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We congratulate the authors of the discussed paper for their valuable contribution with respect to the solution of the problem of calculation of pipe diameter. Their solution is given through the Lambert W-function (Hayes 2005) which can be accurately approximated in the defined relatively short domain of the input parameters using symbolic regression (Dubčáková 2011). Following our experience in Colebrook’s flow friction modelling (Brkić and Praks 2018, and Praks and Brkić 2020) and air-forced flow modelling of fuel cells cooling (Brkić and Praks 2020), we offer very accurate symbolic regression approximations for D_r^* [Eq. (16) of the discussed paper] and D_s^* [Eq. (20) of the discussed paper] which are based on symbolic regression technique.

We noticed that:

- D_r^* [Eqs. (16) and (17) of the discussed paper] depends only on ε^* [Eq. (12) of the discussed paper] which further depends only on the known input parameters, while the domain of the interest is $3.45 \times 10^{-7} < \varepsilon^* < 0.02883$, and

- D_s^* [Eqs. (20) and (21) of the discussed paper] depends only on ν^* [Eq. (13) of the discussed paper] which further depends only on the known input parameters, while the domain of the interest is $7.5 \times 10^{-9} < \nu^* < 0.000828$.

To feed the software Eureqa (Schmidt and Lipson 2009), which generated symbolic regression approximations from the input dataset, we divided the domain $3.45 \times 10^{-7} < \varepsilon^* < 0.02883$, and $7.5 \times 10^{-9} < \nu^* < 0.000828$ in 100 equidistant points [these points can be random or quasi-random as given in Praks and Brkić (2022)] for which we calculated values of D_r^* using Eq. (16) of the discussed paper and D_s^* using Eq. (20) of the discussed paper, respectively. Based on these two sets of 100 pairs each, $\varepsilon^* \rightarrow D_r^*$ and $\nu^* \rightarrow D_s^*$ for D_r^*, Eureqa provided us with the needed approximations, for the rough part D_r^* here given as Eq. (1) and for the smooth part D_s^* here given as Eq. (2).

$$\left.\begin{array}{c} A = \ln(\varepsilon^*) \\ D_r^* = 0.255 + \frac{A}{425.025} - \frac{2.223}{A - 3.421} \end{array}\right\} \qquad (1)$$

Here presented approximation of D_r^* Eq. (1) introduces the relative error of no more than 0.21% compared with Eq. (16) of the discussed paper.

$$\left.\begin{array}{c} B = \ln(\nu^*) \\ D_s^* = 0.3 + \frac{B}{311.526} - \frac{1.7}{B} - 5.06 \cdot \nu^* \end{array}\right\} \qquad (2)$$

Here presented approximation for D_s^* Eq. (2) introduces the relative error of no more than 0.16% compared with Eq. (20) of the discussed paper.

The pattern of the developed approximations uses basic arithmetic operations and in addition, only one more complex logarithmic function, which makes them simple and efficient for execution in the process circuits of computers (Winning and Coole 2015). The performances of the symbolic regression models were recognized by different independent benchmark studies, such as Zeyu et al. (2020), Muzzo et al. (2021), etc.

To illustrate the usage of the proposed approximations, we give the following example which is elaborated in Yetilmezsoy et al. (2021) which refers to Moody (1944). Moreover, the results of the symbolic regression procedure described in this discussion are in the line with those obtained by the procedures described in Swamee and Jain (1976), Swamee and Swamee (2007), and Medina et al. (2017).

<u>Numerical example:</u> For the known input variables: gravitational acceleration g=9.807 m/sec, head loss Δh=1.37 m, length of pipe L=60.66 m, the flow rate of water through the observed pipe Q=0.0324 m^3/sec, absolute roughness of inner pipe surface $\varepsilon=0.12\cdot10^{-3}$ m and kinematic viscosity of water $\nu=1.0974\cdot10^{-6}$ m^2/sec, diameter D of pipe should be calculated used methodology from the discussed paper with the modification proposed in this discussion:

$$\varepsilon^* = \varepsilon \cdot \left(\frac{g \cdot \Delta h}{L \cdot Q^2}\right)^{0.2} = \varepsilon \cdot a = 0.12 \times 10^{-3} \cdot \left(\frac{9.807 \times 1.37}{60.66 \times 0.0324^2}\right)^{0.2} = 3.5 \times 10^{-4} \rightarrow a = 2.91643$$

$$\nu^* = \nu \cdot \left(\frac{g \cdot \Delta h \cdot Q^3}{L}\right)^{-0.2} = 1.0974 \times 10^{-6} \cdot \left(\frac{9.807 \times 1.37 \times 0.0324^3}{60.66}\right)^{-0.2} = 1.16136 \times 10^{-5}$$

$$\left.\begin{array}{c} A = \ln(\varepsilon^*) = \ln(3.5\times 10^{-4}) = -7.957655542 \\ D_r^* = 0.255 + \dfrac{-7.957655542}{425.025} - \dfrac{2.223}{-7.957655542 - 3.421} = 0.431642993 \end{array}\right\}$$

$$\left.\begin{array}{c} B = \ln(1.16136\times 10^{-5}) = -11.36333323 \\ D_s^* = 0.3 + \dfrac{-11.36333323}{311.526} - \dfrac{1.7}{-11.36333323} - 5.06\times 1.16136\times 10^{-5} = 0.41306887 \end{array}\right\}$$

$$D^* = 1.019\cdot\left(D_r^{*20} + 1.9\cdot D_s^{*20.9}\right)^{0.051} = 1.019\cdot\left(0.431642993^{20} + 1.9\cdot 0.41306887^{20.9}\right)^{0.051}$$

$$D = \frac{D^*}{a} = \frac{0.439280904}{2.91643} = 0.150622365\ m = 150.62\ mm$$

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