# ABSTRACT

Qualitative reasoning is a new field of research from artificial intelligence that derives the behavior of a model from a high-level representation of the fundamental principles of the domain, and the geometry and topology of the model. Qualitative reasoning is useful for evaluating conceptual designs of earthquake resistant buildings because it derives values for parameters even with incomplete and imprecise knowledge about the model, which is particularly important for the conceptual design stage. Qualitative reasoning represents the relationships between parameters in a model and a search assigns values associated with intervals and relevant points in the behavior.

The space centered framework is a qualitative reasoning framework suitable for static boundary value problems because it incorporates geometry and spatial relationships. In contrast to previous qualitative reasoning frameworks, it represents three-dimensional geometry and spatial relationships. The space centered framework avoids ambiguity in the qualitative calculus by using parameter relations and maintaining a high-level representation of the symbolic equations for the fundamental principles. This is an important difference compared with previous frameworks that map the symbolic equations into qualitative equations. The qualitative calculus uses four techniques: basic qualitative calculus operations, transitivity relations, linear constant elimination, and consistency checking. The inference scheme has two steps: an elaboration and backward-forward solution propagation. The elaboration enhances the initial description by deriving qualitative values that follow from the specified model. The propagation starts by deriving a qualitative state for a component. The new information added by the component is propagated through the model using the topology. The inference scheme is efficient because it detects invalid combination of component states early in the reasoning process.

The space centered framework is implemented in a program for evaluating the structural behavior of conceptual structural designs. From a high-level description of the structure and a representation of the fundamental principles of equilibrium, compatibility, and force-deformation, the framework infers a set of structural behaviors. The program derives the direction and relative magnitude of forces, moments, rotations, and displacements of the model. Such results can be used by the designer to gain insights into the load transfer characteristics of a structure, deciding which conceptual designs are promising and worth pursuing to a more detailed quantitative design stage. The results are also useful to investigate the redundancy or alternative load transfer characteristics of a conceptual design which is particularly important for earthquake resistant buildings.

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# Chapter 1

# INTRODUCTION

## 1.1.- CONCEPTUAL DESIGN FOR EARTHQUAKE RESISTANT BUILDINGS

Design is a process that, from a set of requirements on function, construction and fabrication, cost requirements, and aesthetics, results in the description of a model. The description of the model varies according to the design context and the level of abstraction or detail. In structural engineering, the description refers to the characteristics of individual components such as frame members, shear walls, connections, and supports. The requirements vary significantly with the scope of the design and the level of abstraction.

There are usually several solutions to a design problem and the design process can be defined as an iterative process which maps the initial set of requirements to several design solutions. Some requirements, such as the resistance function of a building, depend on the design solution because a solution incorporates new requirements. Consider the design of an office building where the resistance function is to transfer the gravity and earthquake loads to the foundation. A design solution augments the requirements, such as the self-weight and dynamic loads caused by the mass, and these new requirements affect the evaluation of the design solution.

The first stage in the design process is the conceptual design. Conceptual design is a process which obtains a qualitative description of design solutions, given an initial set of requirements. In structural engineering, designers develop conceptual designs based on deep knowledge of the principles of equilibrium, compatibility, and material characteristics, and experience. The subsequent stages of the design process start with the conceptual design solutions, adding more detailed information and in some cases modifying the solutions.

Conceptual design is an important stage of the design process because it determines the overall behavior of a structure. Many sound structures such as the Agrippa's dome built in 124 A.D. by the Romans or P. L. Nervi's sports palaces in Rome, have been designed in the past based on good conceptual design, without the precise numerical models available in the present [Billington 90]. Quantitative analysis provides detailed information about the structural demands. However, the detailed information required for a quantitative analysis may not be available at the early stages in the design process. Even if the design is sufficiently refined to construct a model, the influence of factors such as material characteristics, soil conditions, and the influence of non-structural components, among others, may not be investigated. Furthermore, quantitative analysis may obscure fundamental flaws in load path characteristics. An early emphasis on the conceptual design stage rather than on numerical calculations can help to achieve good designs of structures.

