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REVIEW ARTICLE

Advances in discrete element modelling of underground excavations

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Abstract The paper presents advances in the discrete element modelling of underground excavation processes extending modelling possibilities as well as increasing computational efficiency. Efficient numerical models have been obtained using techniques of parallel computing and coupling the discrete element method with finite element method. The discrete element algorithm has been applied to simulation of different excavation processes, using different tools, TBMs and roadheaders. Numerical examples of tunnelling process are included in the paper, showing results in the form of rock failure, damage in the material, cutting forces and tool wear. Efficiency of the code for solving large scale geomechanical problems is also shown.

Keywords Coupling · Discrete element method · Finite element method · Parallel computation · Tunnelling

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1 Introduction

A discrete element algorithm is a numerical technique which solves engineering problems that are modelled as a large system of distinct interacting bodies or particles that are subject to gross motion. The discrete element method (DEM) is widely recognized as a suitable tool to model geomaterials [1, 2, 4, 8]. The method presents important advantages in simulation of strong discontinuities such as rock fracturing during an underground excavation or rock failure induced by a tunnel excavation. It is difficult to solve such problems using conventional continuum-based procedures such as the finite element method (FEM). The DEM makes possible the simulation of different excavation processes [5, 7] allowing the determination of the damage of the rock or soil, or evaluation of cutting forces in rock excavation with roadheaders or TBMs. Different possibilities of DEM applications in simulation of tunnelling process are shown in the paper. Examples include new developments like evaluation of tool wear in rock cutting processes.

The main problem in a wider use of this method is the high computational cost required by the simulations first of all due to large number of discrete elements usually required. Different strategies are possible in addressing this problem. This paper will present two approaches: parallelization and coupling the DEM and FEM.

Parallelization techniques are useful for the simulation of large-scale problems, where the number of particles involved does not allow the use of a single processor, or where the single processor calculation would require an extremely long time. A shared memory parallelization of the DEM algorithm is presented in the paper. A high performance code for the simulation of tunnel construction problems is described and examples of the efficiency of the code for solving large-scale geomechanical problems are shown in the paper.

In many cases discontinuous material failure is localized in a portion of the domain, the rest of it can be treated as continuum. Continuous material is usually modelled more efficiently using the FEM. In such problems coupling of the discrete element method with the FEM can provide an optimum solution. Discrete elements are used only in a portion of the analysed domain where material fracture occurs, while outside the DEM subdomain finite elements can be used. Combining these two methods in one model of rock cutting allows us to take advantages of each method. The paper presents a coupled discrete/finite element technique to model underground excavation employing the theoretical formulation initiated in [5] and further developed in [6].

2 Discrete element method formulation

The discrete element model assumes that material can be represented by an assembly of distinct particles or bodies interacting among themselves. Generally, discrete elements can have arbitrary shape. In this work the formulation employing cylindrical (in 2D) or spherical (in 3D) rigid particles is used. Basic formulation of the discrete element formulation using spherical or cylindrical particles was first proposed by Cundall and Strack [1]. Similar formulation has been developed by the authors [5, 7] and implemented in the explicit dynamic code Simpact. The code has a lot of original features like modelling of tool wear in rock cutting, thermomechanical coupling and other capabilities not present in commercial discrete element codes.

Translational and rotational motion of rigid spherical or cylindrical elements is described by means of the Newton–Euler equations of rigid body dynamics:

$$\mathbf{M}_\mathrm{D}\ddot{\mathbf{r}}_\mathrm{D} = \mathbf{F}_\mathrm{D}, \quad \mathbf{J}_\mathrm{D}\dot{\boldsymbol{\Omega}}_\mathrm{D} = \mathbf{T}_\mathrm{D} \tag{1}$$

where $\mathbf{r}_\mathrm{D}$ is the position vector of the element centroid in a fixed (inertial) coordinate frame, $\boldsymbol{\Omega}_\mathrm{D}$ is the angular velocity, $\mathbf{M}_\mathrm{D}$ is the diagonal matrix with the element mass on the diagonal, $\mathbf{J}_\mathrm{D}$ is the diagonal matrix with the element moment of inertia on the diagonal, $\mathbf{F}_\mathrm{D}$ is the vector of resultant forces, and $\mathbf{T}_\mathrm{D}$ is the vector of resultant moments about the element central axes. Vectors $\mathbf{F}_\mathrm{D}$ and $\mathbf{T}_\mathrm{D}$ are sums of all forces and moments applied to the element due to external load, contact interactions with neighbouring spheres and other obstacles, as well as forces resulting from damping in the system. Equations of motion (1) are integrated in time using the central difference scheme.

