QUADCOPTER WITH THRUST VECTOR CONTROL BASED ON SCOTCH YOKE

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Abstract. People are interested in quadcopters (or quadrotors) because of flight maneuverability and a variety of applications. Standard quadrotor has constant mechanical structure – frame with four motors and four rotors (propellers), and usually, rotors do not change the angle of the rotation axis relative to the frame. However, there are several designs in which the axis of rotation of rotors changes the angle relative to the frame (for example, "Holocopter" and "Tilt-rotor"), and this option changes the thrust vector of the quadcopter. In this work a novel design of thrust vector control based on the Scotch yoke is proposed. All rotor axes incline simultaneously in any direction by only two servos. Construction design is proposed, and general dynamics are described.

1 INTRODUCTION

In conventional quadcopters, the structure is fixed, and the thrust vector is controlled by changing the speed of rotation of different propellers. But there are designs in which the angle of inclination of the rotor axis changes, for example, as in "Holocopter" (where the propellers axles rotate relative to the axes coaxial to the beams of the quadcopter) [1] and "Tilt-rotor" [2-9]. In this case, the change in the thrust vector is carried out not only by the speed of rotation of the rotors but also by tilting their axis of rotation.

A quadcopter is an aircraft that has four motors with propellers that create thrust. Typically, the propeller axes and blade angles are fixed, and only rotational speeds are controlled, which greatly simplifies the design. Vertical movement is carried out by a synchronous change in the rotation speeds of all propellers; for horizontal movement, you need to tilt the quadcopter, which is achieved by a corresponding change in the rotation speeds of different propellers that create the moments necessary for tilt. The opposite direction of the pairs of propellers provides compensation for the moments of drag. Such devices with fixed propeller axes are used quite widely.

In this paper, it is proposed to investigate the possibilities of tilting the propeller axes to create a deflected thrust vector and not tilt the quadcopter for horizontal movement. As a result, – to reduce aerodynamic drag, to increase the speed and maneuverability of flight.

Adding variable axle tilt to the propellers results in additional degrees of freedom for varying the thrust generated by each motor-propeller group. With propellers of this type, the thrust can be changed by changing the angle of inclination of the propeller axles in projection

on the horizontal plane and the vertical. The angles of inclination of the propeller axles and the speed of rotation of the propeller motors - these two factors overlap the range of action, since, for example, for forward motion we can quickly rotate the propellers at the stern, and slow down the speeds of the motors at the bow, and on the other hand we can tilt the axles of propellers for the same purpose. However, in the first case, we have to increase the pitch angle of the quadcopter, and in the second, keeping the pitch angle constant, tilt the propeller axles. The number of possible combinations between the body pitch and pitch of the propellers for forward motion is limited by the maximum pitch angles of the propellers, caused by the maximum engine power, the capabilities of the motion control devices, construction and aerodynamic constraints.

In the quadcopters with a variable angle of inclination of the propeller axles considered in this article, all axles are tilted by the same angle, in the desired direction of movement. So, the quadcopters can quickly accelerate and stop quickly while maintaining the horizontal orientation of the body. The quadcopter can also tilt the body, keeping the barycenter velocity zero, which is impossible in conventional quadcopters.

2   CONSTRUCTION DESIGN

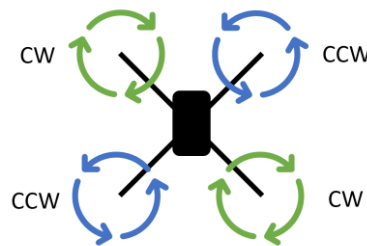There is a standard layout for the quadcopter called X4 (see Fig. 1).

Figure 1: X4 layout of a quadcopter with clockwise pusher propellers (CW) and counter-clockwise normal propellers (CCW)

At the moment we will give preference to the X4 scheme because this scheme is often used in high-speed quadcopters and is easy to control.

Since one of the basic requirements for the quadcopters is the minimum mass, and the other requirement in this article is the deflected thrust vector, it is preferable to use fewer servos, while simultaneously tilting all axes of the propeller group. Structurally, this synchronous tilting of the axes can be achieved using two servo drives and a parallel transfer mechanism. The parallel transfer mechanism, which deflects all axes by one angle, leaving them parallel to each other, consists of the main structural elements:
  • Identical crosses (eight in total)
  • Hooke's joints (eight in total)
  • Scotch yokes (two in total, crossed) united on plate with slots
  • Servos (two in total)

Two servos through levers fix the mechanism in the required angular position of the axes relative to the body.