An optimal design for a building under static loads can be achieved by minimizing an objective function, typically the total weight of materials. For earthquake resistant design, however, there is no simple objective function for an optimal design. Instead, structural engineers design structures that are robust by criteria such as redundant load paths, ductile failure modes, and energy dissipation capacity [Aktan and Bertero 84]. These characteristics are best provided in the conceptual design phase of a structure. Conceptual design is particularly important for earthquake resistant buildings since seismic loads may be severe and codes generally relay on good behavioral characteristics of the design.

Conceptual design is different from the rest of the design process because it is primarily concerned with qualitative or non-numeric representations that are used to evaluate the behavior of a proposed design. The design solution space is typically very large and it is not practical to attempt to investigate each solution in detail to determine that a design may be flawed. There are basically two complementary approaches to reduce this typically very large design space, heuristic criteria and qualitative criteria.

Heuristic criteria, in the form of expert systems, have been the subject of considerably research in recent years [Ganguly et al 90, Subramani et al 89, Fenves and Baker 90, Fenves and Ibarra-Anaya 89]. Expert systems are relatively easy to develop since there are various well known experience-based rules such as:

- Provide symmetry
- Reduce sudden changes in structural stiffness and/or masses
- Reduce unnecessary masses
- Take into account special provisions for short columns/beams
- Take into account changes in structural period caused by non-structural components before and during an earthquake
- Provide enough detailing at final stages of the design

among others. The limiting factor of these rules is that they are very shallow and they do not provide much useful additional information to a designer.

Qualitative criteria have not been previously investigated consistently. They are more difficult to develop than heuristic rules since they require at least a high level description of the design. Some examples of qualitative criteria are:

- Load path characteristics in the form of direction and relative magnitude of forces, moments, rotations and displacements
- Redundant or alternative load paths

These criteria typically provide more useful additional information for a designer than heuristic rules. Qualitative reasoning is one alternative for evaluating these criteria without elaborating the numerical details that may obscure the overall characteristics of the behavior. A typical evaluation of a structural component would state that "this column is in compression and bi-axial flexure, and the magnitude of its drift is greater than for this other column." This is the type of information that is useful to a designer when evaluating conceptual designs.

The conceptual design process outlined in Fig. 1.1 begins with an initial design that satisfies a set of structural, spatial, construction, and aesthetics requirements. Structural requirements are conditions such as transferring the loads to the foundation and the displacements must be within specified limits. Spatial requirements are conditions such as the valid columns locations or minimum distance between floors. Construction and aesthetics requirements are often not explicitly stated. It is likely that the initial solution will not be an efficient or even a realistic solution. The second step, and most important aspect of the conceptual design, is the evaluation of the proposed design. The load transfer and displacement characteristics are identified in this step. This is a critical step because it provides a rational basis for modifying the initial design to achieve a more efficient and realistic solution. For example, a qualitative evaluation can indicate that a design has a non-redundant load path that depends on torsion of the critical member. The designer would then be able to modify the design to provide a redundant load path.

