed-forward neural network (FFNN) is trained to learn the latent space encoded by the encoder part of the autoencoder for corresponding crack locations and depths as inputs. The training phase of the deep-learning framework is schematically represented in Fig. 2 and it can be summarized in the following steps:

1. Generate wave propagation solutions for various crack locations and crack depths.
2. Train the autoencoder and project the wave propagation solutions to the encoded space using the encoder part of the autoencoder.
3. Train an FFNN to learn the encoded space for corresponding crack locations and crack depths.

In the Fig. 2, $U(N, T)$ represents the wave propagation signals, where N is number of sensors (i.e. two in this work) and T corresponds to the time steps (1024 steps). After training the autoencoder and subsequently the FFNN, the trained FFNN can be used to predict the latent space for a given crack depth and location (or set of crack depths and locations). Next, the wave propagation signals can be predicted by using the trained decoder of the autoencoder. The wavefield prediction phase of the framework can be summarized in the following steps:

1. Predict the encoded space using trained FFNN by feeding a new crack location and crack depth.
2. Predict the wave propagation signals using the decoder part of the trained autoencoder.

A simple schematic of the wave propagation signals prediction phase of the deep-learning framework is schematically represented in Fig. 3. The autoencoder is constructed using the Fourier layers [5], convolution layers, batch-normalization layers, pooling layers, and activation layers. The FFNN is constructed via the use of 4 hidden layers, each comprising 256 neurons. The last layer of the FFNN has 64 neurons (latent size).

4 OBJECTIVE FUNCTIONS

Damage indices (DIs) are the tool that compares two signals and provides a scalar number that represents the extent of damage existing in the structure. The DIs are calculated by comparing a reference signal, to the signal supplied by the system in the event of a failure or damage [6]. These DIs are used as objective functions in GAs. Barreto et. al. used the root mean square error (RMSE) or also called the Euclidean norm, mean absolute percentage deviation (MAPD), covariance (COV), and correlation coefficient deviation (CCD) as damage indicators in their comparative study [7]. Xu et. al. used the relative absolute error (RAE) as a metric for spatio-temporal dynamic problems [8]. Sun et. al. implemented the mean absolute error (MAE) in GA for tuning the parameters of a least square support vector machine (LS-SVM) model [9].

In our study, we utilized the RMSE, RAE, CCD, and CD as objective functions given by equations. 1 to 4 respectively, to implement in GA. RMSE and RAE evaluate the deviations at each data point of the signals. However, CCD and CD measure the synchrony between the signals.