A general view of the proposed construction of a quadrocopter in different angles, scales, and details for a better understanding is presented in several figures at once (see Fig. 2-7).
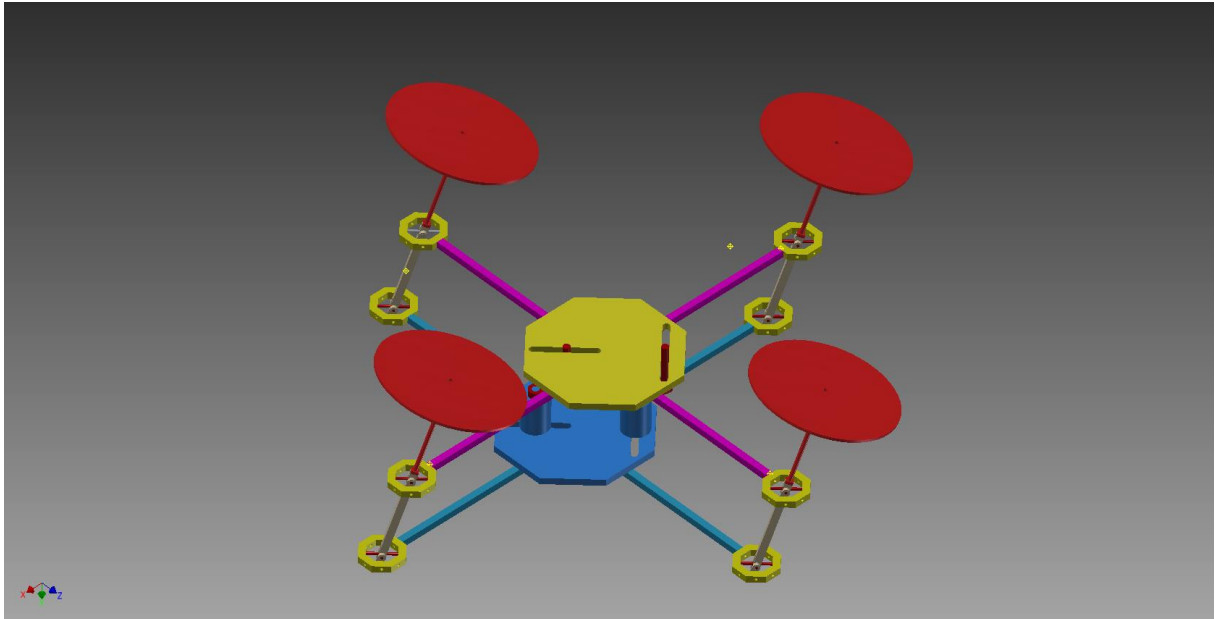

Figure 2: General view for a "left-to-right on the picture" flight

In Fig. 2, we see that the slots on the top plate are made at an angle of 90° to each other, this allows to control the simultaneous tilt of the axes using Scotch yokes in two directions, conventionally speaking "forward-backward" and "left-right", and its combinations.
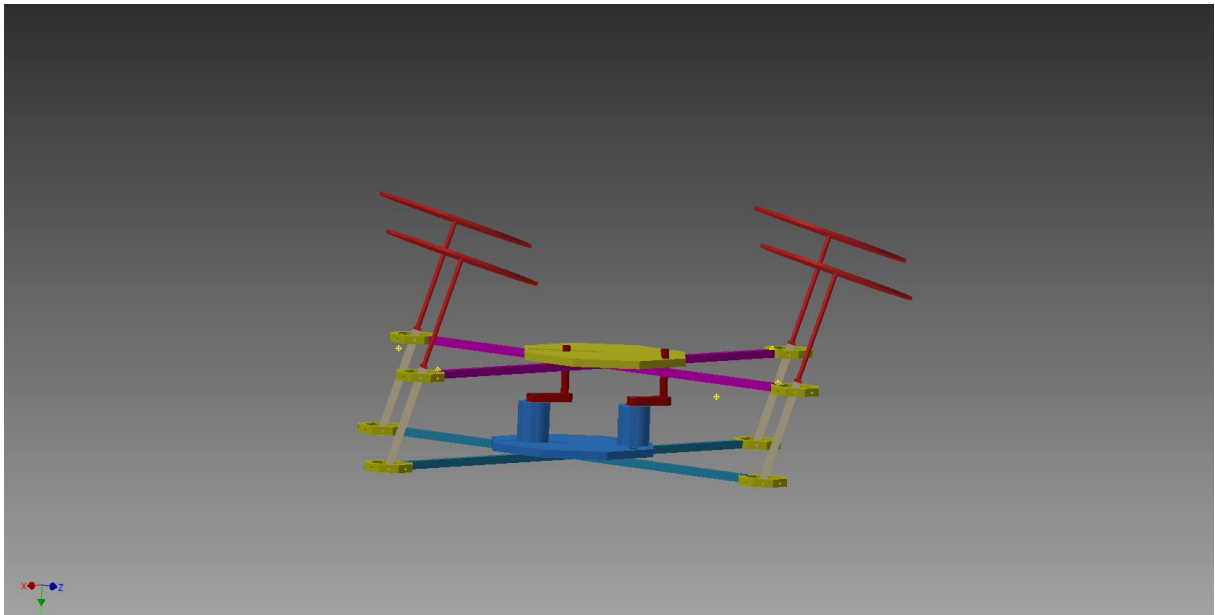