The overall behaviour of the system is determined by the cohesive/frictional contact laws assumed for the interaction between contacting rigid spheres (or discs in 2D). The contact law can be seen as the formulation of the material model on the microscopic level. Modelling of rock or cohesive zones requires contact models with cohesion allowing tensile interaction force between particle. In the present work the simplest of the cohesive models, the elastic perfectly brittle model is used. This model is characterized by linear elastic behaviour when cohesive bonds are active:

$$\sigma = k_\mathrm{n} u_\mathrm{n}, \quad \tau = k_\mathrm{t} u_\mathrm{t} \tag{2}$$

where σ and τ are the normal and tangential contact force, respectively, k_n and k_t are the interface stiffness in the normal and tangential directions and u_n and u_t the normal and tangential relative displacements, respectively. Cohesive bonds are broken instantaneously when the interface strength is exceeded in the tangential direction by the tangential contact force or in the normal direction by the tensile contact force. The failure (decohesion) criterion is written as:

$$\sigma \le R_\mathrm{n}, \quad |\tau| \le R_\mathrm{t}, \tag{3}$$

where R_n and R_t are the interface strengths in the normal and tangential directions, respectively. Breakage of cohesive bonds allows us to simulate fracture of material and its propagation. In the absence of cohesion the frictional contact is assumed with the Coulomb friction model.

3 Coupling the DEM and FEM

In the present work the so-called explicit dynamic formulation of the FEM is used. The explicit FEM is based on the solution of discretized equations of motion written in the current configuration in the following form:

$$\mathbf{M}_\mathrm{F}\ddot{\mathbf{r}}_\mathrm{F} = \mathbf{F}_\mathrm{F}^\mathrm{ext} - \mathbf{F}_\mathrm{F}^\mathrm{int} \tag{4}$$

where $\mathbf{M}_\mathrm{F}$ is the mass matrix, $\mathbf{r}_\mathrm{F}$ is the vector of nodal displacements, $\mathbf{F}_\mathrm{F}^\mathrm{ext}$ and $\mathbf{F}_\mathrm{F}^\mathrm{int}$ are the vectors of external loads and internal forces, respectively. Similarly to the DEM algorithm, the central difference scheme is used for time integration of (4).

It is assumed that the DEM and FEM can be applied in different subdomains of the same body. The DEM and FEM subdomains, however, do not need to be disjoint—they can overlap each other. The common part of the subdomains is the part where both discretization types are used with gradually varying contribution of each modelling method. This idea follows that used for molecular dynamics coupling with a continuous model in [9].

The coupling of DEM and FEM subdomains is provided by additional kinematical constraints. Interface discrete elements are constrained by the displacement field of overlapping interface finite elements. Making use of the split of the global vector of displacements of discrete elements, $\mathbf{r}_{\mathrm{D}}$, into the unconstrained part, $\mathbf{r}_{\mathrm{DU}}$, and the constrained one, $\mathbf{r}_{\mathrm{DC}}$, $\mathbf{r}_{\mathrm{D}} = \{\mathbf{r}_{\mathrm{DU}}, \mathbf{r}_{\mathrm{DC}}\}^{\mathrm{T}}$, additional kinematic relationships can be written jointly in the matrix notation as follows:

$$\chi = \mathbf{r}_{\mathrm{DC}} - \mathbf{N}\mathbf{r}_{\mathrm{F}} = \mathbf{0}, \tag{5}$$

where $\mathbf{N}$ is the matrix containing adequate shape functions.

Additional kinematic constraints (5) can be imposed by the Lagrange multiplier or penalty method. The set of equations of motion for the coupled DEM/FEM system with the penalty coupling is as follows

$$\begin{bmatrix} \bar{\mathbf{M}}_{\mathrm{F}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{M}}_{\mathrm{DU}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \bar{\mathbf{M}}_{\mathrm{DC}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \bar{\mathbf{J}}_{\mathrm{D}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{r}}_{\mathrm{F}} \\ \ddot{\mathbf{r}}_{\mathrm{DU}} \\ \ddot{\mathbf{r}}_{\mathrm{DC}} \\ \dot{\Omega}_{\mathrm{D}} \end{Bmatrix} = \begin{Bmatrix} \bar{\mathbf{F}}_{\mathrm{F}}^{\mathrm{ext}} - \bar{\mathbf{F}}_{\mathrm{F}}^{\mathrm{int}} + \mathbf{N}^{\mathrm{T}}\mathbf{k}_{\mathrm{DF}}\chi \\ \bar{\mathbf{F}}_{\mathrm{DU}} \\ \bar{\mathbf{F}}_{\mathrm{DC}} - \mathbf{k}_{\mathrm{DF}}\chi \\ \bar{\mathbf{T}}_{\mathrm{D}} \end{Bmatrix} \tag{6}$$

