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FORCED VIBRATION OF THE ALUMINUM BEAM USING A PIEZOELECTRIC ACTUATOR – EXPERIMENT AND FINITE ELEMENT ANALYSIS

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Abstract. This paper deals with the forced vibration of the aluminum beam using a piezoelectric actuator. Cantilever beam was excited by thin piezoelectric film placed near the fix support. The oscillation of the free end of the beam was measured using a laser displacement sensor. The beam's eigenfrequency and damping ratio for the first bending vibration mode was determined experimentally. The beam's deflection when the beam was excited by a piezoelectric actuator was also determined experimentally. The actuator was controlled by a signal generator and high-performance power supply and linear amplifier module for driving piezoelectric actuators. Data from experimental measurements were used to validate the finite element model of the beam with piezoelectric actuator. Results from experimental measurements and numerical simulations were compared.

1 INTRODUCTION

The phenomenon of piezoelectricity was discovered in 1880 by Pierre and Paul-Jacques Curie. It occurs in several non-centrosymmetric crystals, such as quartz (SiO_2), in which electric dipoles (and hence surface charges) are generated when the crystals undergo mechanical deformations. The same crystals also exhibit the converse effect, that is, they undergo mechanical deformations when subjected to electric fields. Based on this phenomenon, the piezo materials can serve as both sensors as well as actuators, for being capable of converting one form of energy into another reversibly, whereby they are classified as transducers [1].

Commercial piezoelectric materials are now available as ceramics and polymers, which can be manufactured into a variety of convenient shapes and sizes. Lead (Pb) zirconate titanate (PZT), a stiff and brittle variant, is the most widely used piezoceramic today.

Piezopolymers, on the other hand, are very flexible in nature. The most common commercial piezopolymer is the polyvinvylidene fluoride (PVDF). Traditionally, the piezoelectric materials find their use in accelerometers, strain sensors, emitters and receptors of stress waves, vibration sensors, actuators, and pressure transducers. During the last two decades or so, they have been increasingly deployed in turbo-machinery actuators, vibration dampers and for active vibration control of stationary/moving structures. Other areas of use for piezo materials include structural health monitoring (SHM), biomechanics and biomedical engineering and energy harvesting [1].

Piezoelectricity is described mathematically within a material's constitutive equation, which defines how the piezoelectric material's stress (**σ**), strain (**S**), charge-density displacement (**D**), and electric field (**E**) interact [2, 3].

The piezoelectric constitutive law in strain-charge form is [2, 3]:

$$\begin{aligned}\mathbf{S}&=\mathbf{s_E}\boldsymbol{\sigma}+\mathbf{d}^{T}\mathbf{E}\\ \mathbf{D}&=\mathbf{d}\boldsymbol{\sigma}+\boldsymbol{\varepsilon}_{\boldsymbol{\sigma}}\mathbf{E}\end{aligned} \tag{1}$$

The matrix **d** contains the piezoelectric coupling coefficients (superscript ^{T} denotes matrix transposition), matrix $\mathbf{s_E}$ contains compliance coefficients (measured under at least a constant, and preferably a zero, electric field) and matrix $\boldsymbol{\varepsilon}_{\boldsymbol{\sigma}}$ contains coefficients of electric permittivity (measured under at least a constant, and preferably a zero, stress field) [2, 3].

This paper deals with the creation and validation of the finite element model of cantilever beam with a glued thin piezoelectric film actuator. The actuator serves as a source of forced vibration of the beam. Finite element model is compared with the experimental physical model.

2 PHYSICAL MODEL AND EXPERIMENTAL SETUP

Aluminum beam with rectangular cross-sectional area (30×2 mm) with a length of 262 mm was fixed on one end. Thin piezoelectric film actuator was glued to the beam on one side, 20 mm from fixed support. Schematic drawing and practical realization of the supported beam with actuator is shown in Fig. 1.