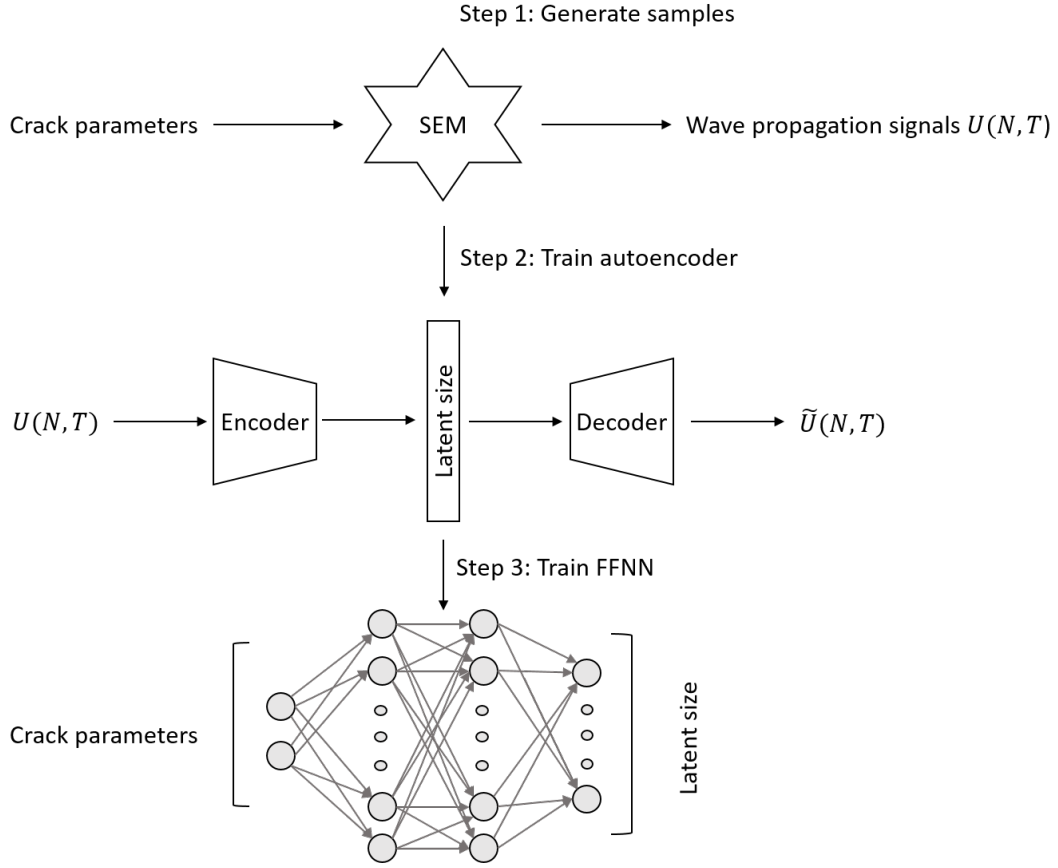


Figure 2: Schematic of training phase of the deep-learning framework.

$$RMSE = \frac{\sqrt{\sum_{i=1}^N (y_i - x_i)^2}}{\sum_{i=1}^N x_i^2} \quad (1)$$

$$RAE = \frac{1}{N} \sum_{i=1}^N \frac{|y_i - x_i|}{\max(|x_i|)} \quad (2)$$

$$CCD = 1 - \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^N (x_i - \bar{x})^2 (y_i - \bar{y})^2}} \quad (3)$$

$$CD = 1 - \cos(\mathbf{x}, \mathbf{y}) = 1 - \frac{\mathbf{x} \cdot \mathbf{y}}{\|\mathbf{x}\| \cdot \|\mathbf{y}\|} = 1 - \frac{\sum_{i=1}^N x_i y_i}{\sqrt{\sum_{i=1}^N x_i^2} \sqrt{\sum_{i=1}^N y_i^2}} \quad (4)$$

Here, \mathbf{x} represents “numerical experiment” signals, \mathbf{y} represents “signals predicted by the deep-learning-based surrogate model” for each generation in GAs.

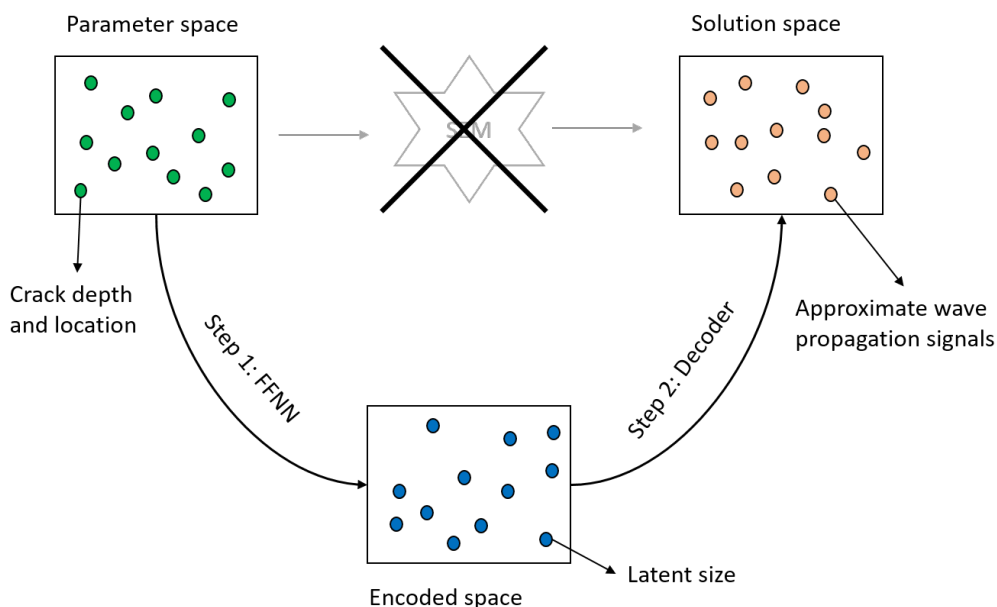


Figure 3: Schematic of prediction phase of the deep-learning framework.