Figure 3: Side view for "left-to-right on the picture" flight

In Fig. 3 you can see how the arms of the servos are rotated, at the ends of which there are rods inserted into the slots of the top plate.
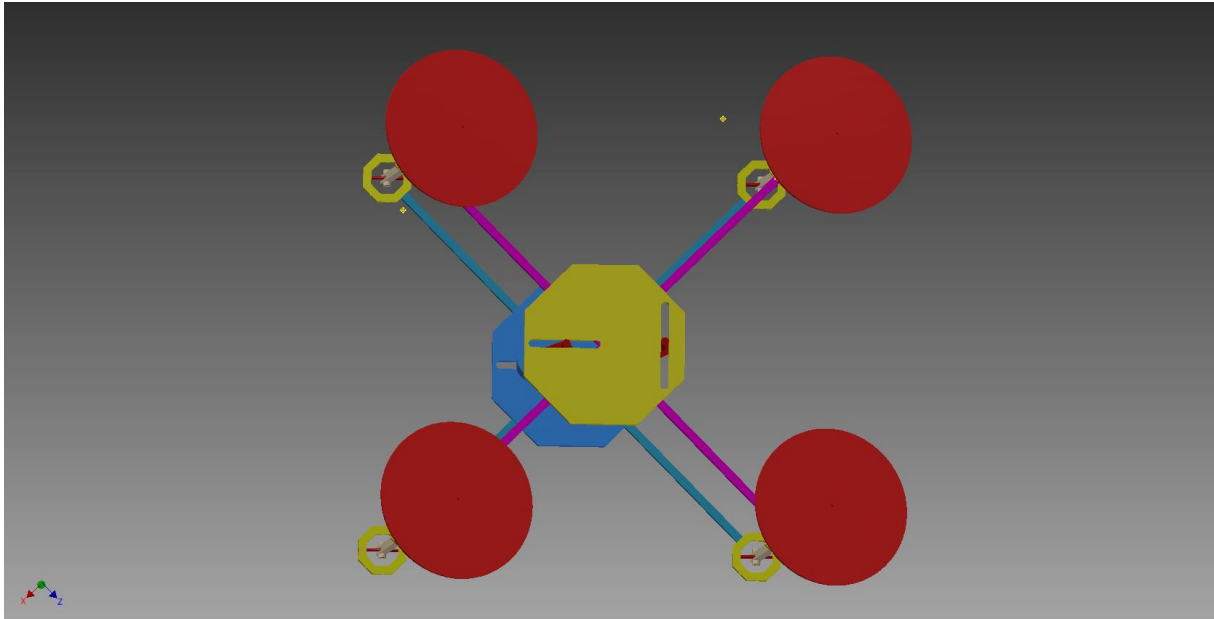