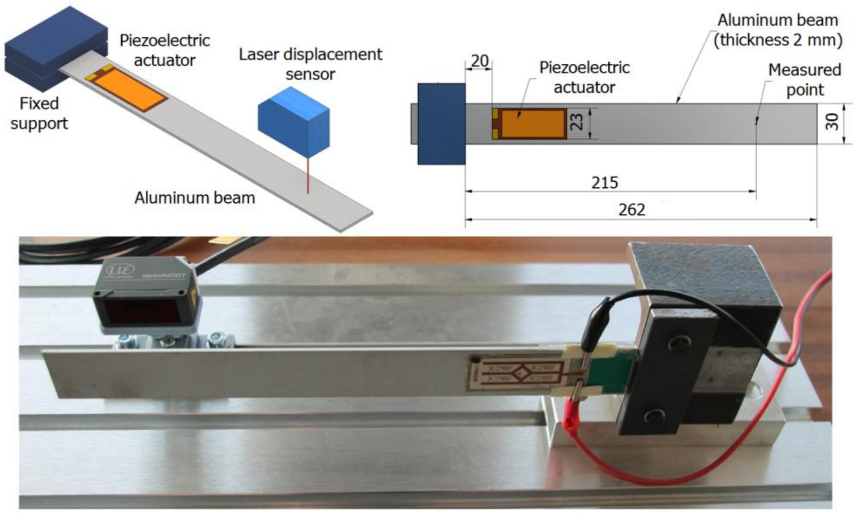

Figure 1: Schematic drawing and practical realization of the supported beam with piezoelectric actuator

Forced vibration of the beam was realized with a piezoelectric actuator powered by a signal generator via a piezo driver with voltage gain of 20. Beam's displacement was measured at a distance of 215 mm from the support using a laser displacement sensor. Measured signal from the sensor was fed to the PC using DAQ device and subsequently processed in software LabVIEW.

The experimental setup scheme is shown in Fig. 2 and the parameters of the individual devices are given in Table 1. The real experimental setup is shown in Fig. 3.

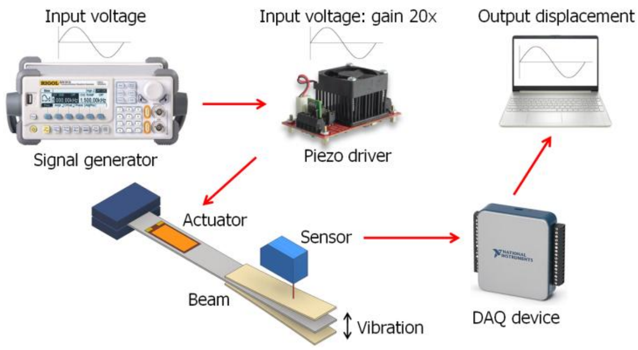

Figure 2: The experimental setup scheme

Table 1: The parameters of the individual devices using in experimental setup

Signal generator	Rigol DG1012
	Function arbitrary waveform generator.
	2 analog channels output, 15 MHz maximum output frequency, 100 MS/s of sample rate, 14 bits of vertical resolution, 4k points of memory depth, max. 10 Vpp
Piezo driver	PDm200B
	High-performance power supply and linear amplifier module for driving piezoelectric actuators.
	power supply voltage +24 V, gain 20 V/V, set to work in bipolar voltage output ±100 V
Piezoelectric actuator	PPA-1001
	Single layer product recommended for energy harvesting and sensing applications. It also exhibits good performance as a resonant actuator.
	capacitance 100 nF, mass 2.8 g, full scale voltage range ±120 V, piezoelectric material PZT-5H
Laser displacement sensor	optoNCDT ILD 1320-25
	Uses the principle of optical triangulation, that is, a visible, modulated point of light is projected onto the target surface.
	power supply voltage +24 V, analog output 4-20 mA, measuring range 25 mm, measuring rate 1 kHz
DAQ device	NI USB-6001
	Simple data logging, portable measurements.
	8 AI (14-Bit, 20 kS/s, ±10 V), 2 AO (5 kS/s/ch), 13 DIO, 32-bit counter

