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MPM DEVELOPMENTS IN SOIL-WATER-STRUCTURE INTERACTION IMPLEMENTED IN ANURA3D

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Abstract. The material point method (MPM) showed to be well suited to study geotechnical applications involving large deformations, non-linear material behaviour, soil-structure interactions, and multiphase (solid, liquid, gas) interactions. This contribution shows the latest numerical developments implemented in the software Anura3D. Particular attention is given to some geotechnical applications, such as the simulation of slope collapse due to earthquake or water pressure changes, erosion problems, installation problems, underground explosions, and soil-structure interaction in liquefied soils.

1 INTRODUCTION

Anura3D MPM Research Community is a group of people belonging to different institutions that share the common goal of advancing the state of the art of numerical simulation of soil-water-structure interaction with the material point method (MPM). Involved institutions are Deltares (NL), TU Hamburg-Harburg (DE), University of Cambridge (GB), Università Politecnica de Catalunia (ES), University of Padua (IT), Virginia Tech (USA), University of Berkeley (USA), Politecnico of Milan (IT), and University of Salerno (IT). The community encourages networking by organizing regular meetings of developers, workshops, and conferences.

The main interest of the community is the study of soil-water-structure interaction with particular emphasis on geomechanical problems. Typical applications are slope instability (landslides, earth embankment, and levee failures), soil excavation, underground explosions, soil-penetration problems. New developments are discussed and shared within the community, and the most interesting advancements are released in the open-source software Anura3D that can be downloaded at www.anura3d.com. Anybody can download and use the software and implement new features that can be incorporated in the forthcoming releases upon request by contacting the core developer team.

Anura3D can solve 2D and 3D problems by applying multiphase formulations for saturated and unsaturated soils. Moreover, useful features such as contact algorithm, moving mesh, rigid body motion, and so on are available. It is a research-oriented software; thus many features are well tested, but others are still under development.

The purpose of this contribution is to present briefly some of the most recent advances of the Anura3D MPM Research Community that are available in the code or will be released soon.

2 RECENT DEVELOPMENTS

2.1 Unsaturated soils

Unsaturated soils in Anura3D can be simulated with a full three-phase formulation presented [1] or with the simplified two-phase formulation recently proposed in [2]. The first one solves the momentum and mass balance equations of the three phases separately (solid grains, liquid, and gas). All inertial terms are accounted, and the principal unknowns are the absolute phase accelerations. Secondary variables are solid stress and fluid pressure. In the open-source code, mass exchange between the phases is not allowed. The second formulation introduces the simplifying assumptions that gas pressure is zero and gas density is negligible, thus removing the momentum and mass balance of the gas from the governing equations. This two-phase formulation is computationally less expensive, and it is well applicable to most geotechnical applications.

Recent advances in the simulation of unsaturated soil problems include the implementation of new hydraulic boundary conditions (HBC) and more advanced constitutive models. Newly developed HBC are:

- Total head: it is very useful to simulate, for example, water levels in reservoirs. A total head value is prescribed on the boundary, which is related to the applied pressure by means of Bernoulli's equation.
- Infiltration/evaporation: it is necessary to simulate rainfall or evaporation by means of a prescribed infiltration rate ($\widetilde{\boldsymbol{w}}$) at the boundary. It is applied with a predictor-corrector scheme: liquid and solid velocities ($\boldsymbol{v}_L, \boldsymbol{v}_S$) are predicted assuming zero pressure at the infiltration boundary and then (eventually) corrected to ensure the

prescribed infiltration rate. If the net infiltration discharge is positive ($q_{net} = \boldsymbol{n} \cdot (\boldsymbol{n_L}(\boldsymbol{v_L} - \boldsymbol{v_S}) - \widetilde{\boldsymbol{w}}) > 0$, $\boldsymbol{n}$ outward normal unit vector) ponding conditions occur, and if fluid accumulation above the boundary is not allowed (it must remain at zero pressure), no correction is necessary, meaning that the maximum soil infiltration capacity is met. If the net infiltration discharge is negative, then the liquid velocity must be corrected to ensure the correct infiltration rate. A flow chart of the algorithm is presented in Fig. 1

- Potential seepage face: this is the interface between soil and atmosphere where the fluid is free to exit at zero pressure when the soil is saturated. It cannot enter when the soil is partially saturated. This condition is solved as a particular case of the previous one, with zero infiltration rate.