Figure 4: Top view for "left-to-right on the picture" flight

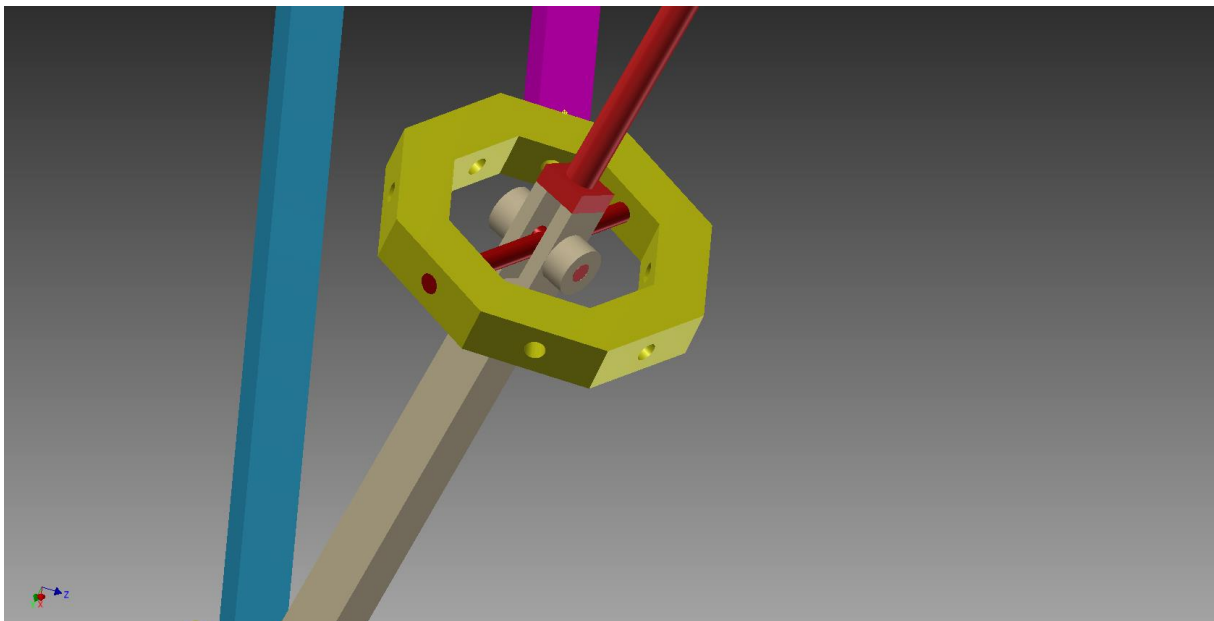In Fig. 4 you can see the parallel shift of the upper frame of the quadcopter relative to the lower one.

Figure 5: Hooke's joint in the connection of the axis with the body

In Fig. 5 you can see that the cross inside the Hooke's joint is attached to the quadcopter beam on one side, and to the quadcopter axis mount on the other side.
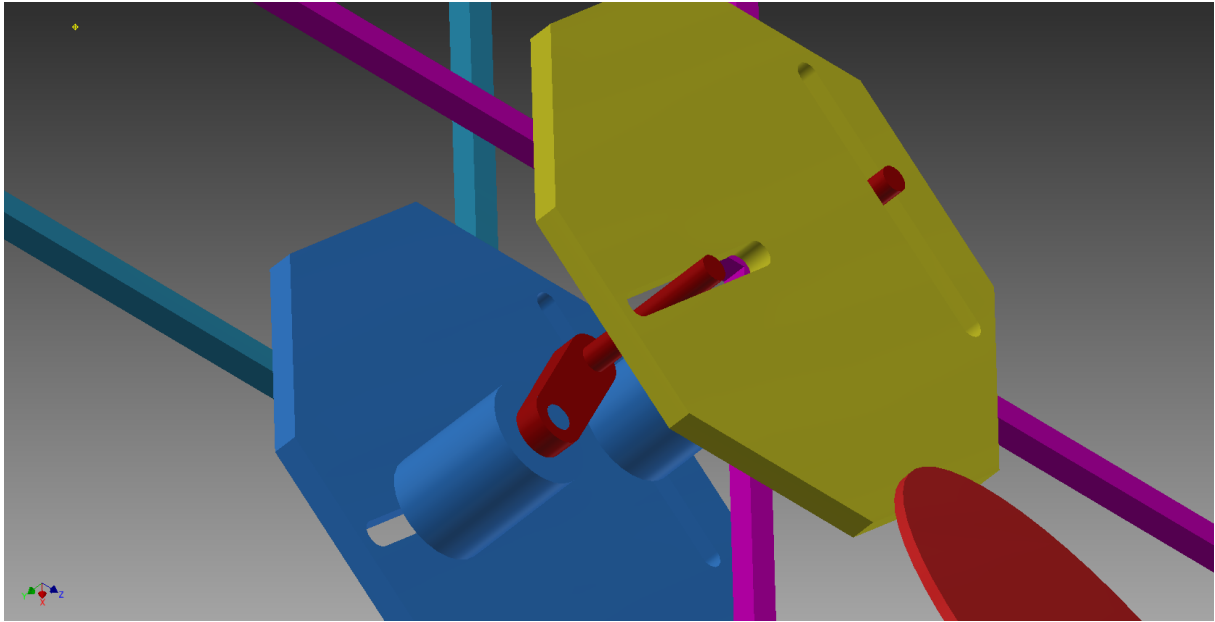