5 GENETIC ALGORITHM: SINGLE-OBJECTIVE

Damage detection constitutes a system identification (inverse) problem, where the damage extent and location are identified using an objective/fitness function through the use of an appropriate optimization method, such as a Genetic Algorithm (GA) [10]. The GA is a search technique, which is formulated based on Darwin’s theory of natural selection and survival of the fittest [11]. The differences between conventional search techniques and evolutionary algorithms, such as the GA can be summarised as follows [3]:

- the GA works with a set of points (population), not a single point.
- the GA does not need derivative information. It only requires a fitness value which is computed via the objective function.
- the GA applies only probabilistic rules of selection.

The GA process initiates with a randomly generated solution set or population and goes through some specific steps. The main steps of the algorithm include initialization, crossover, mutation, and selection. We implemented the real-valued GA as opposed to the commonly used binary-coded GA for the damage detection problem. The binary-coded GA has some difficulties such as the “Hamming cliff” problem, loss of precision, occupying higher computer memory, etc., when dealing with continuous search space [12]. The real-valued GA represents the optimization variables in real values, have fast convergence towards the optima than binary-coded GA, also overcomes the issues of binary-coded GA [13].

6 GENETIC ALGORITHM: MULTI-OBJECTIVE

The presence of multiple objectives in an optimization problem provides a set of optimal solutions (Pareto-optimal solutions) instead of a single optimal solution [14]. These types of problems are known as Multi-objective Optimization Problems (MOOP). By converting the multi-objective optimization prob-

lem into a single-objective optimization problem (i.e., a vector of objective functions is changed to a single function), a MOOP can be solved using traditional optimization techniques such as the Weighting objectives method and the Global criterion method. [15]. However, when some technique of multi-objective evolutionary algorithm is used to solve a MOOP, several solutions can be found at the same time i.e., Pareto-optimal solutions. There are several multi-objective evolutionary algorithms that are based on Pareto-optimal solutions or Pareto front, such as NPGA, NSGA, NSGA-II, SPEA, PESA, etc [16]. In our study, we utilized the computationally fast and elitist multi-objective evolutionary algorithm i.e. Non-dominated sorting genetic algorithm II (NSGA-II) proposed by Deb et. al. [14]. A flowchart of the NSGA-II algorithm is presented in Fig. 4. The combinations of the objective functions (equations. 1 to 4) is used for the multi-objective GA process.