It should be mentioned that in [2] only liquid velocity was corrected at the infiltration/seepage boundary, but later the correction of both solid and liquid velocity as proposed in [3] showed to be more correct [4]. These new HBC will be available in the next release of Anura3D.

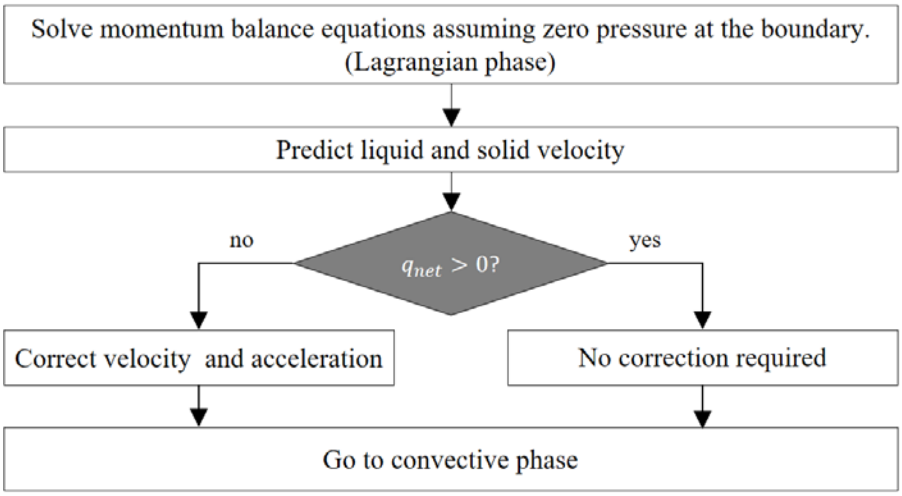

Figure 1 Flow chart of the infiltration BC or seepage BC.

This new set of boundary conditions is applied to the stability of river levees considering (i) rapid river level variation, and (ii) enduring high water level with heavy rainfall [5].

Constitutive models able to deal with saturated soils are implemented. A suction-dependent Mohr-Coulomb model was used in [1,5] to include the increase of cohesion and friction angle with suction. The model was first applied in a wetting-induced slope instability inspired by a real case of road embankment collapse after heavy rainfall [1]. Girardi et al. [5] used it to simulate a levee instability due to rapid increase and decrease of water level. The results are in relatively good agreement with the experimental results presented in [6].

Common features of soils under unsaturated conditions such as suction-dependent stiffness and wetting collapse cannot be reproduced by a simple Mohr-Coulomb model. To overcome such limitation, the extension of Clay and Sand Model [7] to unsaturated conditions presented by [8] has been implemented and validated in Anura3D. To generalize the model, the user can select Bishop's or net stress as constitutive stress for convenience.

2.2 Shearing-induced thermal pressurization

The weakening processes induced by thermal pressurization, which may have a relevant role in landslide acceleration, have been implemented in Anura3D and will be soon available in the open source version. The governing formulations and their implementation into MPM framework are described in [9,10]. The thermo-hydro-mechanical coupling requires the integration of the energy balance equation at the material point level. To avoid the pathological dependence of heat generation by the frictional work dissipated in shear bands, the embedded shear band procedure [11] has been implemented.

A reference case of a slope is shown in Fig. 2. The soil is defined with a perfectly plastic Morh-Coulomb model. Once triggered, the excess pore water pressure generated (Fig. 2b) leads to the drop of the frictional strength that induces the acceleration of the motion. The effect of thermal pressurization can be observed when comparing the calculated run-out when thermal effects are desactivated (Fig. 2a). In this case, an embedded shear band thickness of 4 cm has been considered.