Figure 6: Two crossed Scotch yokes with two servo arms

In Fig. 6 you can see the top slotted plate on an enlarged scale.
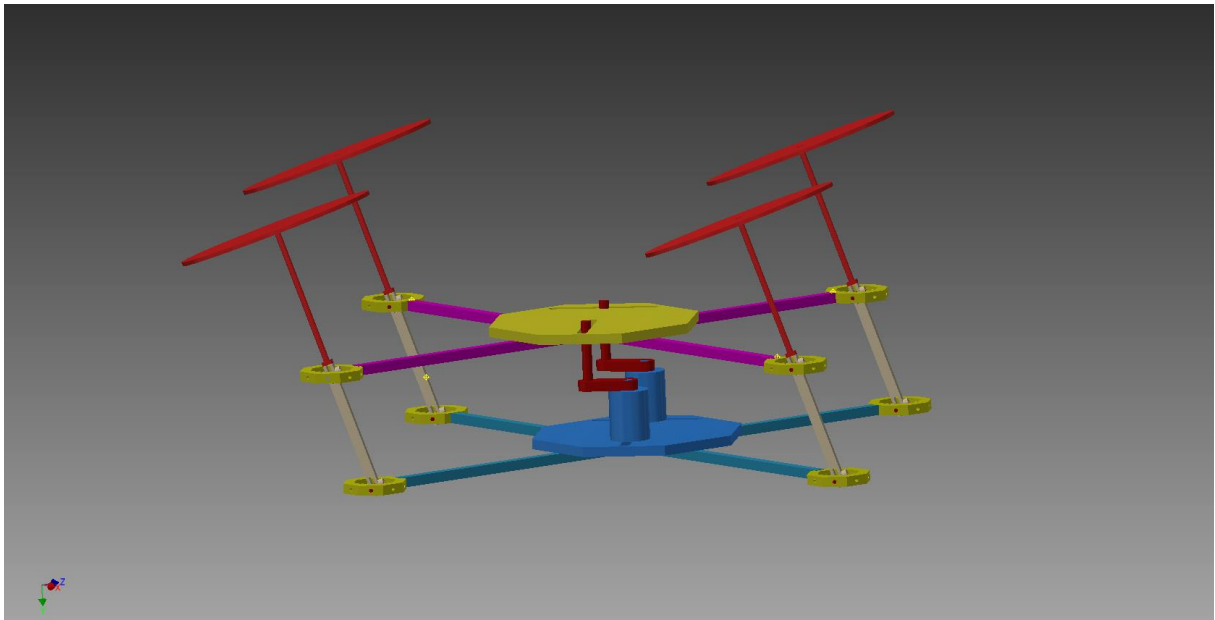

Figure 7: A general view for a "right-to-left on the picture" flight with a different position of the servos while keeping the roll, pitch, and yaw angles of the quadcopter the same as in previous figures

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In Fig. 7 we can see that we can change the direction of flight by setting a different position to the servos, and at the same time, the roll, pitch, and yaw angles of the quadcopter will not change.

Note that the motors that drive the propellers are not shown in the illustrations. This is because there are various options for their placement on tilted axles (under the lower platform, between platforms, above the upper platform).

Also, in this article, the proportions of the construction are not considered in detail, but only the basic concept of a quadcopter with a thrust vector control based on Scotch yoke is proposed. At this conceptual stage, we also do not pay attention to design constraints, such as the maximum possible angles of inclination of the propeller axles, reliability and durability of the mechanism, and so on.

3 GENERAL DYNAMICS

A mathematical model of the dynamics of a quadcopter can be considered using Newton's laws and dynamic Euler equations in the reference frame associated with the body of the quadcopter, where the designation with the subscript "B" means "Body frame" [10]:

$$m\left(\vec{\omega}_B \times \vec{V}_B + \dot{\vec{V}}_B\right) = \vec{f}_B,$$
$$\hat{I}\dot{\vec{\omega}}_B + \vec{\omega}_B \times \left(\hat{I}\vec{\omega}_B\right) = \vec{m}_B,$$
(1)

where m – quadrotor mass, $\vec{\omega}_B$ – angular velocity of quadrotor barycenter, $\vec{V}_B$ – linear velocity of quadrotor, $\hat{I}$ – diagonal inertia matrix of quadrotor, $\vec{f}_B$ – sum of external forces acting on quadrotor, $\vec{m}_B$ – sum of external torques acting on quadrotor.