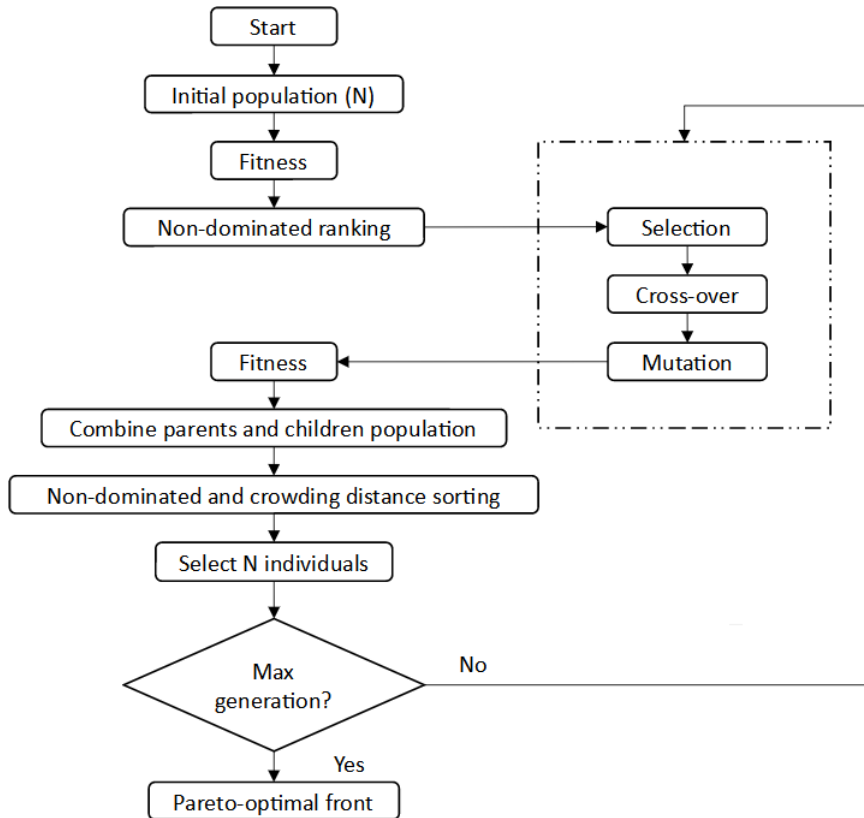


Figure 4: Flow chart of NSGA-II algorithm process. (based on [17])

7 RESULTS

7.1 Single-objective GA

A deep-learning-based surrogate model is used to calculate the objective function for each generation in the GA process. We used pymoo, an open-source Python library for optimization using the GA [18]. For damage identification in isotropic cracked rods, the population size is considered 100 for 100 generations. In this work, we implemented the real-valued GA in which two design variables (crack location

and crack depth) are initialized as random real numbers. We considered 6 different crack locations (0.32 *m*, 0.61 *m*, 0.93 *m*, 1.23 *m*, 1.53 *m*, 1.73 *m*) and 6 different crack depths (7.07%, 12.27%, 18.77%, 24.22%, 29.10%, 34.35%), combination of these gives total 36 crack cases. In Table 1, optimized crack locations and crack depths for 6 crack cases (out of 36) after 100 generations for all four objective functions have been mentioned. The evolution of the objective functions and crack depth values after each generation can be seen in Fig 5. and Fig 6., respectively. To make comparisons in objective functions after each generation, the objective functions are normalized by their respective maximum value. The results obtained from single-objective GA shows that CCD and CD converges rapidly compared to RMSE and RAE around zero as in Figure. 5, and predict better crack depth compared to RMSE and RAE (see Figure. 6). Crack locations are accurately predicted by single-objective GA for all four objective functions seen in Table. 1. Also, single-objective GA predicts the crack depths very accurately, even a very small crack depth of 7.07%.

Table 1: Crack depths [%] at crack location 0.32 *m* along the rod.

Reference		RMSE		CCD		CD		RAE	
location	depth	location	depth	location	depth	location	depth	location	depth
0.32	7.07	0.32	7.24	0.32	7.13	0.32	7.23	0.32	7.26
0.32	12.27	0.32	12.40	0.32	12.40	0.32	12.42	0.32	12.28
0.32	18.77	0.32	18.91	0.32	18.91	0.32	18.90	0.32	18.79
0.32	24.22	0.32	24.35	0.32	24.36	0.32	24.36	0.32	23.61
0.32	29.10	0.32	29.18	0.32	29.15	0.32	29.15	0.32	29.18
0.32	34.35	0.32	34.44	0.32	34.44	0.32	34.44	0.32	34.38

7.2 Multi-objective GA

For the optimization parameters (crack depth and crack location), the number of generations chosen is 100 and the population is 100. The design variable are encoded in real values as in single-objective GA. Total 36 crack cases (as in single-objective GA) are considered for multi-objective GA. For a crack case, the Pareto optimal fronts obtained by NSGA-II after 100 generations in pymoo are presented in Fig 7. After obtaining a set of non-dominated solutions a single solution has to be chosen. This decision-making process for multi-objective problems is also known as Multi-Criteria Decision Making (MCDM). For the MCDM step, the compromise programming (pymoo in-built function) was used which uses decomposition functions. More details about this decision-making by compromise programming can be found in Ref. [18]. After making the decision criteria by giving the equal weights to both objectives (i.e. both objective are equally important), the optimized results for 12 crack cases (out of 36) are given in Table 2. and Table 3. CCD and CD are correlated objective functions, so they have not considered together in multi-objective GA. The results a in Table. 2 & Table. 3 show that, the small crack depths obtained after making decision criteria, have quite a large errors. These variations are directly related to decision criteria. But, crack locations obtained after making decision criteria, are very accurate.