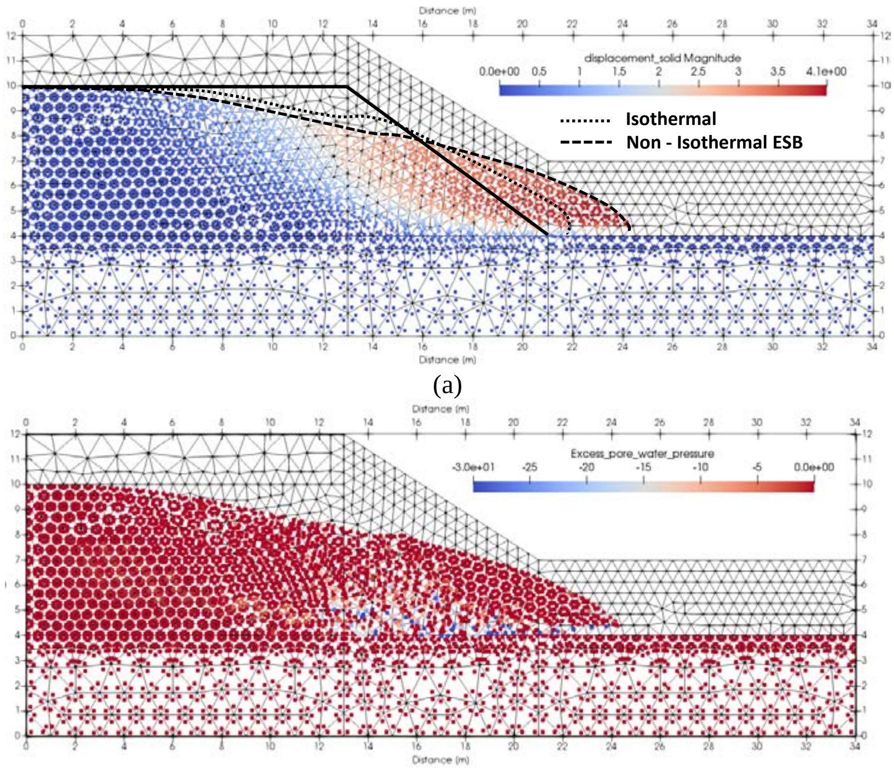

Figure 2 Landslide for a reference slope. (a) Initial and final geometry for the isothermal and non-isothermal cases: (b) Thermal-induced excess pore water pressure.

2.3 Coseismic landslides

The study of earthquake-triggered landslides has been the focus of recent works in the field of MPM [12–17]. The large deformation of the material involved in such events causes immense damage and casualties around the world. When modeling a coseismic event, attention needs to be paid to the ability of the model to (at least) three different aspects: (i) apply the input ground motion at the base of the model, (ii) reproduce the amplification/de-amplification of the seismic waves when traveling through the near-surface soil layers, and (iii) capture failure and post-failure behavior of the mobilized mass in the landslide.

Alsardi and Yerro [12] concentrate on the first aspect and propose to prescribe the seismic motion in terms of the velocity-time history at the boundary nodes combined with a moving computational mesh that displaces according to the input motion. In this manner, the material-point cell-crossing tends to be concentrated only in those areas where the material moves with respect to the seismic action. Note that the whole material domain shakes back and forth if the conventional fixed mesh is used instead, and the cell-crossing increases considerably. A slope instability problem based on a shaking table experiment is modeled, and the MPM results are compared to FEM, FDM, and SPH simulations.