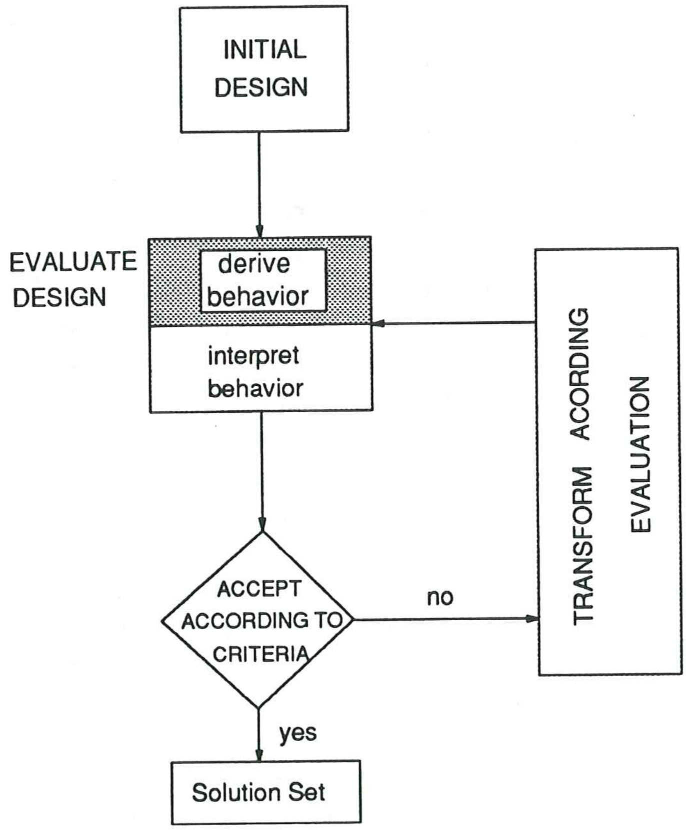



Figure 1.1 A model for the generation of conceptual designs.

The evaluation of a design is decomposed into two stages: (1) derivation of the behavior, and (2) interpretation of the behavior. The derivation of behavior is achieved from a high-level representation of topology, geometry, structural function, structural behavior, and the fundamental principles of equilibrium, compatibility, and material characteristics. Topology and geometry define the form of a structure. Topology refers to the connectivity relationships between the components forming the structure. According to the level of detail and the components' class, topology is represented by the interconnections between components at points, edges, or faces. Geometry refers to the dimensions, position, and orientations of the components. The fundamental principles are represented in a manner consistent with the type of reasoning, qualitative in this case, and the level of abstraction.

The second stage, interpretation of behavior, is typically performed by engineers using experience or heuristic knowledge about the structure and derived behaviors. For this stage there are techniques such as rule-based expert systems which can indicate the likelihood of some failure mode, given the forces and displacements. If the evaluation indicates that the proposed design is satisfactory, the design is added to the set of possible solutions. If the evaluation finds a deficient design, it should be improved according to the information provided by the evaluation. The transformation may be achieved by experience or by production rules which indicate the components to add, delete, or modify to improve the design. The transformed structure is a new design, and the conceptual design process continues until several satisfactory solutions are developed for further consideration in the design process.

Innovation and creativity in the conceptual design phase of a structure are usually the result of changes in the state-of-the-art (for example, a new material or new construction technique), or the result of non-numeric, goal-oriented changes in the initial design. The introduction of the concrete in the beginning of the century enable the possibility of casting in place but it also enable alternative load transfer characteristics that resulted in the creation of concrete shells. The second class of innovation is illustrated by the use of prestressed concrete pioneered by Freyssinet in the '30s. Prestressed beams originated from concrete beams with the qualitative goal of reducing undesired tension stresses in the beams.

The monograph focuses on a new methodology for deriving structural behavior for conceptual designs, based on an abstract and possibly incomplete model and representing the fundamental laws of equilibrium, compatibility, and force-deformation. The evaluation derives the direction and relative magnitude of forces, moments, rotations and displacements of the model. Such results can be used by the designer to gain insights into the load transfer characteristics of the conceptual design, deciding which conceptual designs are promising and worth pursuing to a more detailed quantitative design stage. The evaluation is performed for static loads even for dynamic excitations which is appropriate for conceptual design. Dynamic excitations are a function of the topology, geometry, and structural behavior; however, reasoning about the load transfer characteristics under dynamic excitation is usually accomplished by engineers using equivalent static lateral loads during the conceptual design.

It is possible to derive the initial design solution by heuristic or experiential knowledge. The heuristic knowledge can provide an initial solution closer to a satisfactory design and speedup convergence of the design process. However, if the design problem is not in the scope of the heuristic rules, the initial design may be far from a satisfactory solution. As a consequence of the current work, it is possible to derive the geometry of a design which transfers specified loads to the foundation without using heuristic knowledge. Chapter 7 presents a brief introduction to extensions of the proposed framework for deriving structural designs from fundamental principles.

## 1.2.- QUALITATIVE REASONING

Qualitative reasoning attempts to explain in an automated manner how the model of a device functions by representing the fundamental or first principles for the domain [De Kleer and Brown 84]. Qualitative reasoning derives behavior from the description of a model; it does not rely on heuristic knowledge about the behavior. Compared with the shallow level of knowledge in most heuristic-based systems, qualitative reasoning frameworks are called deep models because they use fundamental principles.