Then angular velocity $\vec{\omega}_B = [p \quad q \quad r]^T$ and linear velocity $\vec{V}_B = [u \quad v \quad w]^T$ can be translated to the inertial reference frame associated with North-East-Down (NED) directions of the Earth local region [10]:

$$\vec{V} = R\vec{V}_B,$$
$$\vec{\omega} = T\vec{\omega}_B,$$
(2)

where $\vec{V} = [\dot{x} \quad \dot{y} \quad \dot{z}]^T$ – linear velocity in NED frame, $\vec{\omega} = [\dot{\phi} \quad \dot{\theta} \quad \dot{\psi}]^T$ – angular velocity in in roll-pitch-yaw Euler angles, R – rotation matrix based on roll-pitch-yaw Euler angles, T – matrix of angular transformations [10]:

$$R = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix},$$

$$T = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi/c_\theta & c_\phi/c_\theta \end{bmatrix},$$
(3)

where $c_* = \cos(*), s_* = \sin(*), t_* = \tan(*)$.

As for forces and torques we can write down the following:

$$\vec{f}_B = mgR^T\vec{e}_z + \vec{f}_t + \vec{f}_w,$$
$$\vec{m}_B = \vec{\tau}_t + \vec{\tau}_g - \vec{g}_a + \vec{\tau}_w,$$

(4)

where $\vec{e}_z$ – the unit vector in the inertial frame directed "down" (to the Earth center), $\vec{f}_t$ – total thrust vector generated by all propellers, $\vec{f}_w = \begin{bmatrix} f_{wx} & f_{wy} & f_{wz} \end{bmatrix}^T$ – aerodynamic force produced by oncoming wind on the quadrotor, $\vec{\tau}_t = \begin{bmatrix} \tau_x & \tau_y & \tau_z \end{bmatrix}^T$ – torque, generated by total thrust from four propellers with different angular velocities, $\vec{\tau}_g$ – torque in the gravity field that appears when propeller axles incline (it is in our construction), $\vec{g}_a$ – gyroscopic moment caused by rotation of four propellers and quadrotor body , $\vec{\tau}_w = \begin{bmatrix} \tau_{wx} & \tau_{wy} & \tau_{wz} \end{bmatrix}^T$ – aerodynamic torque, produced by oncoming wind on the quadrotor.

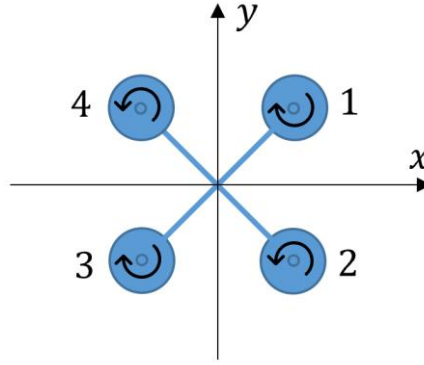In what follows, we will number the propellers as follows (see Fig. 8).

Figure 8: Numeration and rotation of propellers relative to the body frame

3.1 Thrust and its torque

In a conventional quadcopter, the total thrust vector (and thrust of each propeller) is always directed along the z-axis.

In our work, we consider the thrust of each propeller at an angle to the z axis, and to the x and y axes of the body frame. Assuming that the thrust of each propeller is approximately proportional to the square of its rotation speed [11], we write down the total thrust (see Fig. 9):

$$\vec{f}_t = \begin{bmatrix} f_{tx} & f_{ty} & f_{tz} \end{bmatrix}^T = \sum_{i=1}^{4} \vec{f}_i = b\sum_{i=1}^{4}\Omega_i^2 \begin{bmatrix} s(\alpha_i)c(\beta_i) & s(\alpha_i)s(\beta_i) & -c(\alpha_i) \end{bmatrix}^T,$$

(5)

where b – thrust coefficient, $\vec{f}_i$ – thrust of i-propeller, Ω_i – angular speed of the i-propeller, α_i – angle between the i-propeller axle and z axis, β_i – angle between the projection of i-propeller axle on xy-plane and x axis.
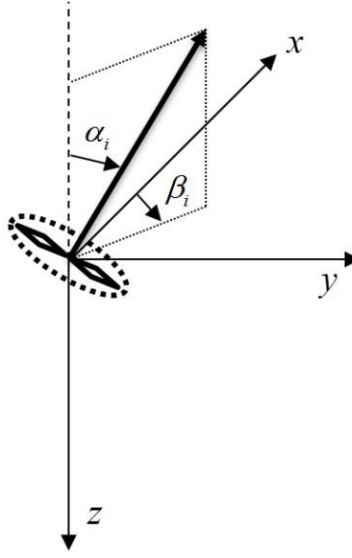
Figure 9: Orientation angles of propeller axle in the body frame

As for the thrust torque we can write down the following:

$$\vec{\tau}_t = \begin{bmatrix} \tau_x & \tau_y & \tau_z \end{bmatrix}^T = \sum_{i=1}^{4} \vec{\rho}_i \times \vec{f}_i, \tag{6}$$

where $\vec{\rho}_i$ – vector from the barycenter of quadrotor to the joint of i-propeller, $\vec{f}_i$ – thrust of i-propeller taken from (5).