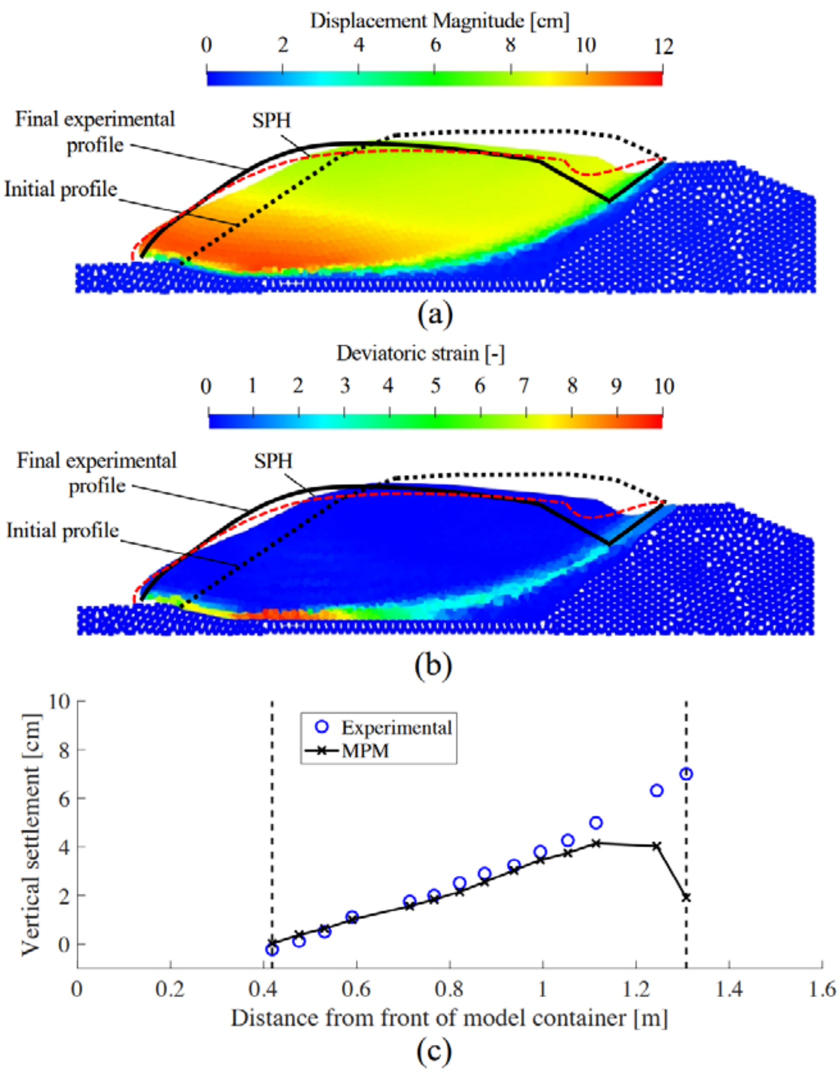

Figure 3 Comparison between experimental and numerical results of the final profile of the slope; (a) displacement, (b) deviatoric strain, (c) final vertical displacement of the ground surface (t = 30 s) (from [13]).

Alsardi et al. [13] validate the previous implementation by reproducing a well-instrumented shaking table test of a slope failure made of a synthetic clay mixture based on [18]. The failure and post-failure responses are relatively well captured by using a Tresca constitutive model with strain softening (Fig. 3). In addition, a parametric analysis performed with MPM, mesh-based, and Newmark-type methods on a theoretical 'rigid' slope highlights the limited capabilities of the mesh-based methods (FEM and FDM) to capture the large strains induced by large intensity ground motions. A good match is obtained between MPM results and selected state-of-the-art simplified Newmark-type methods.

2.4 Interaction between free-water and porous media

Liang et al. [36] apply a two-point two-phase formulation to investigate the dike stability problem under the action of overtopping flows (Fig. 4). This study also allowed for a parametric analysis of commonly used defence mechanisms for dikes. The results obtained using Anura3D were compared against experimental results and it was shown that the deformation of the soil structure was in agreement with the experimental data.

There same formulation have also beenused to detailing the effect that permeable boundaries can have on wave run up [37]. A piston wave generator, which allows for waves of specific heights to be created was modelled. With respect to waves, porous media helps with the dissipation of energy, and thus has potential for usage as sea defences. The results showed that as the permeability increased (from increasing the grain size) the height reached by the solitary wave was reduced [37].

These recent studies contribute to demonstrate the potential of the two-phase two-point MPM to model the interaction between the solid and liquid phases of a model.

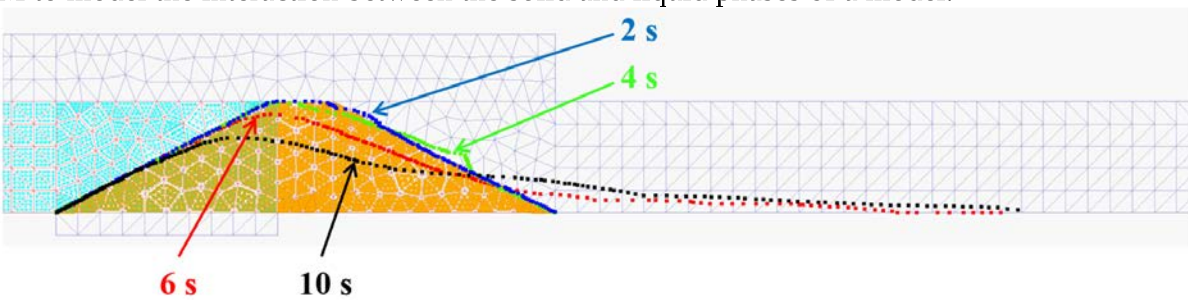

Figure 4 Evolution of dike profile during erosion due to overtopping.

