Moving Mesh Methods for Implicit Moving Boundary Problems

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ABSTRACT

Numerical algorithms for moving boundary problems require both the ability to track the evolution of the boundary and a method for approximating the evolution of the system in the region enclosed by the moving boundary. When the dynamics are governed by partial differential equations they are usually simulated on a computational mesh, so we must deal with a time-dependent computational domain. We could define a fixed mesh over a larger region and subdivide elements which are intesected by the moving boundary at each time-step, but in this work we will define a mesh which conforms to the original boundary position and allow it to follow the boundary as it moves. The latter option has the potential to work extremely well in situations where the computational domain does not become highly distorted.

In particular, we will consider the velocity-based moving mesh framework of Baines, Hubbard and Jimack [1, 2], which was developed for problems in which the boundary movement is not given explicitly. This family of methods consists of two stages:

1. Determine mesh velocities using a conservation principle. Basing this on a physical principle, *e.g.* mass conservation, allows us to obtain the correct velocities for the implicit moving boundary, and this can be combined with a numerical principle, *i.e.* monitor conservation, to preserve equidistribution properties which the initial mesh can be designed to satisfy.

2. Move the mesh according to these velocities and update the solution using a standard time-stepping scheme. This is a step required by all moving mesh methods and is carried out using the general Arbitrary Lagrangian-Eulerian (ALE) approach.

The original versions were finite difference and finite element methods, but these have now been extended to discontinuous Galerkin and virtual elements on arbitrary polygonal moving meshes.

Until recently, this approach (and almost every other ALE-based method) has been limited to second-order accuracy, at best [2]. In this talk I will show how to extend this framework (and other ALE methods) to arbitrary orders of accuracy and present results for the porous medium equation, which demonstrates nonlinear diffusion with an implicit moving boundary.

REFERENCES

[1] M.J.Baines, M.E.Hubbard and P.K.Jimack, A moving mesh finite element algorithm for the adaptive solution of time-dependent partial differential equations with moving boundaries, *Appl Numer Math*, **54**, pp. 450–469 (2005).

[2] M.J.Baines, M.E.Hubbard and P.K.Jimack, Velocity-based moving mesh methods for nonlinear partial differential equations, *Commun Comput Phys*, **10**(3), pp. 509–576 (2011).