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# EULERIAN FORMULATION USING LAGRANGIAN MARKER PARTICLES WITH REFERENCE MAP TECHNIQUE FOR FLUID-STRUCTURE INTERACTION PROBLEM

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**Key words:** Fluid-Structure Interaction, Eulerian Method, Eulerian Unified Formulation, Finite Volume Method, Reference Map Technique

**Abstract.** Full Eulerian methods constitute a family of numerical techniques used to simulate fluid-structure interaction problems. In a full Eulerian method, the velocity gradient tensor is used to compute deformation of solid. However, it is difficult to compute solid stress accurately near the interface, where the velocity between fluid and solid changes drastically. In this work, we propose an Eulerian formulation for fluid-structure interaction problems using Lagrangian marker particles with the Reference Map Technique to compute the deformation of solid accurately near material interfaces without using the gradient of the velocity. We illustrate and validate the proposed method through the presentation of various benchmark problems.

# 1 INTRODUCTION

Full Eulerian methods[1, 2, 3] using a fixed mesh to compute motions of fluids and solids have been developed to solve fluid-structure interaction (FSI) problems. These methods are suitable for high-performance computing and computing large deformation of solids. In these methods, color functions like the volume-of-fluid (VOF) method[4] are used to express spatial distribution and shape of materials. Moreover, advection equations are used to compute not only color functions but also internal variables of solid. However, the numerical dissipation of color functions and internal variables of solid is unavoidable due to solving the advection equations in the Eulerian way[3].

We have introduced Lagrangian marker particles into the full Eulerian FSI formulation proposed by Nishiguchi *et al.*[3] to avoid solving advection equations[5]. In this method, Lagrangian marker particles represent solid regions and have internal variables of solids. We have confirmed our method obtains better accuracy results than results by the conventional Eulerian method. However, it is difficult to compute solid stress accurately near the interface with our method, when the velocity near the interface between fluid and solid changes drastically. The reason is that the velocity gradient tensor is used to compute solid stress in our method.

The Reference Map Technique (RMT)[6] is one of the methods to compute solid stress with a full Eulerian method. In the RMT, the Reference Map which means the spatial distribution of the initial position vector (the position vector of the reference configuration) of solid is used to compute the deformation gradient tensor of solid. Thus the RMT can overcome the problem about the computation of solid stress near the interface due to using a velocity gradient tensor. However, in the original RMT[6], the advection equation is solved to update the Reference Map in the Eulerian manner. In short, the numerical dissipation of the Reference Map is unavoidable.

From these backgrounds, we propose an Eulerian formulation for fluid-structure interaction problems using Lagrangian marker particles with the Reference Map Technique to compute solid stress accurately without using the advection equation of the Reference map. We have confirmed that the proposed method overcomes the problem mentioned above. We explain our proposed method in this paper, but we are going to show the results of benchmark tests in the presentation.

# 2 BASIC EQUATIONS

## 2.1 Mixture equations

In this research, the incompressible mixture equations[1, 2, 3] shown below are used to compute the motion of incompressible fluids and incompressible solids.

$$\nabla \cdot \boldsymbol{v}_{\mathrm{mix}} = 0 \tag{1}$$

$$\frac{\partial \rho_{\mathrm{mix}} \boldsymbol{v}_{\mathrm{mix}}}{\partial t} + \nabla \cdot (\rho_{\mathrm{mix}} \boldsymbol{v}_{\mathrm{mix}} \otimes \boldsymbol{v}_{\mathrm{mix}}) = \nabla \cdot \boldsymbol{\sigma}_{\mathrm{mix}} + \rho_{\mathrm{mix}} \boldsymbol{b} \tag{2}$$

Equation (1) is the mixture equation of continuity and equation (2) is the mixture equation of motion. In these equations, $\boldsymbol{v}_{\mathrm{mix}}$ means the mixture velocity, $\boldsymbol{\rho}_{\mathrm{mix}}$ means the mixture density, $\boldsymbol{\sigma}_{\mathrm{mix}}$ means the mixture stress, and $\boldsymbol{b}$ is the body force. The single velocity field and pressure field are computed with these equations. In short, the velocity and pressure of each material are not obtained.

### 2.2 Constitutive equations

In this research, in order to compare the results of the benchmark problems by Zhao *et al.*[7] and by Nishiguchi *et al.*[3] with the result by the proposed method we use the same constitutive equations in their papers.

The constitutive equation of the fluid is an incompressible Newtonian fluid,

$$\boldsymbol{\sigma} = 2\mu\boldsymbol{D} - p\boldsymbol{I}, \tag{3}$$

where, $\mu$ is the viscosity, $\boldsymbol{D}$ is the deformation rate tensor, $p$ is the pressure, and $\boldsymbol{I}$ is the second-order unit tensor.