2.5 Internal erosion problems

In the geotechnical field, "internal erosion" is used as a generic term to describe the erosion of soil particles by water passing through a porous soil matrix. This mechanism is the leading cause of failure of water retaining structures such as dikes and dams. The same phenomenon controls the amount of sand production in oil-producing wells. The modeling of the initiation, evolution, and consequences of internal erosion is highly challenging.

Two different MPM formulations are proposed to study the internal erosion mechanism in bimodal soils and they will be included in future releases of the open source version of Anura3D. In bimodal soils, the solid phase is formed by a mix of coarse grains and fine grains. When the system is subjected to a certain hydraulic pressure, the fine fraction is susceptible to erosion. The first approach proposed by [19], uses the single-point formulation (i.e., one set of material points is used to represent the saturated soil), while the second approach proposed by [20] uses the double-point formulation (i.e., two sets of material points represent solid and liquid independently).

Fig. 5 shows the evolution of the eroded mass carried by the liquid through the liquid material points. In both formulations, the internal erosion mechanism is incorporated by the addition of the mass transfer between the solid and liquid phases. To ensure mass conservation of the system, the mass balances of the solid phase, water, and eroded grains are posed at the material points. Additionally, an erosion law, traditionally based on empirical relationships, controls the rate of mass transfer.

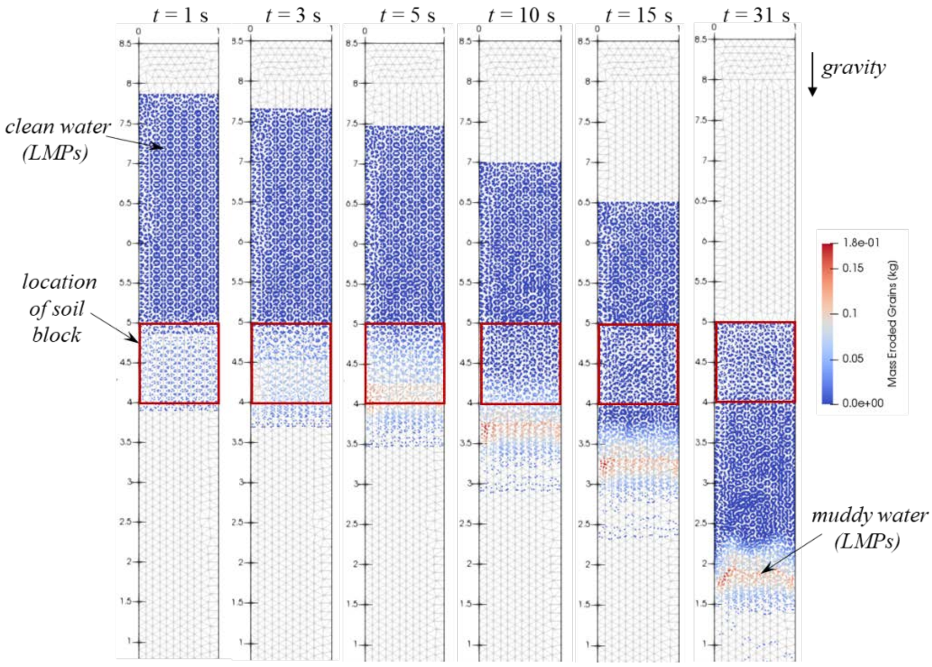

Figure 5 Evolution of eroded mass through a soil block using the double-point MPM approach for the simulation of internal erosion of bimodal soils (from [20]).

2.6 Soil penetration problems

The explicit 3D MPM formulation was successfully used to study soil penetration problems such as anchor pullout [21], pile jacking [22], impact-driven piles [23], vibratory-driven piles [24], and cone penetration tests in fine-grained soils both for drained and undrained conditions [25] and also considering partially drained conditions [26].

