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MODELLING LANDSLIDE DEBRIS FLOW WITH ENTRAINMENT: DEVELOPMENT AND VALIDATION

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Abstract. *The volume and mobility of a debris flow could increase with distance travelled as it has the potential to entrain a substantial amount of channel-bed material along its travel path. This entrainment effect renders the debris flow more devastating to downslope populations and facilities. Over the past two decades, the Geotechnical Engineering Office (GEO) of Hong Kong has expended considerable effort to develop debris mobility modelling tools for use in routine engineering practice for forward prediction purposes. Recently, GEO has completed a study to enhance an in-house debris mobility code. Physical parameters which can be estimated from the field by engineers or geologists are incorporated in the code to predict entrainment effects in a simple and rational manner. This allows the modelling of varying entrainment potential along a debris flow path. The code has been checked against simplified analytical solutions and validated against field observations in a major historical landslide event involving high-mobility debris flows in Hong Kong. The numerical modelling results indicated that simulated entrainment volume and mobility characteristics are broadly consistent with geological field mapping records.*

1 INTRODUCTION

Landslides are major natural hazards that can pose significant risk to human lives and properties in mountainous areas. In Hong Kong, over 60 % of the land is occupied by natural terrain, and over 30 % of which are steep hillsides with gradients of over 30°. Coupled with the dense population and high seasonal rainfall, Hong Kong is considered as one of the places facing such a landslide risk. Since 1977, the Government of Hong Kong has established the Geotechnical Engineering Office (GEO) to manage landslide risk in the territory. A holistic landslide risk management system has been developed by the GEO (Cheung, 2021)[1].

In modern landslide risk management, being able to predict the characteristics of debris flows is essential for the design of mitigation measures against landslide hazards. Such need has led to the development of a variety of debris mobility models around the world in recent times. Such modelling tools are commonly used for estimating the extent of the run-out zone, debris depth and velocity hydrographs at particular chainage along the flow path. This enables the estimation of debris impact load for designing landslide mitigation measures.

Currently, there are many computer software for debris mobility analysis. Kwan et al. (2021)[15] reported the development of debris mobility models in Hong Kong. A suite of numerical tools with different levels of sophistication have been developed to meet the need of landslide risk management. Examples are two-dimensional debris mobility models 2d-DMM (Kwan & Sun, 2006)[12] and various three-dimensional models 3d-DMM formulated using different numerical techniques including the particle-in-call method (Kwan & Sun, 2007)[13], the smoothed particle hydrodynamics method (Law et al., 2017)[16] and the arbitrary Lagrangian-Eulerian method (Koo et al., 2018)[11]. Together with other numerical tools, such as DAN-W (Hungr, 1995)[5] and DAN3D (McDougall & Hungr, 2004)[17], back analyses of historical landslides have been conducted to validate the prediction capability of the debris mobility models (e.g. Hungr et al., 2007[8]; Pastor et al., 2018[19]). These tools have been shown to produce results consistent with site observations. Some of the numerical models have also been applied in routine engineering practice for forward prediction purposes (Kwan et al., 2021)[15].

Among the above-mentioned studies, there have been relatively few studies in modelling entrainment. In practice, simplifications by assessing a constant active landslide volume during the debris run-out process are often adopted. However, this kind of analysis neglects the physical role of surficial materials, where entrainment of these materials could in fact increase the debris volume, alter the composition, and ultimately enhance the mobility of the landslide (McDougall & Hungr, 2005)[18]. In this paper, a review of the state-of-the-art theory and methodologies of entrainment modelling was carried out. This review led to the enhancement of the in-house debris mobility program 2d-DMM developed by the Geotechnical Engineering Office (GEO) by incorporating a semi-empirical and practical method for entrainment modelling with simple inputs required that can be determined from conventional geological mapping results.

2 REVIEW ON MODELLING APPROACHES

In the modelling of debris mobility, continuum dynamic models based on hydrodynamic theory are most commonly adopted. An early 2D model in Lagrangian framework was presented by Hungr (1995)[5] in the development of DAN-W. The effect of deposition or entrainment in DAN-W is modelled by changing the volume of the flowing debris at each time step by a prescribed amount proportional to the distance travelled. The rate of erosion increases with the flow depth, resulting in a depth-proportional distribution of entrained material and natural exponential growth of the landslide with displacement. Although largely empirical, this method has a physical basis where the changes in stress conditions leading to failure within the path material can be related to changes in the total bed-normal stress and therefore the flow depth (McDougall & Hungr, 2005)[18].