There are major differences between quantitative and qualitative models. Quantitative models represent the fundamental principles by algebraic or differential equations. The parameters for quantitative models usually have an infinite range of values, such as the set of real numbers. Given complete information about the input parameters, a *procedural computation* produces a unique solution which is only valid for the specified values of input parameters. Even a small variation in the model requires a new analysis or an evaluation of the sensitivity coefficients. For qualitative models, in contrast, parameters are represented by a small set of intervals and relevant values such as positive, zero, and negative. Given possibly incomplete information about the qualitative values of the parameters, a *search* results in a usually non-unique set of solutions. Qualitative models represent the relationships between parameters and the distinction between input and output parameters, such as loads and responses for a structure does not exist.

Four frameworks for reasoning about qualitative models have been developed by researchers in artificial intelligence: (1) the component centered framework, (2) the process centered framework, (3) the constraint centered framework, and (4) the unifying framework. Reasoning about structural behavior requires representing the geometry and spatial relationships between components, such as frame members, supports, shear walls, connections, and plates. Unfortunately, the representation of geometry and spatial relationships in the frameworks is limited because they focus on one-dimensional, initial value problems. The modeling of even simple one-dimensional structural engineering problems, such as springs, is cumbersome and inefficient.

The new framework developed in the monograph, named *the Space Centered Framework,* overcomes these deficiencies and it is suitable for multi-dimensional, boundary value problems. The framework is implemented in the program *Agrippa*. The name *Agrippa* comes after the largest shell of the antiquity built by the Romans. Representing the fundamental principles of equilibrium, compatibility, and material characteristics, *Agrippa* provides information about the load transfer characteristics and deformations for nonlinear elastic structures. An interpretation of the structural behavior, derived by *Agrippa*, provides the engineer with insights about the loads paths and type of failure modes that may be expected in the structure.

## 1.3.- REASONING ABOUT STRUCTURAL BEHAVIOR FOR CONCEPTUAL DESIGNS

The reasoning about how a device achieves its function may be performed in several ways. This section discusses two approaches: investigate the effects caused by changes in parameters of a model, or evaluate behavior from a high-level description of the device. In structural engineering the first approach is useful when a constraint is exceeded and the designer wants to determine how changing parameters will satisfy the constraint. For example, the proposed design of a building to resist earthquake ground motion may exceed the drift limit at the first story, and the designer needs to select and modify various parameters such that the drift constraint is satisfied. The designer needs to know *a priori* which parameters affect the drift. Much of the research in qualitative reasoning has concentrated in this approach for dynamic systems that evolve in time.

The second approach for reasoning about a device is to evaluate behavior from a high-level description of the problem. This approach is also used by designers whom, at the conceptual stage of the design process, would like to derive the behavior. Based on the evaluations, a designer disregards some designs and continue to pursue others. The current research focuses on the derivation of structural behavior, not on the interpretation of the behavior or its acceptability according to design criteria.

As an example to illustrate reasoning about structural behavior, consider a shear wall supported by two columns resisting a lateral load, as shown in Fig. 1.2. At the conceptual design stage the precise load distribution over height and the sections characteristics are not available. However, a qualitative derivation of the behavior indicates that the columns are subjected to axial force, shear, and bending moments independent of specific assumptions. The axial forces in the columns have the same magnitude but opposite direction, and they are caused by the overturning moment on the shear wall. An interpretation of the derived behavior would indicate that the axial forces may be large enough to induce a compression or tension failure in the columns and affect their flexural ductility.