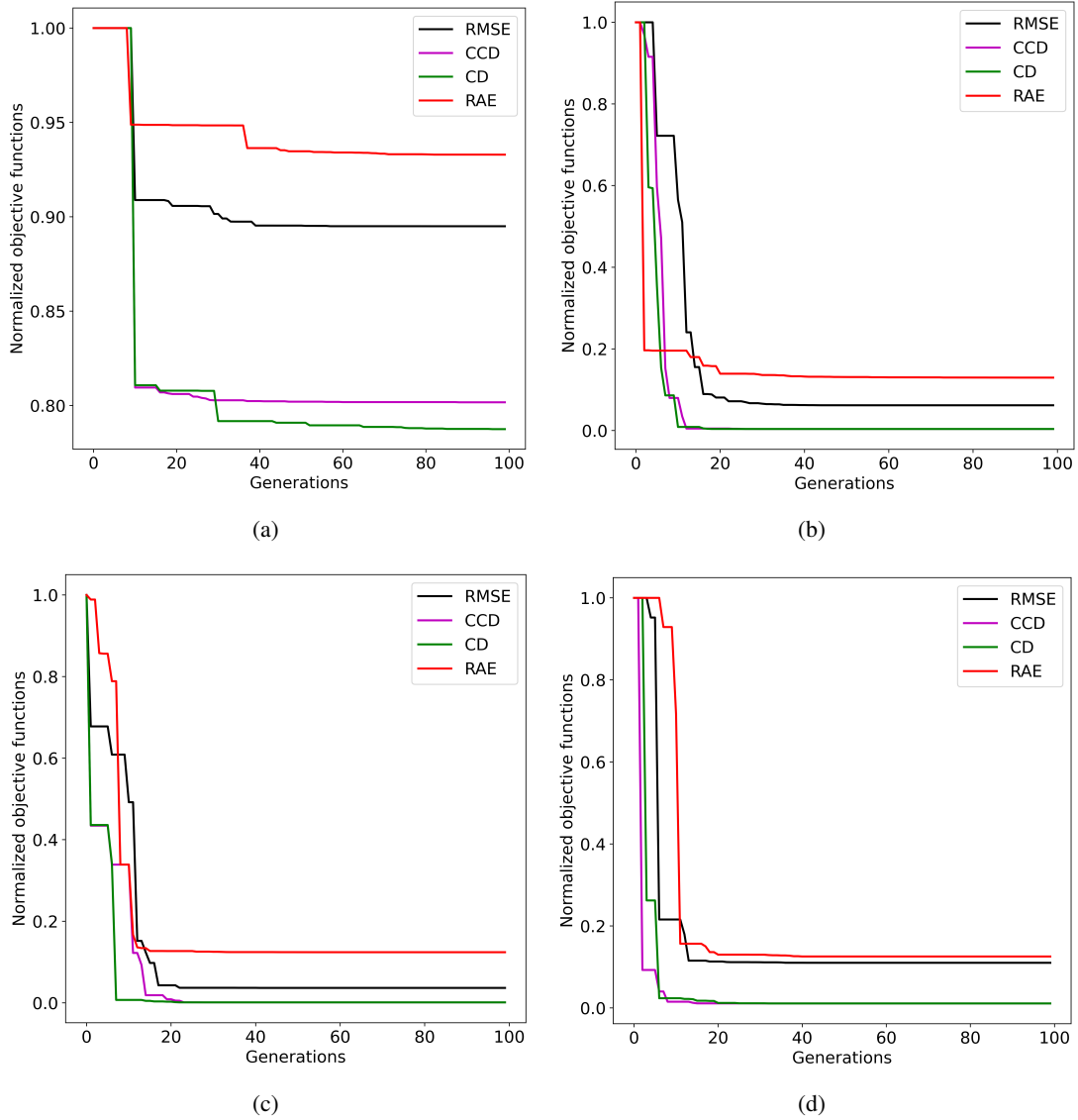


Figure 5: Fitness curves of GA for crack depth of a) 7.07%, b) 18.77%, c) 24.22%, and d) 34.35% at crack location 0.32 m along length of the rod.

8 CONCLUSIONS

In this work, a guided-wave-based approach for damage detection in an isotropic cracked rod is presented, relying on the use of genetic algorithms. A time domain-spectral element model is used as a reference for simulating wave propagation scenarios (called “numerical experiment”) and a deep-learning-based surrogate is exploited for computing the objective/fitness functions in the genetic algorithm process. We utilized four types of objective/fitness functions in the study of single-objective genetic algorithms and multi-objective genetic algorithms. In multi-objective GA, the combination of these objective functions have used to obtain the optimized parameters (crack depth and crack location).