Further development in entrainment modelling was reported by McDougall & Hungr (2004)[17] in association with the development of DAN3D, an extension to DAN-W. In DAN3D's formulation, the effect of entrainment is expressed as a "bed-erosion" term ($E_t = \partial b/\partial t$), or known as "erosion velocity" as defined by Takahashi (1991)[24]. E_t is incorporated into the governing equation of motion, i.e. the depth averaged mass balance equation (Equation (1)) and depth averaged momentum balance equations (Equations (2) and (3)) in Lagrangian form.

$$\frac{\partial h}{\partial t}+h\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}\right)=\frac{\partial b}{\partial t} \tag{1}$$

$$\rho h\frac{\partial v_x}{\partial t}=\rho h g_x+k_x\sigma_z\left(-\frac{\partial h}{\partial x}\right)+k_{yx}\sigma_z\left(-\frac{\partial h}{\partial y}\right)+\tau_{zx}-\rho v_x\frac{\partial b}{\partial t} \tag{2}$$

$$\rho h\frac{\partial v_y}{\partial t}=\rho h g_y+k_y\sigma_z\left(-\frac{\partial h}{\partial y}\right)+k_{xy}\sigma_z\left(-\frac{\partial h}{\partial x}\right)+\tau_{zy}-\rho v_y\frac{\partial b}{\partial t} \tag{3}$$

where h = flow depth, v = flow velocity, b = bed-normal erosion depth, ρ = density of both landslide and path materials, g = gravitational acceleration, τ = basal shear stress and k_x, k_{yx}, k_y, k_{xy} = lateral stress coefficients normalised by bed-normal stress σ_z.

This results in a velocity-dependent inertial resistance additional to the basal shear resistance, which is consistent with Perla et al. (1980)'s[20] formulation. DAN3D adopted an empirical approach to determine the erosion rate which is similar to DAN-W by introducing an "entrainment parameter" (McDougall & Hungr, 2005)[18]. The entrainment parameter E is regarded as the growth rate representing the bed-normal depth eroded per unit flow depth per unit displacement. By relating E_t to h and v, the following expression in Equation (4) is obtained:

$$E_t=\frac{\partial b}{\partial t}=Ehv \tag{4}$$

As a preliminary assessment, McDougall & Hungr (2005)[18] recommended the parameter E to be computed by using an average growth rate assuming a natural exponential growth in Equation (5):

$$\bar{E}=\frac{\ln\left(V_f/V_o\right)}{\bar{S}} \tag{5}$$

where V_o = total volume entering the zone, V_f = total volume existing the zone and $\bar{S}$ = approximate average path length of the zone.

Pirulli & Pastor (2012)[21] reviewed different entrainment rate formulae published between 1963 to 2008 and found that most of the formulae relate the entrainment rate to flow velocity and / or to flow depth, and the most frequently used erosion laws are of empirical type. The method after McDougall & Hungr (2005)[18] is mainly used for back analysis and requires one user-specified parameter (i.e E in Equation (4)) which is usually obtained through calibration. Given sufficient geological information for describing other events of similar nature, Pirulli & Pastor commented that the method proposed by McDougall & Hungr (2005)[18] is simple yet effective.

Another documentation and review on entrainment modelling was carried out by Iverson & Ouyang (2015)[9] on literatures published between 1987 to 2014. They critically evaluated modelling methods of erosive mass flows by comprehensively deriving depth-integrated mass and momentum conservation equations for a two-layer model that can exchange mass and momentum with adjacent layers. From the derivation, they reported that many existing entrainment rate formulae lack explicit dependence on boundary tractions, including the method proposed by McDougall & Hungr (2005)[18] (i.e. Equations (4) and (5)). However, inclusion of boundary traction into erosion rate formula must be accompanied by knowledge of the constitutive behaviour of the bed and flow materials to account for the shear and normal tractions at the eroding surfaces. As mentioned by Iverson & Ouyang, a critical issue concerns the identification of the magnitude and location of basal slip in complicated sediment beds that contain natural grains with varying shapes and sizes.