More recently, Zambrano and Yerro [27] simulated a free-fall penetrometer (FFP). A new algorithm for the simulation of the rigid body has been implemented, which enhances the performance of the computation and reduces its computational cost. Indeed, the explicit time integration scheme implemented in Anura3D is conditionally stable, and the critical time step size decreases with the increase of material elastic modulus; as a result, the simulation of stiff bodies results in very small time steps and long computational time. By introducing the rigid body algorithm, the stability criterion only depends on the bulk modulus of the soil, and the calculation time is minimized. Numerical results of FFP are validated with experimental results obtained in the calibration chamber.

Martinelli and Galavi [28] focused on CPT penetration in sandy soils, and Yost et al. [29] simulated CPT in layered soils using the 2D axisymmetric formulation [30]. Due to much limited computational time compared to a full-3D formulation.

Martinelli et al. [28] investigated the effect of the different numerical parameters (e.g., computational mesh size, element type, etc.) and different constitutive models on the accuracy and the stability of the MPM simulations. A well-documented chamber test on sandy soil was selected as a benchmark, where not only the evolution of cone resistance with penetration is recorded, but also the displacement field around the penetrometer is measured.

Fig. 6 shows that the MPM results are in good agreement with the chamber test data. In conclusion, the study showed that MPM provides accurate results not only in terms of cone resistance but also it can give a good assessment of the soil displacements around the penetrometer.

Yost et al. [29] compared the numerical results of CPT in layered soils with experimental tests in calibration chamber and show that MPM is capable of accurately simulating the tip resistance in soil profiles with multiple layers as thin as 20 mm.

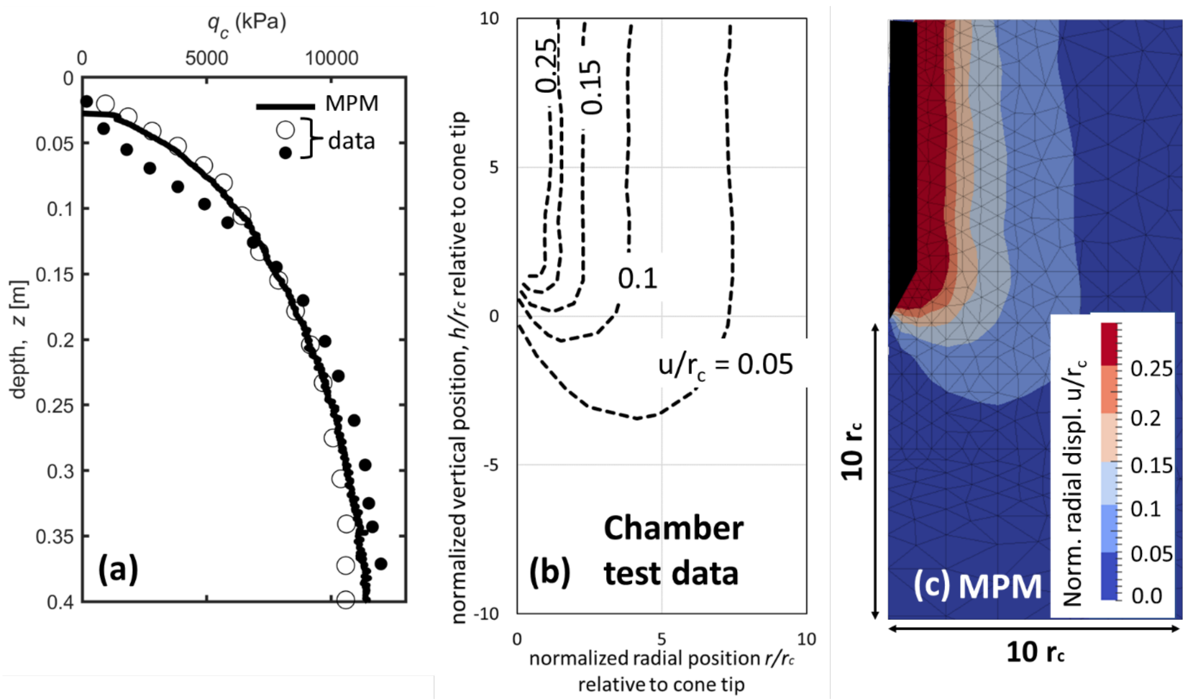

Figure 6 cone penetration in sand [28] (a) cone resistance; (b) normalized radial displacement in chamber test and (c) in MPM. Cone radius is r_c.