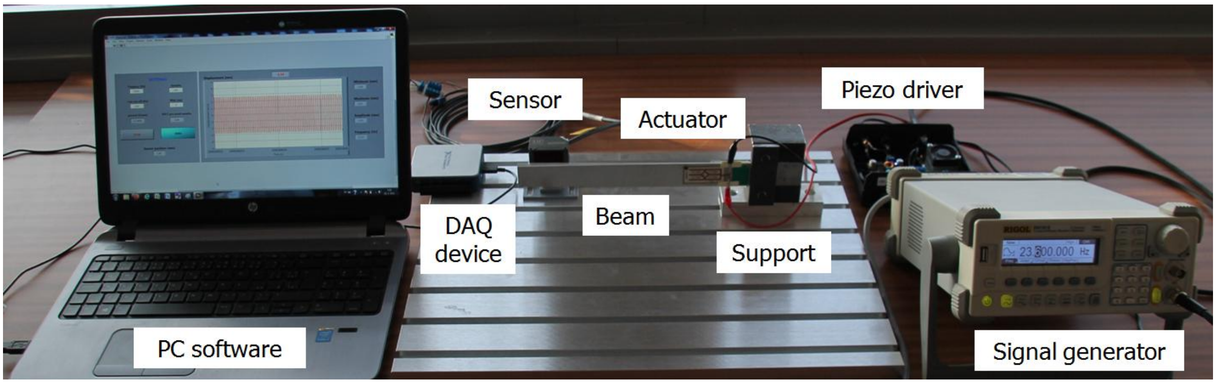

Figure 3: The real experimental setup

3 EXPERIMENTAL MEASUREMENTS

Two types of experimental measurements were performed for the purpose of creating and validating the finite element model.
In the first case, the beam was manually deflected and its deflection during free vibration was measured - free vibration test.
In the second case, the beam's vibrations were caused by the action of the piezoelectric actuator and beam's deflection was measured - forced vibration test.

3.1 Free vibration test

The beam's free end was manually deflected and then released which caused the beam to start vibrating freely. Time course of the beam's displacement at the measuring point was measured (Fig. 4). Piezoelectric actuator was not active.

The eigenfrequency f_0 = 23.6 Hz and damping ratio ζ = 0.6 % of the beam's 1st bending vibration mode were determined from the measured data using FFT analysis and damping envelope.

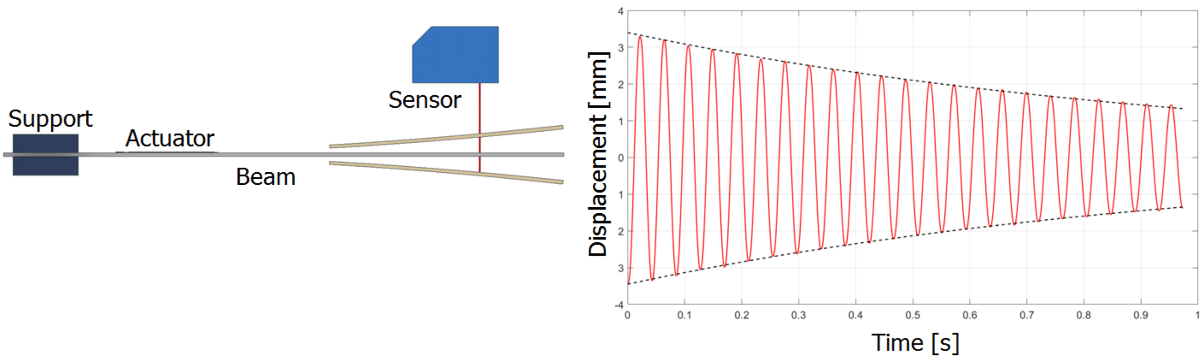

Figure 4: Free vibration test: 1st bending vibration mode
(black dashed line - damping envelope: damping ratio ζ = 0.6 %)

3.2 Forced vibration test

Forced vibration of the beam was realized with a piezoelectric actuator powered by a signal generator via a piezo driver with voltage gain of 20. Sinusoidal input voltage signal with different amplitude level was generated and fed to the piezoelectric actuator, while the beam's output displacement amplitude was measured. Frequency of the input signal was tuned to the beam's eigenfrequency f_0 = 23.6 Hz. Table 2 presents beam's displacement amplitudes A for different amplitudes of the input voltage signal U_{amp}.