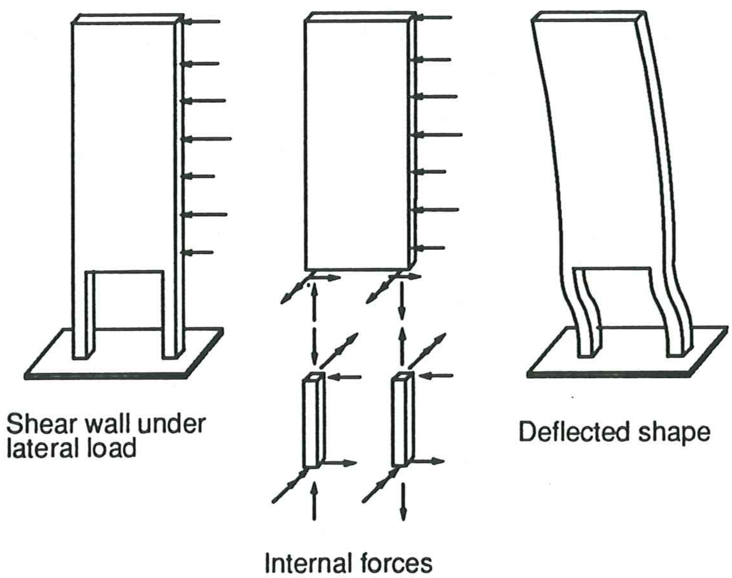



Figure 1.2 Structural behavior of a shear wall supported on two columns and carrying lateral loads.

As a second example, consider the one bay frame in Fig. 1.3. According to the soil conditions, it may be necessary to investigate the effects of support displacements on the internal bending and forces. A derivation of the qualitative behavior indicates that the axial forces in the columns do not change, but the columns develop shear forces and bending moments, as the right support displaces.

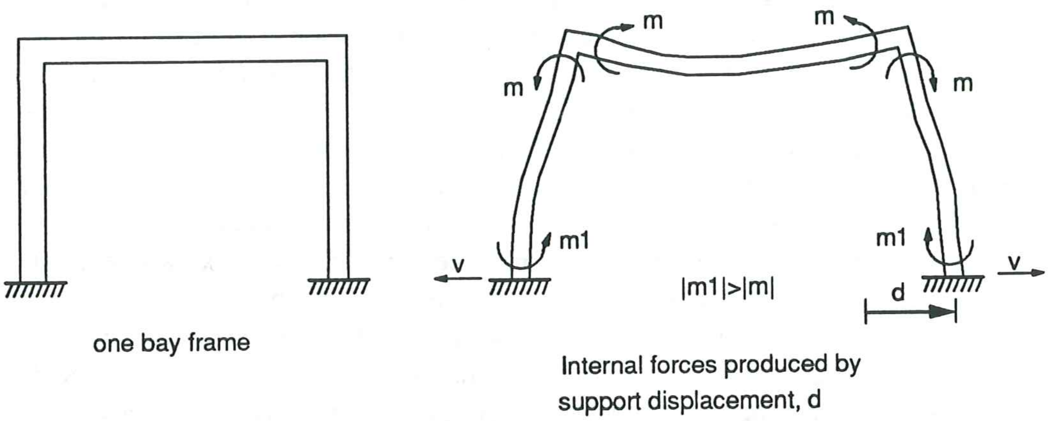



Figure 1.3 Qualitative behavior of a one bay frame subjected to a support displacement.

As a final introductory example, consider the three-dimensional frame illustrated in Fig. 1.4. By taking moments about a line between the supports, it is clear that the structure is not stable and that the design should not be further pursued. A designer can modify the design by changing the simple supports to fixed supports so that the structure is stable. A quantitative analysis of the modified frame provides the numeric value for the parameters in the model but it cannot indicate if under a different length, section dimensions or material characteristics the columns would be in tension or the beams would not transfer torsion.

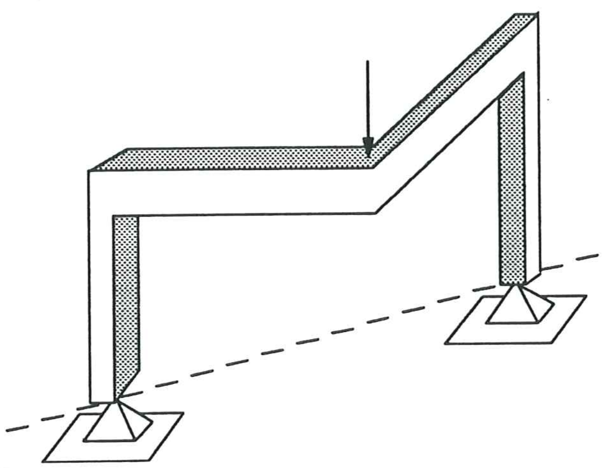


Figure 1.4 Unstable structure detected at the conceptual design stage.