where $\mathbf{k}_{\mathrm{DF}}$ is the diagonal matrix containing on its diagonal the values of the discrete penalty function, and global matrices $\bar{\mathbf{M}}_{\mathrm{F}}, \bar{\mathbf{M}}_{\mathrm{DU}}, \bar{\mathbf{M}}_{\mathrm{DC}}$ and $\bar{\mathbf{J}}_{\mathrm{D}}$, and global vectors $\bar{\mathbf{F}}_{\mathrm{F}}^{\mathrm{int}}, \bar{\mathbf{F}}_{\mathrm{F}}^{\mathrm{ext}}, \bar{\mathbf{F}}_{\mathrm{DU}}, \bar{\mathbf{F}}_{\mathrm{DC}}$ and $\bar{\mathbf{T}}_{\mathrm{D}}$ are obtained by aggregation of adequate elemental matrices and vectors taking into account appropriate contributions from the discrete and finite element parts. Equation (6) can be integrated in time using the standard central difference scheme.

4 Application of DEM to simulation of tunnelling process

Fracture of rock or soil as well as interaction between a tunnelling machine and rock during an excavation process can be simulated by means of the DEM. This kind of analysis enables the comparison of the excavation process under different conditions.

4.1 Simulation of tunnelling with a TBM

Simplified models of a tunnelling process must be used due to a high computational cost of a full-scale simulation in this case. We assume that the TBM is modelled as a cylinder with a special contact model for the tunnel face is adopted. Figure 1 presents a simplified tunnelling process. The rock sample, with a diameter of 10 m and a length of 7 m, is discretized with randomly generated and densely compacted 40,988 spheres. Discretization of the TBM geometry employs 1,193 rigid triangular elements. Tunnelling process has been carried out with prescribed horizontal velocity 5 m/h and rotational velocity of 10 rev/min.

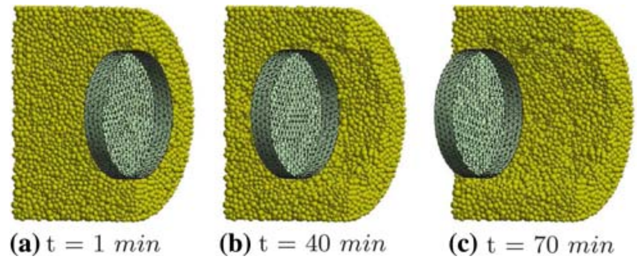

Fig. 1 Simulation of TBM excavation: Evolution and elimination of material

Rock properties of granite are used, and the microscopic DEM parameters corresponding to the macroscopic granite properties are obtained using the methodology described in [10]. A special condition is adopted to eliminate the spherical particles in the face of the tunnel. Each particle, which is in contact with the TBM and lacks cohesive contacts with other particles, is removed from the model. Thus, the advance of the TBM and the absorption of the material in the shield of the TBM is modelled.

Figure 1a, c presents the displacement of the TBM and the elimination of the rock material. The area affected by the loss of cohesive contacts, resulting in material failure is shown in Fig. 2. This loss of cohesion can be considered as *damage*, because it produces the change of the equivalent Young modulus.

4.2 Simulation of linear cutting test of single disc cutter

Simulation of the linear cutting test was performed. A rock sample with dimensions of 135 × 10 × 5 cm is represented by an assembly of randomly generated and densely compacted 40,449 spherical elements of radii ranging from 0.08 to 0.60 cm. The granite properties are assumed in the simulation and appropriate DEM parameters are evaluated. The disc cutter is treated as a rigid body and the parameters describing its interaction with the rock are as follows: contact stiffness modulus k_{n} = 10 GPa, Coulomb friction coefficient μ = 0.8. The velocity of the disc cutter is assumed to be 10 m/s.