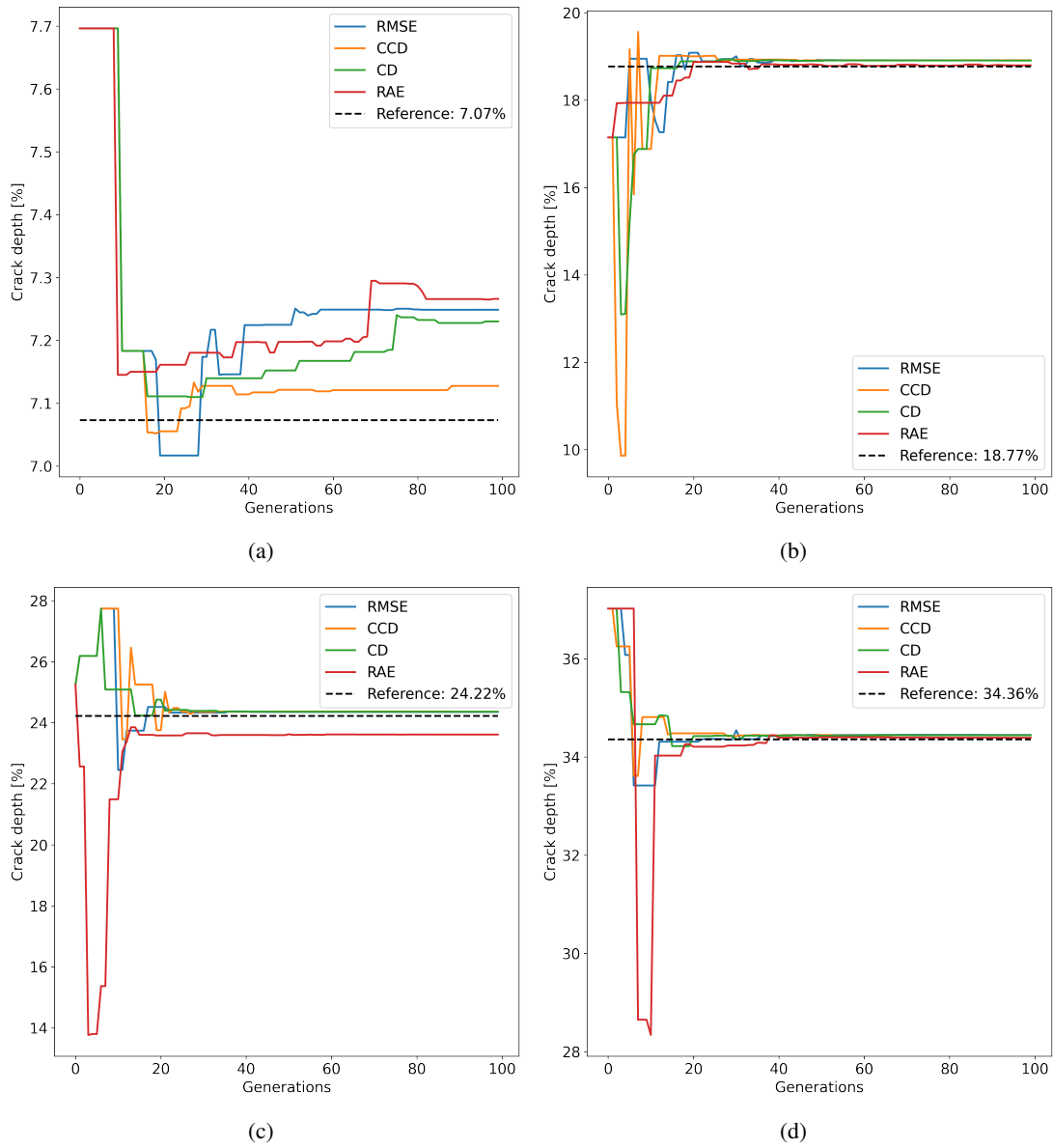


Figure 6: Evaluation of crack depths for crack depth of a) 7.07%, b) 18.77%, c) 24.22%, and d) 34.35% at crack location 0.32 m along length of the rod.

Six crack depths (7.07% 12.27% 18.77% 24.22% 29.10%, 34.35%) and six crack locations (0.32 m, 0.61 m, 0.93 m, 1.23 m, 1.53 m, 1.73 m) combined gives 36 crack scenarios considered for this work. The obtained results from single-objective GA show that the approach is capable of detecting early defects (small cracks). But in case of multi-objective GA, it is essential to choose wisely, the objective functions and decision criteria to obtain trade-off solution from Pareto front. For the MCDM step, the compromise programming was used by giving equal weights to both objectives. In the future, we are planning to work with beam-like models to capture multiple wave reflections and dispersion. Currently, the deep-learning