The constitutive equation of the solid is an incompressible neo-Hookean solid with viscosity,

$$\boldsymbol{\sigma} = G\left(\boldsymbol{B} - \boldsymbol{I}\right) + 2\mu\boldsymbol{D} - p\boldsymbol{I}. \tag{4}$$

Here, $G$ is the shear modulus, $\boldsymbol{B}$ is the left Cauchy-Green deformation tensor, $\boldsymbol{I}$ is the second-order unit tensor $\mu$ is the viscosity, $\boldsymbol{D}$ is the deformation rate tensor, and $p$ is the pressure. The left Cauchy-Green deformation tensor $\boldsymbol{B}$ is given by

$$\boldsymbol{B} = \boldsymbol{F} \cdot \boldsymbol{F}^T, \tag{5}$$

where, $\boldsymbol{F}$ is the deformation gradient tensor. The deformation gradient tensor is defined as

$$\boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}. \tag{6}$$

Here, $\boldsymbol{x}$ is the position vector of the current configuration and $\boldsymbol{X}$ is the position vector of the reference configuration.

## 3 NUMERICAL METHOD

Following our previous method[5], the governing equations and spatial differentials are solved on an Eulerian mesh and the physical quantities of solid are calculated and carried on Lagrangian marker particles in our proposed method in this paper.

The difference between the method in our previous paper and the proposed method is in the evaluation method of the left Cauchy green deformation tensor. In our previous method[5], the time evolution equation of the left Cauchy–Green deformation tensor is calculated on Lagrangian marker particles using the velocity gradient tensor calculated on the Eulerian mesh. However, in the proposed method, the deformation gradient tensor is

obtained on a Euler grid using the Reference Map Technique[6], and then the left Cauchy-Green deformation tensor is computed. In the proposed method, instead of solving the advection equation of the Reference Map, Lagrangian marker particles carry the Reference Map to prevent the numerical dissipation of the Reference Map.

Based on our previous method[3, 5], the finite volume method and the fractional step method[8] was used to solve the governing equations (1,2) on the Eulerian mesh. The mixture velocity and pressure are computed at cell centers of the Eulerian mesh by a collocated variable arrangement, and density and volume fraction are also defined at the cell centers. The deviation stress of an incompressible neo-Hooke solid is defined at cell faces of the Eulerian mesh, while the deviation stress of the viscosity of a fluid and a solid is defined at the cell centers. In addition, the Reference Map $\boldsymbol{X}$ and the deformation gradient tensor $\boldsymbol{F}$ are defined at the cell centers.

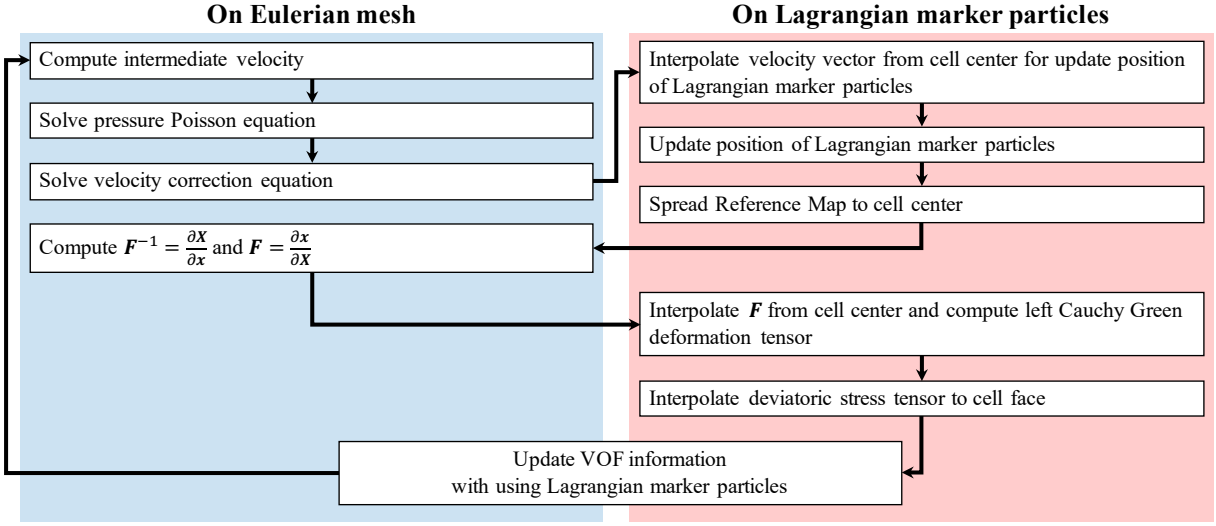


**Figure 1**: Computational flowchart of the proposed method

### 3.1 Computation of intermediate velocity

As described above, to solve the governing equation with the fractional step method, the intermediate velocity $\boldsymbol{v}^*_{\mathrm{mix}}$ is calculated based on the following equation.

$$\rho^n_{\mathrm{mix}} \frac{\boldsymbol{v}^*_{\mathrm{mix}} - \boldsymbol{v}^n_{\mathrm{mix}}}{\Delta t} + \nabla \cdot (\rho_{\mathrm{mix}} \boldsymbol{v}_{\mathrm{mix}} \otimes \boldsymbol{v}_{\mathrm{mix}}) = \nabla \cdot \boldsymbol{\sigma}'_{\mathrm{mix}} + \rho_{\mathrm{mix}} \boldsymbol{b} \tag{7}$$

In this study, the second-order Adams-Bashforth method is used for the time integration scheme of the advection term and stress term and the second-order central difference scheme is used for the spatial discretization scheme of the advection term.