Apart from the above continuum-based formulation, a coupled computational fluid dynamics and discrete element method (CDF-DEM) was reported by Kong et al. (2018)[10] to simulate a debris flow as a mixture of a gap-graded particle system and viscous fluid. The erodible bed was simulated by bonded particles and the erosion criterion depended on the debonding thresholds. It however remains a preliminary pilot study and that development on systematic calibration and verification procedures is needed as commented by Kong et al.

In a more recent review on modelling flowslides and debris, Cuomo (2020)[3] also reported that analytical bed entrainment analysis requires a proper constitutive model for the behaviour of the interface between the propagating landslide and the ground surface. Cuomo also reported that there are very few analytical models for bed entrainment in the current literature, and therefore their application to real case histories is still limited.

Based on the above review, McDougall & Hungr (2005)[18]'s method was found to be widely adopted (Cuomo et al., 2014[[2]; Iverson & Ouyang, 2015[9]; Shen et al., 2018[23]; Pirulli et al., 2018[22]; Cuomo, 2020[3]) owing to its simplicity and ability to produce results consistent with field observation through calibration. The calculation of erosion velocity based on overburden and debris velocity also explains some physics behind the process.

2 IMPROVEMENTS TO COMPUTER PROGRAM "2D-DMM"

The GEO developed its first debris mobility model 2d-DMM following the finite-difference scheme (DAN-W) proposed by Hungr (1995)[5] but with some modifications (Kwan & Sun, 2006)[12]. The scheme is based on a two-dimensional dynamic analysis of landslide debris travelling on a user-specified run-out profile using an explicit Lagrangian solution of the equations of unsteady non-uniform flow in an open channel. The debris mass is discretised into a number of connecting blocks (Figure 1), and the formulation calculates the velocities of the blocks at time-step advances.

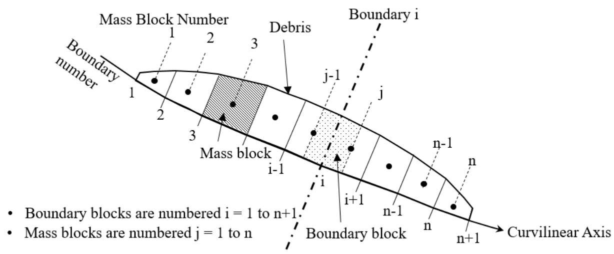


Figure 1: Connecting debris blocks in Lagrangian mesh in 2d-DMM (Modified from Kwan et al., 2006)

The first version of 2d-DMM (i.e. Version 1.0) was coded using Visual Basic for applications on Microsoft Excel. The current version of 2d-DMM is Version 2.0 (GEO, 2015)[4], which is upgraded from Version 1.0 as a standalone Microsoft Windows application programmed using C# language (Kwan et al., 2021)[15]. In the current Version 2.0, the change of landslide volume along the flow path due to entrainment is simulated by specifying the "channel yield rate", which is defined as the volume entrained per unit time and similar to the formulation in Hungr & Evans (1997)[6]. Users are required to determine the entrainment rate by trial-and-error such that the entrained volume matches that suggested in geological mapping or hazard assessments.

Based on the review in Section 1, considering the complex nature and mechanisms involved in material entrainment with limited application using full analytical approach, it is considered that a semi-empirical approach with physical basis that relates entrainment rate to flow depth and flow velocity remains more favourable at this stage. The entrainment modelling in the enhanced version of 2d-DMM (i.e. Version 3.0) will incorporate a semi-empirical method similar to McDougall & Hungr (2005)[18] owing to its simplicity and ability to produce reliable results. The depth-averaged, 2D momentum balance equation in Lagrangian form in 2d-DMM (Version 3.0) is shown in Equation (6).

$$\rho h \frac{\partial v}{\partial t} = \rho h g \sin\alpha + k_x \sigma_z \left(-\frac{\partial h}{\partial x}\right) + \tau_x - \rho v \frac{\partial b}{\partial t} \tag{6}$$

where α = slope inclination, and the bed erosion rate, $\partial b/\partial t$, is related to the flow depth, h and velocity, v, via an entrainment parameter, E as shown in Equation (7) below:

$$\frac{\partial b}{\partial t} = Ehv \tag{7}$$

After rearranging, the volume change due to entrainment can be related to the initial debris volume and the distance travelled by the debris along the channel section as given by Equation (8):

$$\begin{aligned} \Delta V_i &= (\partial b/\partial t) A_i \Delta t \\ &= (\partial b/\partial t) A_i (\Delta s_i / v_i) \\ &= E_i V_i \Delta s_i \end{aligned} \tag{8}$$

where ΔV_i = volume change in debris block *i* due to entrainment, A_i = basal area of debris block *i*, v_i = velocity of debris block *i*, Δt = time step, E_i = entrainment parameter for debris block *i* and Δs_i = displacement of debris block *i*.