The vector $\vec{\rho}_i$ depends on the geometry of the quadrotor. Let us assume that on Fig. 8 we have square in propeller's joints 1-2-3-4, and the side of the square is a. Then we have:

$$\vec{\rho}_1 = \begin{bmatrix} a/\sqrt{2} & a/\sqrt{2} & 0 \end{bmatrix}^T, \qquad \vec{\rho}_2 = \begin{bmatrix} a/\sqrt{2} & -a/\sqrt{2} & 0 \end{bmatrix}^T,$$
$$\vec{\rho}_3 = \begin{bmatrix} -a/\sqrt{2} & -a/\sqrt{2} & 0 \end{bmatrix}^T, \quad \vec{\rho}_4 = \begin{bmatrix} -a/\sqrt{2} & a/\sqrt{2} & 0 \end{bmatrix}^T. \tag{7}$$

Then, considering (5)-(7) and the fact that in our construction all propeller axles are inclined the same angles, we obtain:

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha \in \left[-\alpha^{\max}, \alpha^{\max}\right], \quad \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta \in \left[-\pi, \pi\right],$$
$$\vec{f}_t = b\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right)\begin{bmatrix} s(\alpha)c(\beta) & s(\alpha)s(\beta) & -c(\alpha) \end{bmatrix}^T,$$
$$\tau_x = \frac{ab}{\sqrt{2}}\left(-\Omega_1^2 + \Omega_2^2 + \Omega_3^2 - \Omega_4^2\right)c(\alpha),$$
$$\tau_y = \frac{ab}{\sqrt{2}}\left(\Omega_1^2 + \Omega_2^2 - \Omega_3^2 - \Omega_4^2\right)c(\alpha), \tag{8}$$
$$\tau_z = \frac{ab}{\sqrt{2}}\left(\left(-\Omega_1^2 + \Omega_2^2 + \Omega_3^2 - \Omega_4^2\right)c(\beta) + \left(\Omega_1^2 + \Omega_2^2 - \Omega_3^2 - \Omega_4^2\right)s(\beta)\right).$$

3.2 Gravity torque

Torque in the gravity field generated by axles inclination can be determined by this expression:

$$\vec{\tau}_g = \sum_{i=1}^{4} \vec{\sigma}_i \times \mu \vec{g},$$ (9)

where $\vec{\sigma}_i$ – vector from the barycenter of fixed part of quadrotor to the barycenter of i-th propeller-motor group, $\mu \vec{g}$ – gravity force, acting on i-th propeller-motor group.

The value of $\vec{\tau}_g$ characterizes the disbalance of total barycenter of quadrotor, and because of the masses shift it creates incline torque. By the way, this also changes the inertia matrix of quadrotor. However, if masses and its shift are not so big, we can neglect it. For this, it is necessary that the barycenter of each propeller-motor group should be fixed in the body frame when the propeller axles are inclined.

3.3 Gyroscopic torque

Gyroscopic torque can be determined as follows:

$$\vec{g}_a = \sum_{i=1}^{4} J_p \left(\vec{\omega}_B \times \vec{e}_i \right) (-1)^{i+1} \Omega_i,$$ (10)

where J_p – propeller moment of inertia, $\vec{e}_i$ – unit vector along the angular velocity of the i-th propeller.

According to [12], J_p is small compared to any moment of inertia of the quadcopter body. In this regard, the gyroscopic torque can be removed from consideration in the final system. In addition, the numerous aerodynamic influences observed on the aircraft body and propellers have a much larger impact on dynamics.

3.4 Aerodynamic effects

Aerodynamic effects can be divided into body-related effects and propeller-related effects.