### 3.2 Computation of pressure Poisson equation

Next, the pressure $p^{n+1}$ at the next time step $(n+1)$ is obtained by solving the following pressure Poisson equation using the intermediate velocity $\boldsymbol{v}^*_{\mathrm{mix}}$.

$$\frac{\nabla \cdot \boldsymbol{v}^*_{\mathrm{mix}}}{\Delta t} = \nabla \cdot \left( \frac{1}{\rho^n_{\mathrm{mix}}} \nabla p^{n+1} \right) \tag{8}$$

Equation (8) is discretized with the second-order central difference scheme. The discretized equation is solved using the successive overrelaxation method color-coded by red and black ordering[9]. It is known that a collocated variable arrangement cannot avoid checkerboard pressure instability occurs because a pressure difference between adjacent cells cannot be evaluated. Thus, in this study, the Rhie-Chow method[10] is used to refrain from the pressure instability.

### 3.3 Computation of velocity correction equation

Finally, the velocity $\boldsymbol{v}^{n+1}_{\mathrm{mix}}$ at the next time step $(n+1)$ is obtained by solving the velocity correction equation which is given by

$$\boldsymbol{v}^{n+1}_{\mathrm{mix}} = \boldsymbol{v}^*_{\mathrm{mix}} - \frac{\Delta t}{\rho^n_{\mathrm{mix}}} \nabla p^{n+1}. \tag{9}$$

The pressure gradient $\nabla p^{n+1}$ in equation (9) is discretized with the second-order central difference scheme.

### 3.4 Interpolation of velocity vector to Lagrangian marker particles

The velocity of Lagrangian marker particles $\boldsymbol{v}_{\mathrm{Lag}}$ which is necessary for calculating their motion is interpolated from the cell centers.

### 3.5 Update position of Lagrangian marker particles

The position of each Lagrangian marker particle in the next step $\boldsymbol{x}^{n+1}_{\mathrm{Lag}}$ is discretized with the second-order Adams-Bashforth method and given by

$$\boldsymbol{x}^{n+1}_{\mathrm{Lag}} = \boldsymbol{x}^n_{\mathrm{Lag}} + \Delta t \left( \frac{3}{2} \boldsymbol{v}^n_{\mathrm{Lag}} - \frac{1}{2} \boldsymbol{v}^{n-1}_{\mathrm{Lag}} \right). \tag{10}$$

### 3.6 Interpolation Reference Map to Eulerian mesh

The Reference Map is interpolated from the Lagrangian marker particles to the cell centers. Since the Reference Map is the position vector of the reference configuration of solid, the position vector at the initial computational step is uniquely assigned to each Lagrangian marker particle.

### 3.7 Computation of deformation gradient tensor

Using the Reference Map value $\boldsymbol{X}$ interpolated to the cell centers, the inverse tensor of the deformation $\boldsymbol{F}^{-1}$ gradient tensor is obtained as follows.

$$\boldsymbol{F}^{-1} = \frac{\partial \boldsymbol{X}}{\partial \boldsymbol{x}} \tag{11}$$

In our method, the inverse of the deformation gradient tensor is defined at the cell centers and calculated using the second-order central difference. Finally, the deformation gradient tensor at the cell centers $\boldsymbol{F}$ is obtained by calculating the inverse of $\boldsymbol{F}^{-1}$ obtained by the above equation (11).

### 3.8 Interpolation of deformation gradient tensor

The deformation gradient tensor $\boldsymbol{F}$ computed on the cell centers is interpolated to the Lagrangian marker particles.

### 3.9 Computation of solid deviation stress and interpolation to the cell face

Using the deformation gradient tensor $\boldsymbol{F}$ interpolated to the Lagrangian marker particles, the left Cauchy green deformation tensor is obtained on each Lagrangian marker particle from equation (5). After that, the solid deviation stress of the first term on the right hand side of equation (4) is calculated on the Lagrangian marker particles. Finally, the deviation stress on Lagrangian marker particles are interpolated to the cell centers.

### 3.10 Computation of volume fraction of solids

At the end of the computation in one time step, the volume fraction of solids in each cell is obtained by using positional relationship between Lagrangian marker particles and each cell.

## 4 NUMERICAL EXAMPLES

To verify the proposed method, we conducted the numerical benchmark tests by Zhao *et al.*[7] and by Nishiguchi *et al.*[3] The results of them are going to be shown in the presentation.

## 5 CONCLUSIONS

In this paper, we proposed the Eulerian formulation method for FSI problems using the Reference Map Technique and Lagrange marker particles to solve the problem that stress oscillation occurs near an interface of materials. In order to confirm the effectiveness and validity of the proposed method, the numerical benchmark tests[7] have been conducted.

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