In the new version of 2d-DMM (Version 3.0), users are allowed to specify the entrainment depth, d_k, at each segment *k* along the flow path (Equation (9)). This flow path dependent input parameter allows the entrainment characteristics to be modelled in a practical way in which the input data are commonly readily available from conventional geological mapping results. An example is given in Section 3.3. The volume growth rate for each segment, E_k, is then determined as follows:

$$E_k = \frac{d_k B_k}{V_k} \tag{9}$$

It follows that

$$V_k = V_{k-1} + E_k V_{k-1} \Delta x_k \tag{10}$$

where d_k = entrainment depth within k^{th} segment, B_k = base width of landside trail within k^{th} segment, V_k = volume of landslide debris after passing through k^{th} segment and Δx_k = length of k^{th} flow path segment.

In other words, the value of E for each flow path segment is estimated based on the input value of d_k before the time marching calculation commences. It should be noted that the total entrainment volume estimated from geological mapping or hazard assessments may not be equal to the simulated final entrainment volume. This is because a debris block may stop without passing through a channel segment for which an entrainment depth d_k has been specified. In addition, the calculation does not consider entrainment arising from the side slopes of the debris run-out.

3 VALIDATION OF 2D-DMM (VERSION 3.0)

3.1 Comparison with simplified analytical lumped-mass solutions

The 2d-DMM results were compared with an analytical solution with a simple frictional lumped-mass as shown in Figure 2. The landslide trail modelled has a channel width of 5 m and gradient, θ of 20°. An entrainment depth of 0.2 m was modelled along an arbitrary chainage of the channel. The initial source volume was 100 m^3 and the basal friction angle was taken as 25°. The entrained volume, ΔV_k, of the lumped-mass at each time step, Δt, as shown in Figure 2 is calculated using Equations (9) and (10).

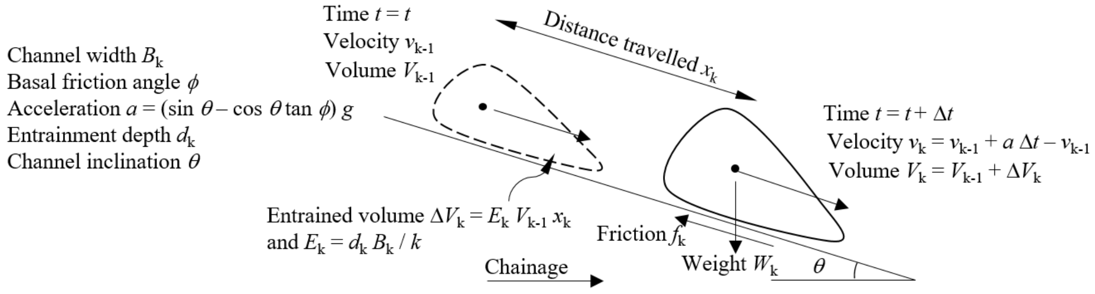


Figure 2: Frictional lumped-mass model

As shown in Figure 3, the velocity of the debris blocks in 2d-DMM is consistent with that of the lumped-mass. The volume entrained is within 2 % of the value in the lumped-mass model.