## 1.4.- ORGANIZATION OF THE MONOGRAPH

Chapter two introduces the frameworks for qualitative reasoning that have been previously developed in the artificial intelligence field. These frameworks are presented in a structural engineering context, but their limitations for multi-dimensional, static boundary value problems are discussed.

Chapter three presents the general requirements for a knowledge representation formalism and it introduces a high-level description for conceptual structural design. The description includes knowledge about topology, geometry, structural behavior, and structural function of a design. A major contribution of the current work is that it includes a representation of structural behavior in terms of the fundamental laws of equilibrium, compatibility, and material characteristics. The representation for material characteristics incorporates elastic nonlinear behavior of a material.

Chapter four presents the space centered framework which is the central contribution of the monograph. The framework is suitable for static boundary value problems, for which geometry and spatial relationships are important. From a possibly incomplete high-level description of three-dimensional structures, and from a representation of the fundamental principles, the framework derives the existence, direction, and relative magnitude of internal forces and displacements. An engineering interpretation of the forces enables a designer to gain insights into the load transfer characteristics of the design.

Chapter five presents the implementation of the space centered framework in the computer program *Agrippa*. The program is developed using the Prolog language. Efficiency is essential for qualitative reasoning programs, and *Agrippa* derives the behavior for complex planar structures and simple three-dimensional structures in a relatively rapid manner.

Chapter six illustrates various examples of the evaluation of the load transfer characteristics for conceptual structural designs. The behavior of systems such as planar frames and three-dimensional structures is derived using *Agrippa*.

Finally, Chapter seven summarizes the conclusions of the current work. This chapter also presents an introduction to future applications of the space centered framework for design synthesis. In particular *Agrippa* is used to rediscover an arch based on the qualitative behavior of zero bending moments. The monograph provides several contributions, but it also introduces new questions and future research areas.

# Chapter 2

## QUALITATIVE REASONING ABOUT PHYSICAL BEHAVIOR

"A qualitative physics predicts and explains the behavior of mechanisms in qualitative terms. The goals for the qualitative physics are (1) to be far simpler than the classical physics and yet retain all the important distinctions (e.g., state, oscillation, gain, momentum) without invoking the mathematics of continuously varying quantities and differential equations, (2) to produce causal accounts of physical mechanisms that are easy to understand, and (3) to provide the foundations for commonsense models for the next generation of expert systems."

by Johan De Kleer and John Seely Brown [De Kleer and Brown 84].

### 2.1.- INTRODUCTION

Qualitative reasoning attempts to derive and explain the physical behavior of a device in non-numeric, or qualitative, terms by representing first principles for the domain. This chapter introduces four qualitative reasoning frameworks proposed by researchers in the field of artificial intelligence. The chapter also introduces fundamental concepts that are necessary to develop the new framework proposed in the monograph. Most of the ideas discussed in this chapter are standard qualitative reasoning concepts, although they are primarily explained in a structural engineering context.

Initial attempts to automate the reasoning about physical problems suggested that pure symbolic manipulation or numeric techniques are not adequate as a general problem solving tool [De Kleer 90]. Apparently simple questions such as, what would it happen if water on a container is placed on a heater, are difficult to answer by the symbolic manipulation because the mathematical equations that represent this simple problem are complex. A class of knowledge is not included in the mathematical equations and one of the first persons to characterize the missing information as qualitative or non-numeric knowledge was De Kleer [Forbus 90a].

An early computer program that incorporated qualitative knowledge as a fundamental strategy for problem solving is `NEWTON` [De Kleer 90]. This program solves roller coaster problems associated with the movement of a car on a track under gravity. Qualitative knowledge is generated from a graph of the possible distinct behaviors, such as shown in Fig. 2.1, for a block sliding over a frictionless surface. In

Fig. 2.1 points and intervals are represented by nodes in the graph. For example, the block is initially located at node C1 in the graph; the segment between the point C1 and the lowest point in the trajectory, point C2, is the node S1. A block sliding through the segment S2 either goes up to point C3 or returns down to point C2, and therefore the node S2 has two leaves.