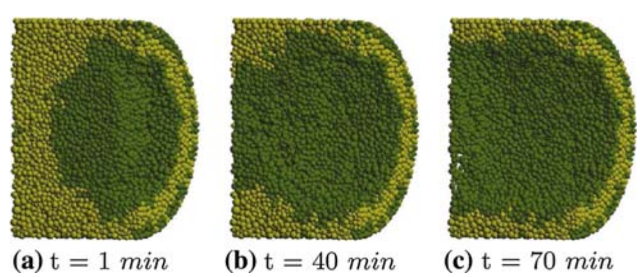

Fig. 2 Simulation of TBM excavation: Damage over tunnel surface

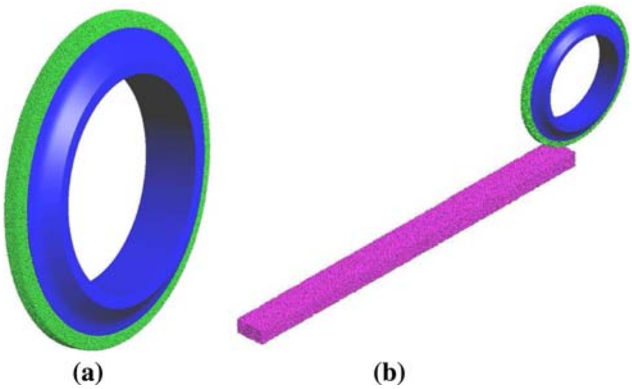

Fig. 3 Linear cutting test simulation: **a** cutter ring with partial discretization; **b** full discretized model

Figure 3a shows the discretization of the disc cutter. Only the area of the cutter ring in direct interaction with the rock is discretized with discrete elements due to the computational cost reasons. The whole model is presented in Fig. 3b.

The evolution of the normal cutting force during the process is depicted in Fig. 4a. The values of the forces should be validated, because the boundary condition can affect the results. The evolution of the wear, using the formulation presented in [5], can be seen in Fig. 4b. The elimination of the discrete elements, where the wear exceed the prescribed limit, permit the modification of the disc cutter shape, which leads to a change of the interaction forces. In the present case, a low value of the wear constant is considered, in order to maintain the initial tool shape. Accumulated wear indicates the areas where the removal of the tool material is most intensive. An acceleration of the wear process using higher values of the wear constant is required in order to obtain in a short time considered in the analysis the amount of wear equivalent to real working time.

Table 1 Times for different number of processors

Time (s) versus processors	1	2	4
Total	404.31	272.93	156.85
Static contacts (per step)	0.1279	0.0692	0.0351
Dynamic contacts (per step)	0.0059	0.0057	0.0055
Time integration (per step)	0.0426	0.0357	0.0344
Speed up	1.00	1.84	2.58

5 High performance simulations

One of the main problems with the DEM simulation is the computational cost. The contact search, the force calculation for each contact, and the large number of elements necessary to resolve a real life problem requires a high computational effort. High performance computation, and parallel implementation could be necessary to run simulations with large number of time steps.

The advances of the computer capabilities during last years and the use of multiprocessors techniques enable the use of parallel computing methods for the discrete element analysis of large scale real problems. A shared memory parallel version of the code is tested. The main idea is to make a partition of the mesh of particles and use each processor for the contact calculation at different parts of the mesh. The partition process is performed using a specialized library [3].

The calculation of the cohesive contacts requires most of the computational cost. A special structure for the database, and the dynamic load balance is used in order to obtain a good performance for the simulations. Two different structures for the contact data are used in order to have a good management of the information. The first data structure is created for the initial cohesive contacts, where a static array can be used. The other data structure is

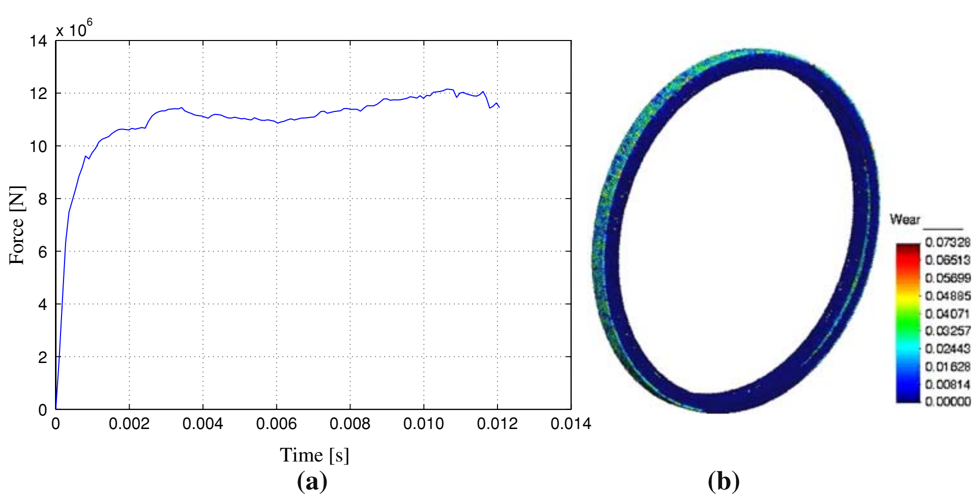

Fig. 4 Linear cutting test simulation: **a** normal force over disc cutter; **b** accumulated wear on the disc cutter