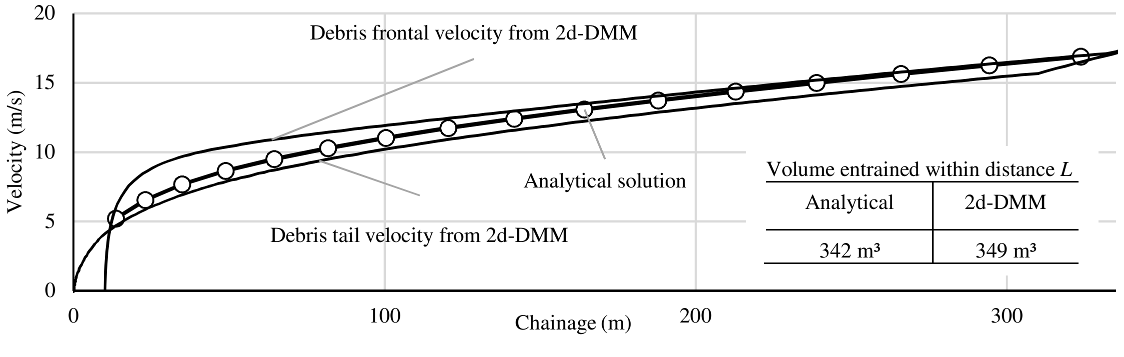


Figure 3: Comparison between analytical and 2d-DMM results

3.2 Comparison with DAN-W

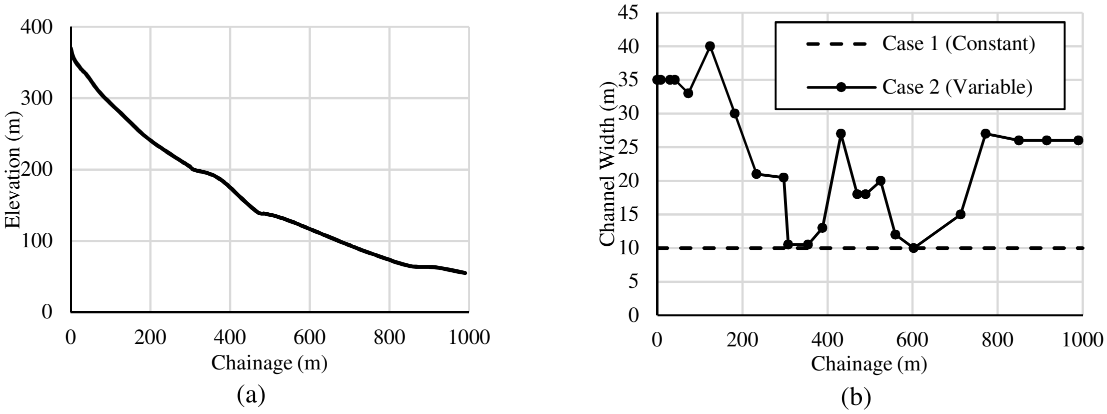
In this section, results from 2d-DMM (Version 3.0) are compared with those from DAN-W, which has been well validated against many historical landslide cases (Hungr et al., 1999[7]; Hungr et al., 2007[8]). DAN-W allows the user to specify the entrainment depth as a debris material property, instead of as a channel property. For simplicity, an arbitrary constant entrainment depth of 0.1 m was assumed throughout the channel.

In order to simulate different flow characteristics, two rheological models, namely, the friction model (Equation (12)) and the Voellmy model (Equation (13)) were adopted for comparing the effects of entrainment in both 2d-DMM (Version 3.0) and DAN-W.

$$\tau_x = -\sigma_z \tan \emptyset_a \tag{12}$$

$$\tau_x = -\sigma_z \tan \emptyset_a + \frac{\gamma v^2}{\xi} \tag{13}$$

where τ_x = basal shear resistance in Equation (6), $\emptyset_a$ = apparent friction angle, γ = unit weight of debris, v = debris velocity and ξ = turbulence coefficient. In this validation exercise, the run-out path used follows the profile of the Shek Pik debris flow event (Kwan et al., 2011)[14] as shown in Figure 4(a). In addition, two cases using different channel configurations were modelled (Figure 4(b)). The first configuration (Case 1) is a rectangular channel with a constant width of 10 m. The second configuration (Case 2) is a rectangular channel with variable widths determined from field geological mapping varying from 10 m to 40 m. Both friction model and Voellmy model were adopted for analysis in Case 1 and only Voellmy model was adopted for Case 2 analysis. A summary of the key input parameters for debris mobility modelling in this validation exercise is shown in Table 1.