Body-related effects mainly consist of drag force, lift force, and lateral force. All these forces can be described by approximate expressions:

$$f_D = c_D \frac{\rho \upsilon^2}{2} S, \; f_L = c_L \frac{\rho \upsilon^2}{2} S, \; f_Y = c_Y \frac{\rho \upsilon^2}{2} S,$$ (11)

where $\rho \upsilon^2/2$ – air flow pressure, ρ – air density, υ – air flow velocity, c_D – drag coefficient, c_L – lift coefficient, c_Y – lateral force coefficient, S – area of the largest cross-section of the quadcopter body, perpendicular to the incoming air flow.

In our quadcopter design, due to which it is possible to fly at a zero pitch angle (as well as at pitch angles of different signs), firstly, it is possible to reduce the cross-sectional area, and therefore to reduce the drag force, and secondly, to create the effect of a lifting wing, thanks to which to compensate for the effect of gravity, and direct the thrust to move forward, and thereby increase the speed and duration of the flight.

As for propeller-related aerodynamic effects, many of them are described [11], but most of

all we will pay attention to the drug torque, which allows the quadrotor to change yaw-angle. The *i*-th propeller-drag torque can be approximately described by:

$$\vec{\tau}_{D(i)} = -\text{sgn}(\Omega_i) k \Omega_i^2 \vec{e}_i, \tag{12}$$

where k – constant, $\vec{e}_i$ – unit vector along the direction of the propeller axle.

The total propeller-drag torque will be:

$$\vec{\tau}_D = k\left(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2\right)\left[s(\alpha)c(\beta) \quad s(\alpha)s(\beta) \quad -c(\alpha)\right]^T. \tag{13}$$

All these aerodynamic effects must be included in $\vec{f}_w$ and $\vec{\tau}_w$ in (4).

4   CONCLUSIONS

- Newest Scotch yoke thrust vectoring quadcopter design introduced.
- The design allows the quadcopter to fly with zero pitch and roll angles.
- The absence of an inclination of the quadrocopter body reduces the characteristic cross-section of the aircraft relative to the incoming air flow. This reduces the aerodynamic drag of the quadcopter. As a consequence, this leads to an increase in flight speed and a decrease in power consumption.
- If we make the body in the form of a wing, so that there is a lift force, then it can be achieved that it will compensate for the force of gravity and the entire thrust of the quadcopter can be directed towards forward movement. Based on this, it is possible to provide the "Cruise" mode for high-efficiency flight of the quadcopter.

REFERENCES

[1] Ryll, M., Bülthoff, H.H. and Giordano, P.R. A novel overactuated quadrotor unmanned aerial vehicle: modeling, control, and experimental validation. *IEEE Transactions on Control Systems Technology* (2015) **23(2)**:540-556 (DOI: 10.1109/TCST.2014.2330999).

[2] https://youtu.be/7FuMzUPm6s4 (the link is available 2021/03/22).

[3] Cutler, M. *Design and control of an autonomous variable-pitch quadrotor helicopter*. Ph.D. dissertation, Citeseer (2012).

[4] Cutler, M. and How, J.P. Actuator constrained trajectory generation and control for variable-pitch quadrotors. *Proceeding of the AIAA Guidance, Navigation, and Control Conference (GNC)*, Minneapolis, MN, USA (2012).

[5] Fresk, E. *Modeling, control and experimentation of a variable pitch quadrotor*. Master's Thesis, Luleå University of Technology, Luleå, Sweden (2013).

[6] Hua, M.-D., Hamel, T., and Samson, C. Control of VTOL vehicles with thrust-direction tilting, *Proc. of the 19th IFAC World Congress* (2014).

[7] Nemati, A., and Kumar, M. Modeling and control of a single axis tilting quadcopter. *2014 American Control Conference*, pp. 3077–3082 (2014).

[8] Riccardi, F., Panizza, P., Lovera, M. Identification of the attitude dynamics for a variable-pitch quadrotor UAV. *Proceeding of the European Rotorcraft Forum (ERF)* 2014, Southampton, UK (2014).

[9] Sheng, S. and Sun, C. Control and optimization of a variable-pitch quadrotor with minimum power consumption. *Energies* (2016) **9(4)**: 232.

[10]   Sabatino, F. *Quadrotor control: modeling, nonlinear control design, and simulation*. Master's degree project. Stockholm, Sweden (2015).

[11]   Bristeau, P.J., Martin, P., Salaun, E. and Petit, N.. The role of propeller aerodynamics in the model of a quadrotor UAV. *Proceedings of the European Control Conference* (2009) 683-688.

[12]   Mellinger, D., Lindsey, Q., Shomin, M. and Kumar, V. Design, modeling, estimation and control for aerial grasping and manipulation. *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)* (2011) 2668 –2673.