Fig. 5 shows the beam's measured displacement amplitude-frequency characteristics in the vicinity of eigenfrequency for two input voltage signals with amplitude U_{amp} = 1 and 1.5 V.

Table 2: Beam's displacement amplitudes A for different amplitudes of the input voltage signal U_{amp}, measured data

Input sinusoidal voltage signal (23.6 Hz)		Output sinusoidal displacement signal (23.6 Hz)
U_{amp} [V]	U_{amp_GAIN} [V]	A [mm]
0.50	10	1.04
0.75	15	1.51
1.00	20	2.04
1.25	25	2.55
1.50	30	3.01
1.75	35	3.41
2.00	40	3.78

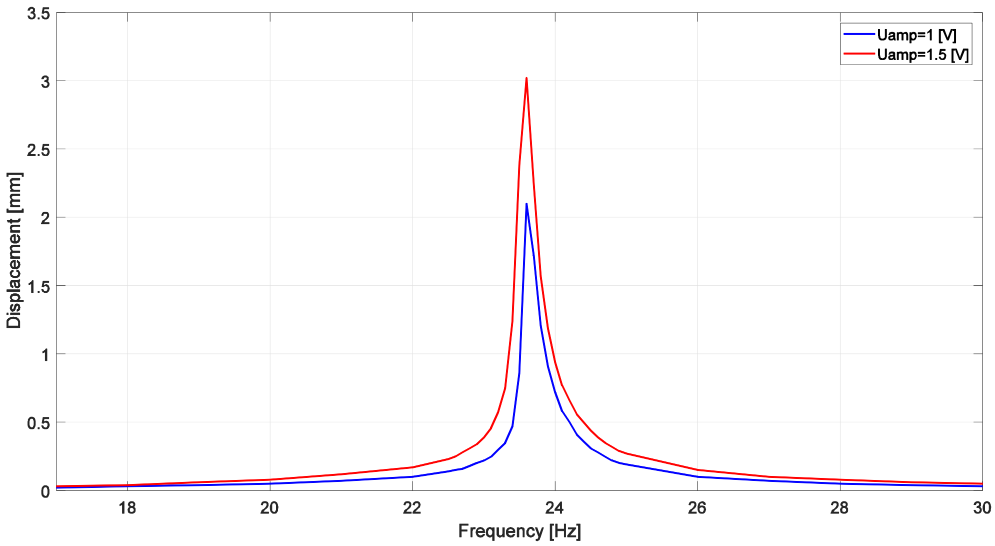

Figure 5: Forced vibration test: displacement amplitude-frequency characteristics

4 FINITE ELEMENT MODEL

Geometries of the cantilever beam, support and piezoelectric actuator were created in Design Modeler of Ansys Workbench according to the dimensions of the real model structures (except for support, which has no effect on the results of finite element analyses).

Piezoelectric actuator was modelled as one thin layer (0.15 mm) of the material with piezoelectric properties according to its datasheet [4]. Other support layers of the actuator were neglected to simplify the model.

Mesh of the finite element model is shown in Fig. 6. Hexahedral elements with uniform size (but with different size for individual solids) were used for the mesh since the model consists of only simple orthogonal solids. For the beam and the piezoelectric actuator 3 layers of elements were used (as shown in detail view in Fig. 6) to better represent their behaviour in simulated deflections. For the piezoelectric actuator, element type SOLID226 with coupled physics capabilities was used, which has piezoelectric coupled-field option enabled via custom Ansys APDL command. For other solids, element type SOLID186 was used.