(a) (b)

Figure 4: (a) Ground profile used and (b) Channel configuration used for validation against DAN-W

Table 1: Input parameters for debris mobility modelling

	Friction Model	Voellmy Model
Source volume (m^3)	1,000	
Debris density (kg/m^3)	2,200	
$k_x = k_o$ (at-rest pressure coefficent)	1.0	
$k_x = k_a$ (active pressure coefficient)	0.8	
$k_x = k_p$ (passive pressure coefficeint)	2.5	
Entrainment depth (m)	0.1 (constant)	
$\emptyset_a$ (degrees)	20	8
ξ (m/s^2)	-	500

A comparison of the results in terms of total entrained volume and run-out distance of the debris obtained from 2d-DMM and DAN-W is presented in Table 2 and Table 3 respectively for Case 1 and Case 2. The results indicated both 2d-DMM (Version 3.0) and DAN-W using both friction and Voellmy model under different channel configurations produced consistent results. Among the analyses carried out, the differences in entrained volume and run-out distance obtained from the two computer program generally range from 0.7 % to 5.8 %.

Table 2: Comparison between 2d-DMM (Version 3.0) and DAN-W results (Case 1)

Rheological Model	Friction Model			Voellmy Model		
Program	2d-DMM	DAN-W	Difference (%)	2d-DMM	DAN-W	Difference (%)
Total volume entrained (m^3)	569	565	0.70	1739	1819	4.60
Run-out distance (m)	666	643	3.58	877	885	0.91

Table 3: Comparison between 2d-DMM (Version 3.0) and DAN-W results (Case 2)

Rheological Model	Voellmy Model		
Program	2d-DMM	DAN-W	Difference (%)
Total volume entrained (m^3)	1719	1819	5.81
Run-out distance (m)	870	876	0.69

A comparison of the velocity profile against path chainage also indicated that 2d-DMM and DAN-W produced consistent results using both friction model (Figure 5) and Voellmy model (Figure 6 and Figure 7).

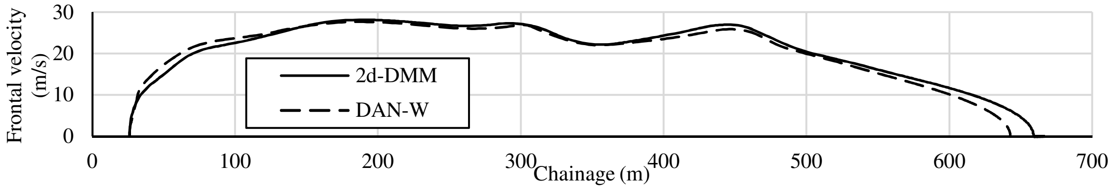


Figure 5: Case 1 – Velocity profile from 2d-DMM and DAN-W using friction model

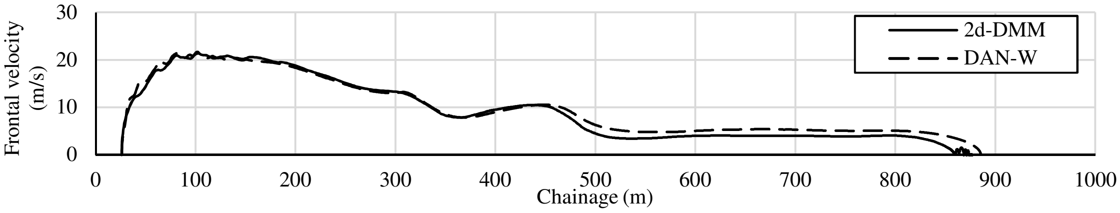


Figure 6: Case 1 – Velocity profile from 2d-DMM and DAN-W using Voellmy model

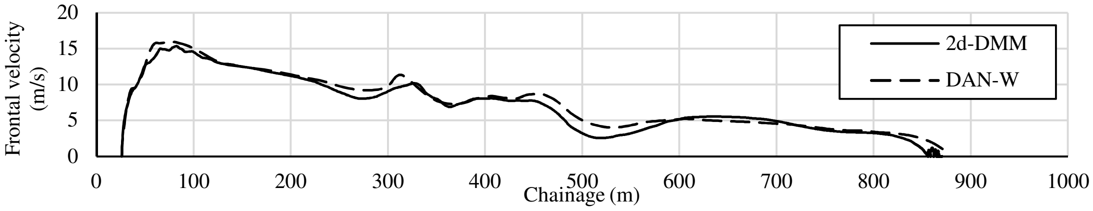


Figure 7: Case 2 – Velocity profile from 2d-DMM and DAN-W using Voellmy model

3.3 Comparison with Historical Case – Yu Tung Road Debris Flow

The new version of 2d-DMM has also been used to simulate the Yu Tung Road debris flow (Figure 8). The debris flows, initiated by the 7 June 2008 extreme rainstorm, involved a substantial amount of entrainment. In order to validate the new entrainment algorithm embedded in 2d-DMM (Version 3.0), the input values of the entrainment depth follow those documented in the detailed landslide field. The width of the landslide trail recorded by the landslide mapping was also adopted in the simulation. The apparent basal friction angle, $\emptyset_a$, and the Voellmy coefficient, ξ, were taken as 8° and 500 *m/s²* respectively, following results of the previous back analysis by Kwan et al. (2011)[14].