2.7 Underground explosions

The MPM method is suitable for simulating the large deformations that occur during underground explosions [31]. Here, the explosive can be described by means of the Friedlander equation as a time-dependent pressure boundary condition:

$$p(t) = p_0 + p_{max}\left(1 - \frac{t}{t_d}\right)^{-\frac{t}{t_d}}$$

We use the following values for the Friedlander equation, $p_0 = 0, p_{max} = 70\text{kPa}, t_d = 100\text{ms}$, which results in the curve shown in Fig. 7.
We consider the experiment from [32] to demonstrate the application of MPM. In this experiment, a barrel with a height of 85 cm and a diameter of 63 cm was used. The barrel was filled with silica sand and an explosive charge was placed at a depth of 5 cm. A steel plate was placed 20 cm above the barrel, which was implemented as a fixed boundary condition in the simulation. Fig. 8 shows the simulation results in MPM, which are in good agreement with the high-speed camera images from [32].

In geotechnical applications, simulations of this kind are of particular interest, for example when assessing the dangers of unexploded ordnance from the Second World War. The unexploded bombs lying underground can detonate independently or through external influences. The simulations thereby provide an assessment of the potential damage that can occur to buildings, tunnels, bridges, and other structures.

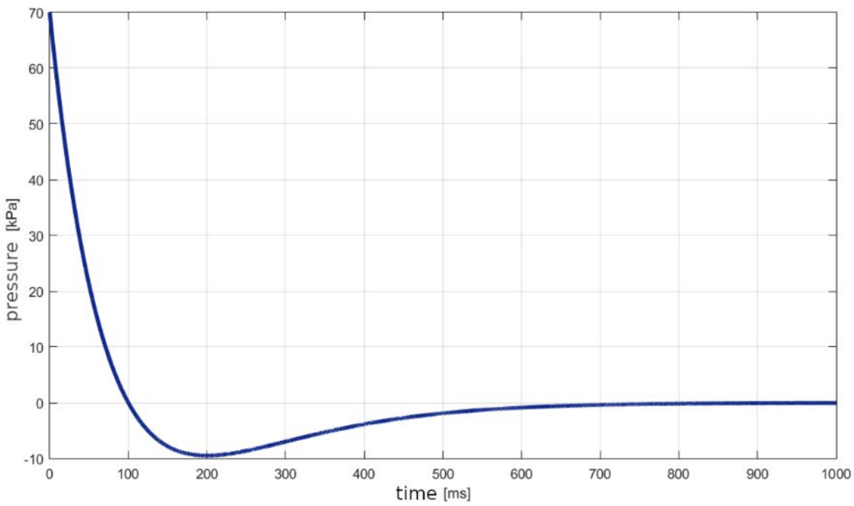

Figure 7 The resulting curve of the Friedlander equation.

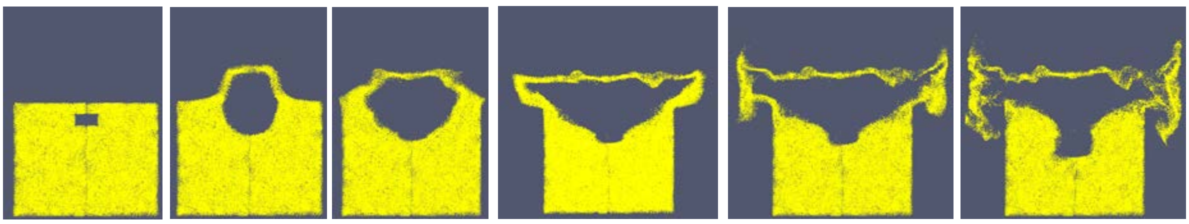

Figure 8 The simulation results at different instants: *t*=0s, *t*=2e-4s, *t*=5e-4s, *t*=7e-4s, *t*=1e-3s, *t*=2e-3s and *t*=5e-4s