This graph is called the envisionment of the system and it is useful for answering simple questions about the behavior of the block. If the initial height for point C1 is greater than zero, a block placed at C1 must pass through the interval S1, the lowest point C2, and the interval S2. Therefore, the question, "would the mass reach the point C2?" is answered affirmatively without examining the governing equations. There are questions that cannot be resolved by envisionment, or whose results are ambiguous. For such cases NEWTON uses equations for conservation of energy.

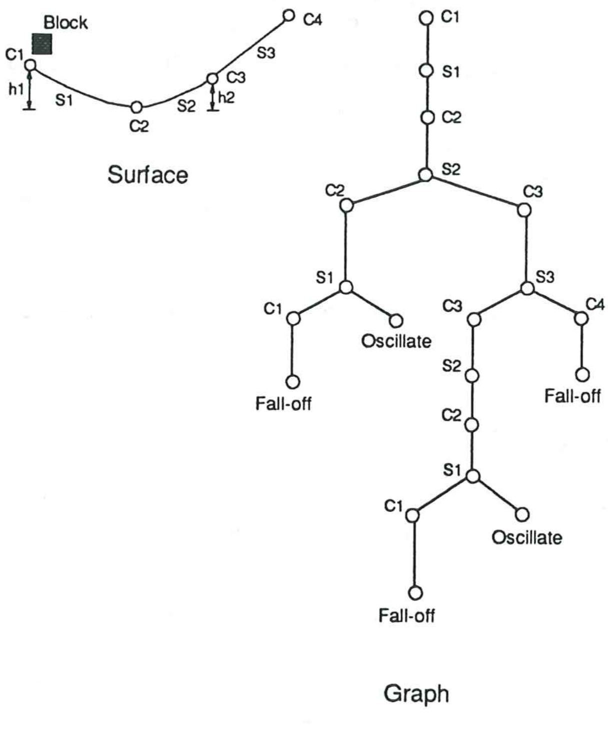



Figure 2.1 Envisionment graph for a small mass sliding on a frictionless surface. Ci represents points and Si represents segments on the surface.

Human thinking, at least for this class of problems, is not initially performed through equations, and commonsense questions are difficult to answer by a problem solving tool that does not incorporate qualitative knowledge. For example, it is hard to explain why the mass reaches point C2 because the governing equations obscure the physics of the problem. Qualitative reasoning began as an attempt to incorporate commonsense knowledge into intelligent problem solving tools. The field was profoundly influenced by a paper written by P. Hayes [90], where he postulated a formalization for commonsense knowledge. A central concept is the history or a "connected piece of space-time in which something happens." Following the ideas from

Hayes about histories, much of the work in qualitative reasoning focused on initial value problems for dynamic systems, where object instances and object states change over time. Further readings about the beginning of qualitative reasoning can be found in Cohn [87] and Forbus [90a].

Qualitative reasoning, however, is no longer concerned with just commonsense reasoning, rather it is "devoted to automated reasoning about the physical world using qualitative representations" [De Kleer and Brown 84]. Reasoning about physical behavior is achieved by at least two approaches, as discussed in Section 1.3. One approach is to determine the effect of changes in the parameters of a system. In the context of structural engineering, qualitative reasoning would determine the change in the behavior of a structure caused by variations in material properties, external loads, or dimensions of the structural components. The second approach corresponds to the derivation of behavior from the description of the problem, without considering variation of the parameters. The established frameworks described in this chapter determine the influence of changes in the parameters of a system. The understanding of physical behavior is restricted to the explanation of causal relations such as, "an increment in the stiffness of spring_1 causes an increment in its internal load."

Four qualitative reasoning frameworks developed by researchers in artificial intelligence for reasoning about physical systems are:

(1) The component centered framework provides component states as the fundamental modeling tools to represent physical laws. A component has a specified number of states that indirectly represent the fundamental principles of the domain.
(2) The process centered framework uses processes as the fundamental tool to represent physical laws. Changes in the states of the components are caused by active processes among the components.
(3) The constraint centered framework represents physical laws directly as constraints which are expressed in qualitative or non-numeric form.
(4) The unifying framework represents physical laws as component states as well as processes. Changes in the model state are caused by components and by active processes between components.