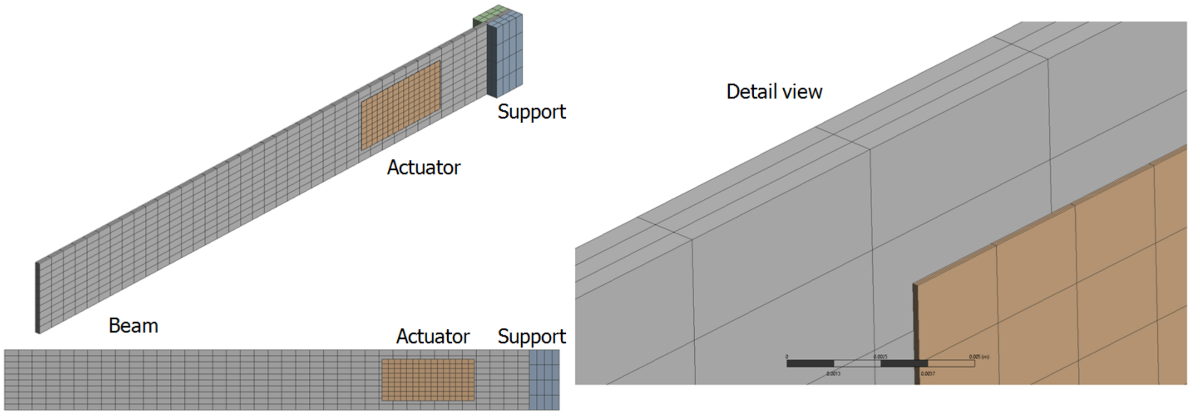

Figure 6: Mesh of the finite element model

5 MATERIAL PROPERTIES

In engineering data of Ansys Workbench material properties were created and subsequently assigned to solids of the finite element model. Structural steel was assigned to support structure and aluminum alloy was assigned to the beam. Material of the piezoelectric actuator is PZT-5H (according to manufacturer's datasheet [4]). Basic mechanical properties for all materials are present in Table 3. Material dependent damping ratio ζ_{alu} = 0.4 % was assigned to aluminum alloy. For PZT-5H dielectric loss factor $\tan \delta_e$ = 2 % has been defined.

Table 3: Material properties of the support, beam and actuator solids

	Young's modulus [GPa]	Density [kg/m^3]	Poisson's ratio [-]
Support (steel)	200	7850	0.3
Beam (aluminum)	66	2750	0.3
Actuator (PZT-5H)	50	7800	0.31

Mechanical and piezoelectric properties of the actuator's material were inserted and assigned using custom APDL commands in form of coefficients, which form matrices **d**, $\mathbf{s_E}$ and $\boldsymbol{\varepsilon_\sigma}$ used in mentioned constitutive equations of piezoelectricity (1). Material properties for piezoelectricity of PZT-5H were taken from engineering database with some constants modified according to data given by piezoelectric actuator's manufacturer [4, 5]:

Compliance matrix:

$$\mathbf{s_E} = \begin{vmatrix} 16.5 & -4.78 & -8.45 & 0 & 0 & 0 \\ -4.78 & 16.5 & -8.45 & 0 & 0 & 0 \\ -8.45 & -8.45 & 20.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 43.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 43.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 42.6 \end{vmatrix} \times 10^{-12} \ [\mathrm{m^2/N}] \tag{2}$$

Piezoelectric coupling:

$$\mathbf{d} = \begin{vmatrix} 0 & 0 & 0 & 0 & 741 & 0 \\ 0 & 0 & 0 & 741 & 0 & 0 \\ -320 & -320 & 650 & 0 & 0 & 0 \end{vmatrix} \times 10^{-12} \ [\mathrm{C/N}] \tag{3}$$

Relative permittivity:

$$\boldsymbol{\varepsilon_\sigma}/\varepsilon_0 = \begin{vmatrix} 3130 & 0 & 0 \\ 0 & 3130 & 0 \\ 0 & 0 & 3800 \end{vmatrix}, \quad \varepsilon_0 = 8.854 \times 10^{-12} \ [\mathrm{F/m}] \tag{4}$$

6 FINITE ELEMENT ANALYSES

To verify results of experimental measurements three types of finite element analyses (FEA) were computed: modal, transient response and harmonic response. Modal analysis verified eigenfrequency of the beam's 1st bending vibration mode. Transient response analysis verified attenuation of beam's free vibration from initial deflection while the actuator was not active. Harmonic response analysis verified beam's maximal deflection when the beam was excited with a sinusoidal voltage signal on the piezoelectric actuator.

6.1 Modal analysis

In the modal analysis, the solver range was set to compute eigenfrequencies of the system in range from 0 to 200 Hz. Fixed support (zero displacement) was assigned to one of the support's solid faces. Voltages on nodes of top and bottom faces of the piezoelectric actuator were coupled to form electrodes and then grounded using custom APDL commands. Computed 1st bending vibration eigenfrequency was 23.64 Hz, which is very close to the measured value 23.6 Hz. Shape of the beam's deflection for 1st vibration mode is shown in Fig. 7.