2.8 Soil-structure interaction in liquefied soils

In many geotechnical applications, the material may experience a phase transition (solid- to fluid-like and viceversa). Phase transition is expected to occur when the material is subject to very low effective confining pressure or to very large strain rates. To properly reproduce the material response, the use of (i) a constitutive modelling approach capable of reproducing phase transition and (ii) large displacement-based approaches is mandatory. To analyze the uplift of a buried pipeline in "quasi-liquefied" soil, a single-point model developed in Anura3D was employed [33]. During uplift, some parts of the domain (below the pipeline) experience liquefaction whereas others (above the pipeline) re-solidification. To properly reproduce the material behavior, both soil and pore water are accounted for. The simplest constitutive relationship capable of reproducing soil phase transitions is an elastic-viscoplastic (Perzyna type) with a Mohr Coulomb yield surface and a bilinear viscous nucleus [34]. Despite the simplicity of the constitutive law, the numerical model can qualitatively reproduce the experimental test results from [35] (Fig. 9).

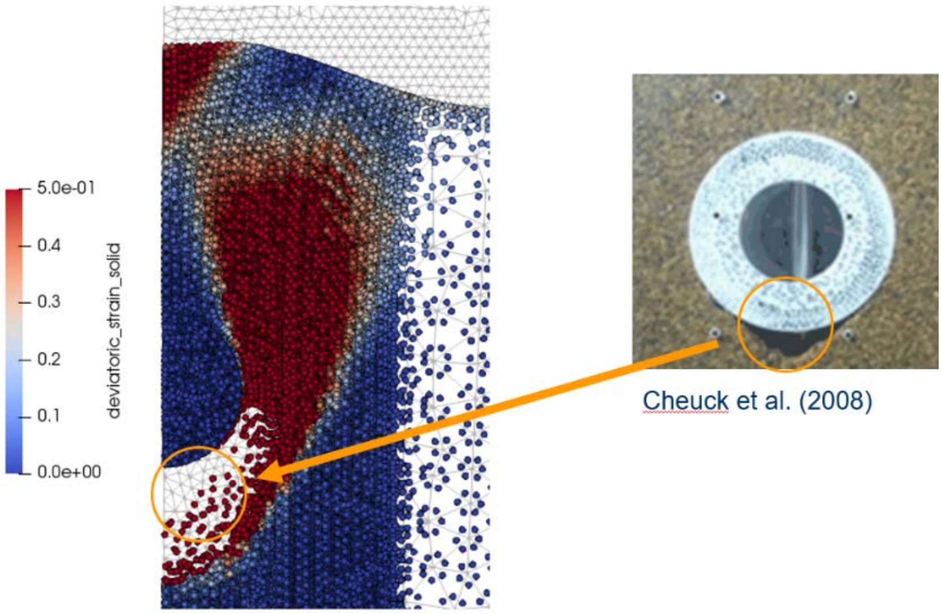

Figure 9 Local soil detachment beneath the pipe: MPM simulations and experimental tests from [35].

3 FUTURE DEVELOPMENTS

In the near future, the Anura3D MPM research community will work on the improvement of the accuracy and stability of space and time integration scheme, in particular B-spline shape function, and moving least square approximation will be considered. Implicit time integration schemes for one-phase single-point formulation will be improved and linked to the explicit scheme.

Further development of the boundary conditions is needed to perform seismic response analysis and to reproduce large-scale real-field conditions during earthquake events accurately. For this reason, the implementation of boundary conditions capable of avoiding the reflection of outward waves such as tied, viscous, or free-field is being investigated. More advanced constitutive models specific to earthquake engineering applications are also essential to capture the soil response during seismic loading conditions.

In the field of internal erosion, the validation and comparison of both formulations, as well as the application of advanced in/out flow boundary conditions, will soon be available in the open source software [30,31]. Additionally, further developments include the simulation of large-scale problems and the implementation of constitutive soil laws that account for the soil degradation resulting from erosion processes. New advanced constitutive models will be developed to account for soil fluidization and mechanical response at a high-strain rate.

In the field of slope stability and levee response, the effect of material heterogeneity will be included.

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