Figure 8: Aerial photo of Yu Tung Road debris flow

The Yu Tung Road event had a source volume of 2,350 *m³* and a debris run-out path of about 550 *m* (Kwan et al., 2011)[14]. An extract of the detailed field mapping record is shown in Table 4. Detailed geological field mapping conducted after the event established the volume of eroded material in each section along the flow path as well as the basic geometry of the flow including the flow width. The average entrainment depth could be calculated from the entrained volume, section length and average trail width for input into 2d-DMM.

Table 4: Extract of field mapping record of Yu Tung Road debris flow

Chainage (m)	Gradient (degrees)	Trail Width (m)	Erosion (m^3)	Calculated Average Entrainment Depth (m)
0-40	35	32.5	2502	- (Source area)
40-55	35	32.5	73	0.18
⋮	⋮	⋮	⋮	⋮
470-510	25	14	110	0.22
510-545	20	16	70	0.13

From field geological mapping, the total volume of debris entrained along the trail was estimated at 1,768 *m³*. A similar entrainment volume of 1,814 *m³* was calculated by 2d-DMM (Version 3.0). The calculated velocity profile is also broadly consistent with that determined from super-elevation data obtained in the field (Figure 9).

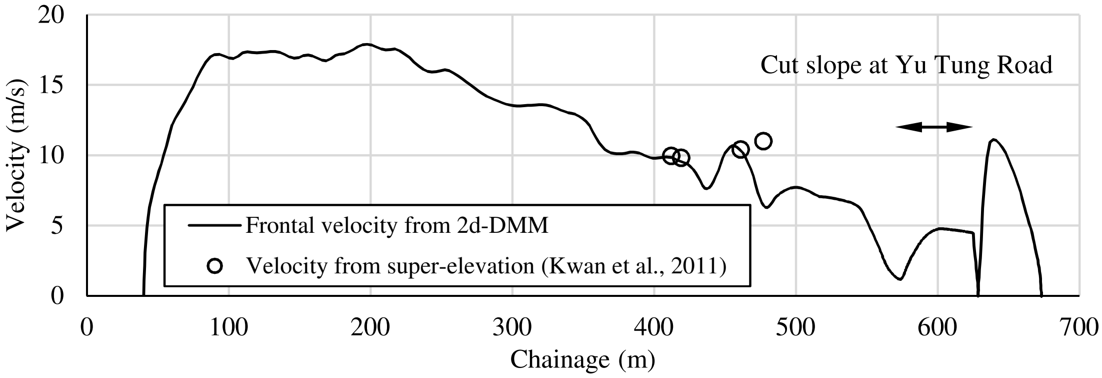


Figure 9: Debris velocity along Yu Tung Road Debris Flow

4 CONCLUSIONS

- A review of the current practice in modelling entrainment effect in debris mobility was carried out. Given the challenges in modelling the complex phenomenon of material entrainment using full analytical approach, a semi-empirical approach should be favored at this stage.
- The method of correlating entrainment rate with debris flow depth and flow velocity was used as a basis to enhance the entrainment modelling in the computer code 2d-DMM developed by the GEO. This method, although largely empirical, contains some

physical basis where the changes in stress conditions leading to failure within the path material can be related to changes in the total bed-normal stress and therefore the flow depth.

- The capability of modelling entrainment in the enhanced version of 2d-DMM (i.e. Version 3.0) was validated against different alternative methods in estimating entrainment (i.e. lump mass model, DAN-W and field mapping records of a high mobility debris flow event). The validation results indicated that the enhanced 2d-DMM was capable to produce consistent results in terms of entrainment volume and mobility. An example is also provided to demonstrate the required input parameters in the enhanced version of 2d-DMM can be readily determined from conventional geological mapping results which facilitates the practical modelling of entrainment.

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