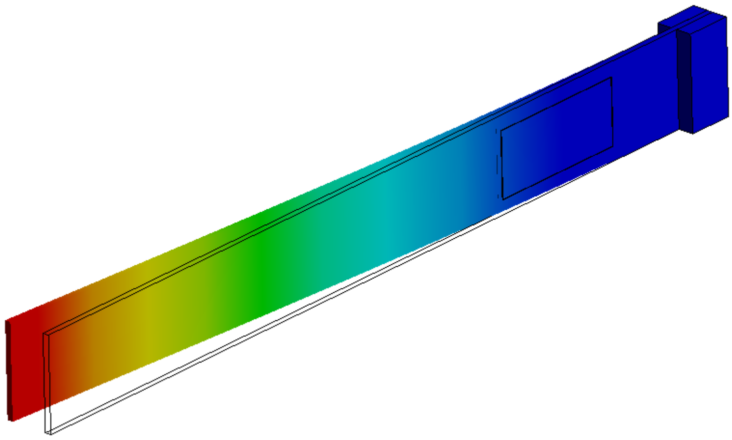

Figure 7: Modal analysis: beam's 1st vibration mode at 23.64 Hz

6.2 Transient analysis

In the transient analysis, the solver was set to simulate one second of beam's free vibration from initial deflection. Deflection was measured in sensor distance of 215 mm from the support. Initial deflection was set to -3.45 mm, same as in the experimental free vibration test. Fixed support was set the same as in modal analysis. Damping ratio in analysis setting was set to 0.6 %. Voltages on nodes of top and bottom faces of the piezoelectric actuator were coupled to form electrodes. Only the bottom electrode was grounded. Comparison between results of the free vibration from experimentally measured and Ansys simulated data is shown in Fig. 8. Data from measured free vibration test are a harmonic exponentially decayed signal with calculated frequency 23.6 Hz and damping ratio 0.6 %, whereas simulated data from Ansys are a harmonic exponentially decayed signal with frequency 23.5 Hz and damping ratio 0.55 %.