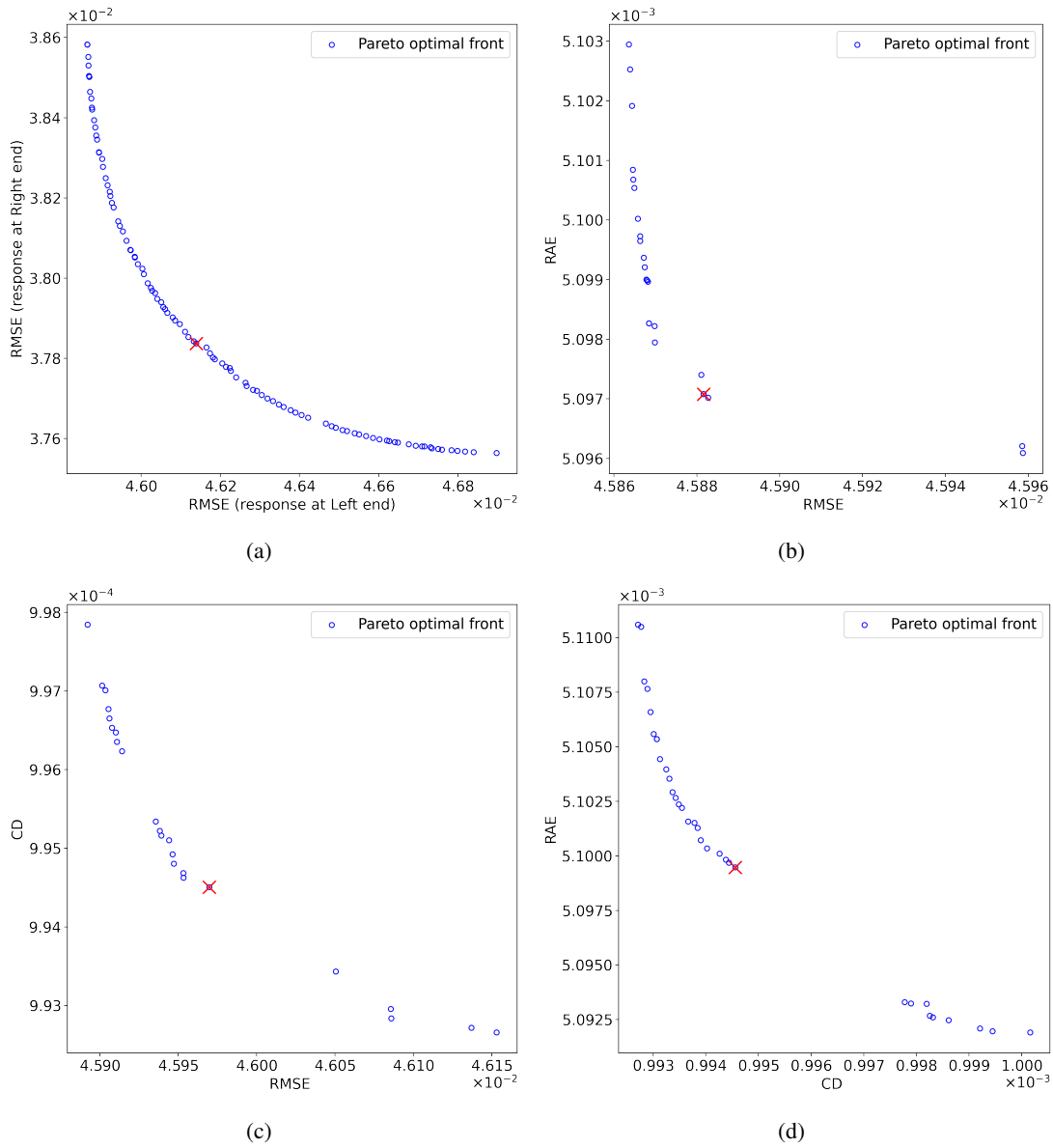


Figure 7: Pareto front obtained by NSGA-II for a crack depth of 29.10% at crack location of 0.61 m. \times represents the trade-off solution.

framework is limited to predicting problem-specific solutions because it was trained for only particular material, the geometry of the rod, and the excitation signal. Also, it has another limitation of generating the signals for a specific type of damage scenario as presented in Fig. 1. To implement it in real-life problems, this framework will be built to generate signals for different damage scenarios, various geometries, materials, and excitation signals. Also, the “numerical experiment” signals will be replaced with the real experimental signals (“measured signals”) where the deep-learning-based wave propagation surrogate will be coupled.

Table 2: Crack depths [%] at crack location 0.61 *m* along the rod.

Reference		RMSE-RMSE		RMSE-RAE		RMSE-CD		RAE-CD	