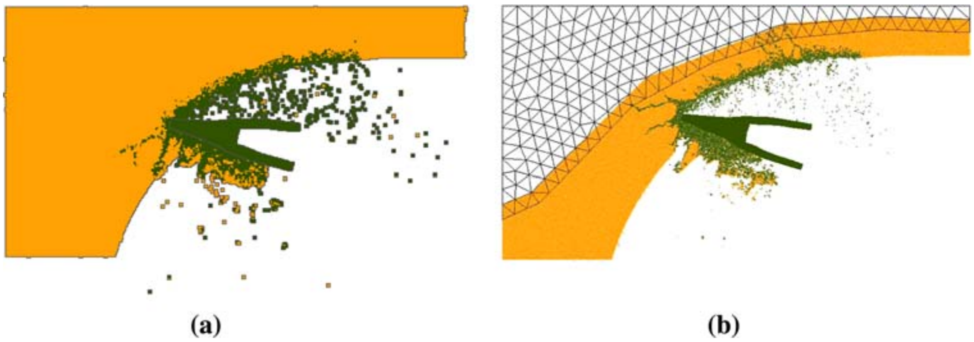

Fig. 5 Simulation of rock cutting: **a** DEM model, **b** DEM/FEM model

designed for the dynamic contacts, occurring in the process of rock fragmentation, and the interaction between different bodies. The management of this kind of contact is completely dynamic, and it is not necessary to store variables with the history information.

Table 1 presents the times of parallel simulations of a tunnelling process, which was described earlier. The main computational cost is due to the cohesive contacts evaluation. The results shown in the table confirm that a good speed-up has been achieved.

6 DEM and DEM/FEM simulation of rock cutting

A process of rock cutting with a single pick of a roadheader cutter-head has been simulated using discrete and hybrid discrete/finite element models. In the hybrid DEM/FEM model discrete elements have been used in the part of rock mass subjected to fracture, while the other part have been discretized with finite elements. In both models the tool is considered rigid, assuming the elasticity of the tool is irrelevant for the purpose of modelling of rock fracture.

Figure 5 presents results of DEM and DEM/FEM simulation. Both models produce similar failures of rock during cutting. Cutting forces obtained using these two models are compared in Fig. 6. Both curves show oscillations typical for cutting of brittle rock. In both cases similar values of amplitudes are observed. Mean values of cutting forces agree very well. This shows that combined DEM/FEM simulation gives similar results to a DEM analysis, while being more efficient numerically—computation time has been reduced by half.

7 Conclusions

- Discrete element method using spherical or cylindrical rigid particles is a suitable tool in modelling of underground excavation processes.

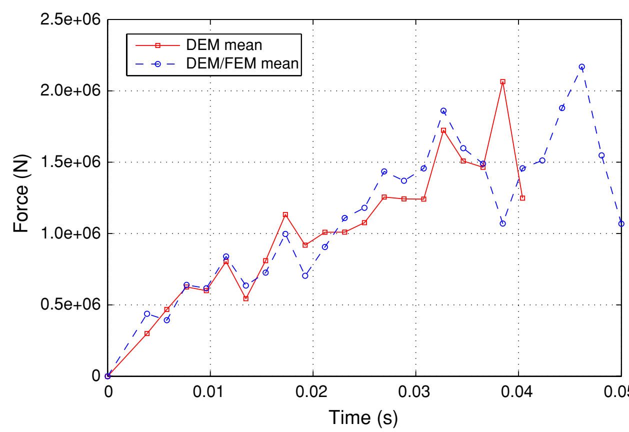

Fig. 6 Cutting force histories for DEM and hybrid DEM/FEM simulations of rock cutting

- Use of the model in a particular case requires calibration of the discrete element model using available experimental results.
- Discrete element simulations of real engineering problems require large computation time and memory resources.
- Efficiency of discrete element computation can be improved using technique of parallel computations. Parallelization makes possible the simulation of large problems.
- The combination of discrete and finite elements is an effective approach for simulation of underground rock excavation.

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