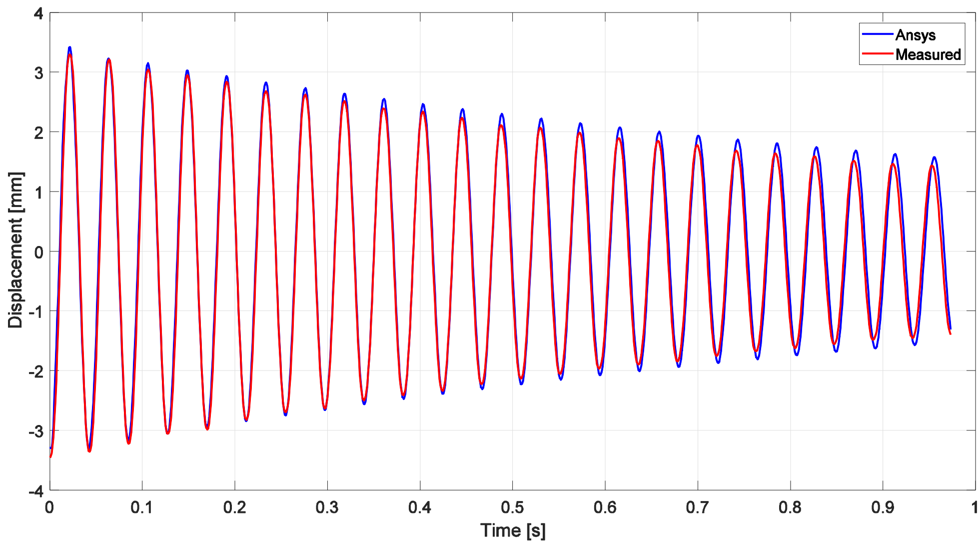

Figure 8: Comparison between free vibration of measured and simulated system

6.3 Harmonic analysis

In the harmonic analysis, the solver range for frequency response was set in range from 17 to 30 Hz to compute deflection of the beam in sensor distance of 215 mm from the support. Fixed support was set the same as in the modal and transient analyses. Damping ratio in analysis setting was set to 0.6 %. Voltages on nodes of top and bottom faces of the piezoelectric actuator were coupled to form electrodes. Sinusoidal voltage on the top electrode was set to the same amplitudes (U_{amp_GAIN}) as in experimental forced vibration test (Table 2) using custom APDL commands. The bottom electrode was grounded. Comparison of the measured and simulated beam's displacement amplitudes A for different amplitudes of the input voltage signal U_{amp} is presented in Table 4.

Table 4: Beam's displacement amplitudes A for different amplitudes of the input voltage signal U_{amp}, measured and simulated data

Input sinusoidal voltage signal (23.6 Hz)		Output sinusoidal displacement signal (23.6 Hz)		
U_{amp} [V]	U_{amp_GAIN} [V]	A [mm]		ΔA [%]
		measured	simulated	
0.50	10	1.04	1.05	0.96
0.75	15	1.51	1.54	1.99
1.00	20	2.04	2.01	-1.47
1.25	25	2.55	2.51	-1.57
1.50	30	3.01	2.98	-1.00
1.75	35	3.41	3.48	2.05
2.00	40	3.78	3.94	4.23

Comparison between results of frequency response from experimentally measured and Ansys simulated beam deflection data in measured point for piezo driver input voltage amplitudes 1 and 1.5 V is shown in Fig. 9.

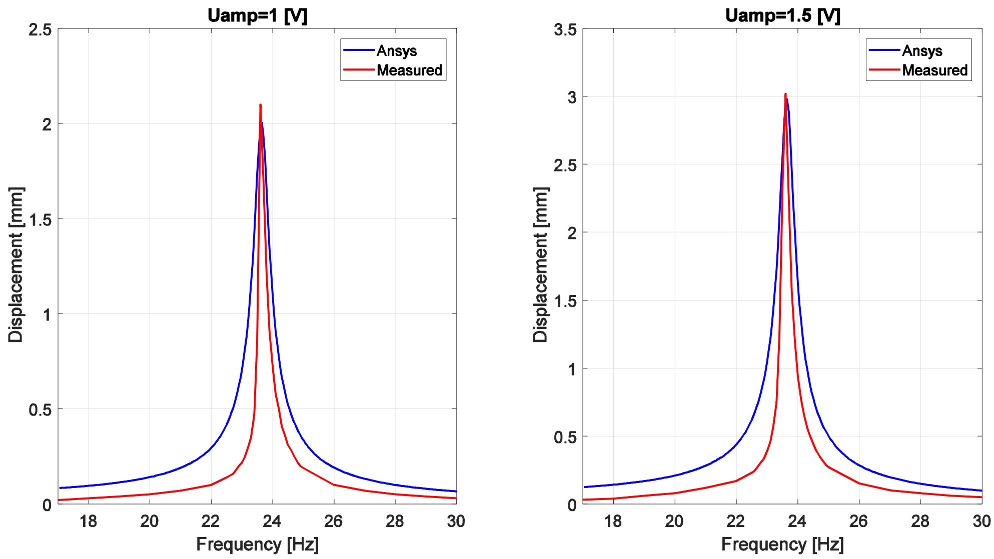

Figure 9: Comparison between frequency response of measured and simulated system

7 CONCLUSIONS

A complex numerical finite element model of the cantilever beam with piezoelectric actuator was created in software Ansys. This model was verified based on the experimental measurements. Results of the real and modeled system were compared using three types of analyses: modal, transient and harmonic. In modal analysis, the eigenfrequency of the beam's 1st bending vibration mode was verified. Damping properties of the system were monitored in free vibration test and transient analysis. Beam's displacement amplitudes vs. driving frequencies of the piezoelectric actuator were evaluated in the harmonic analysis. All analyses show that the finite element model is in very good agreement with the real system. This model will be used in the future for numerical analyses of the actuator as an active damping element and in the study of its energy harvesting capability.

ACKNOWLEDGEMENT

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