location	depth	location	depth	location	depth	location	depth	location	depth
0.61	7.07	0.61	8.17	0.61	8.23	0.61	8.18	0.61	8.25
0.61	12.27	0.61	12.24	0.61	12.25	0.61	12.26	0.61	12.24
0.61	18.77	0.61	18.78	0.61	18.82	0.61	18.82	0.61	18.84
0.61	24.22	0.61	24.44	0.61	24.46	0.61	24.46	0.61	24.46
0.61	29.10	0.61	29.24	0.61	29.16	0.61	29.18	0.61	29.16
0.61	34.35	0.61	34.42	0.61	34.49	0.61	34.55	0.61	34.49

Table 3: Crack locations [*m*] with crack depth of 12.27% along the rod.

Reference		RMSE-RMSE		RMSE-RAE		RMSE-CD		RAE-CD	
location	depth	location	depth	location	depth	location	depth	location	depth
0.32	18.77	0.31	18.90	0.31	18.90	0.31	18.85	0.31	18.89
0.61	18.77	0.61	18.77	0.61	18.82	0.61	18.82	0.61	18.84
0.93	18.77	0.93	18.78	0.93	18.81	0.93	18.79	0.93	18.81
1.23	18.77	1.23	18.91	1.23	18.91	1.23	18.94	1.23	18.92
1.53	18.77	1.53	18.76	1.53	18.69	1.52	18.70	1.52	18.69
1.73	18.77	1.73	18.70	1.73	18.73	1.73	18.75	1.73	18.74

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