The next section discusses theoretical aspects of qualitative reasoning frameworks and compares quantitative and qualitative models of a device. Section 2.3 explains the aforementioned frameworks in detail.

## 2.2.- THEORETICAL ASPECTS OF QUALITATIVE REASONING

An early objective of qualitative reasoning research was to provide non-numeric representations for obvious or commonsense knowledge. Much of the initial work deals with how to represent, at the appropriate level of abstraction, apparently simple causal relations between quantities. Parameters were abstracted to intervals and the calculus was defined by simple rules such as, the addition of two positive numbers is a positive number. Qualitative reasoning was later extended to represent physical problems, but ambiguity in the calculus produces solutions that do not satisfy the laws of the domain. Inconsistent solutions have been studied by several researchers [Struss 90a, Struss 90b, Williams 90, Lee and Kuipers 90, Dormoy and

Raiman 90, Bredeweg et al. 90]. This section presents concepts and techniques useful for understanding and preventing this undesirable behavior.

### 2.2.1.- Qualitative and quantitative models

A quantitative model is defined as a tuple $(D_{eqs}, R, +, x)$, where $D_{eqs}$ is a set of algebraic or differential equations, R is a set of numbers, which is usually the infinite set of the real numbers, and "+, x" are addition and multiplication operators (subtraction and division are defined from these). By providing precise values for the input parameters, a quantitative simulation derives a unique and precise set of output parameters.

A qualitative model is also defined as the tuple $(Q_{eqs}, Q_{sp}, \oplus, \otimes)$, where $Q_{eqs}$ is a set of qualitative equations, $Q_{sp}$ is a small set of intervals and points called a quantity space, such as {negative, zero, positive}, and "⊕, ⊗" are addition and multiplication operations defined for qualitative quantities. Qualitative models represent relations between parameters, so the traditional distinction between input and output parameters does not exist in qualitative models. By providing an imprecise and possibly incomplete knowledge of parameters, a qualitative simulation provides a range of behaviors.

Consider Fig. 2.2, a variant from the diagram presented by Struss [90b]. In this diagram $D_{eqs}$ and $Q_{eqs}$ are quantitative and qualitative equations, respectively. The mapping pd transforms the quantitative equations $D_{eqs}$ to the qualitative equations $Q_{eqs}$. The symbol $Sim_{quan}$ represents procedures, such as matrix inversion, which map a set of equations to their unique solution, $Sol_{quant}$. The symbol $Sim_{qual}$ represents searching procedures which map a set of qualitative equations to their solution space, $Sol_{qual}$. The mapping p transforms a quantitative solution to a qualitative solution, and the mapping q transforms a qualitative solution to a quantitative solution. For example a mapping p, from the set of real numbers to the quantity space used by the component centered framework is:

mapping p: R $\rightarrow$ S where
$\forall x \in R$,
[x] = positive if $x > 0$
[x] = zero if $x = 0$
[x] = negative if $x < 0$

A similar mapping q transforms a qualitative solution into the quantitative intervals (-∞,zero), [zero,zero], and (zero, +∞).

Two definitions about completeness and soundness based on Fig. 2.2 are [Struss 90b]:

(1) *Completeness:* A qualitative reasoning framework is complete if the mapping p exists for all quantitative solutions in $Sol_{quant}$. This definition implies the coverage of all the possible physical behaviors by the qualitative reasoning method.

(2) *Soundness:* A qualitative reasoning framework is sound if the mapping q exists for all the qualitative solutions in the set $Sol_{qual}$.

> Soundness is a stronger condition than completeness. It implies the elimination of all qualitative solutions which do not correspond to any quantitative model.

To illustrate these concepts consider the extended addition operation presented in Table 2.1. Since this operation is weaker than the quantitative addition operation, any numerical solution that satisfies the quantitative addition operation must satisfy the qualitative addition operation. The extended addition operation is complete because there is no numerical solution that would not satisfy the extended addition operation. The converse of this statement is not true, and it is easy to find real numbers that satisfy the extended addition table but not the classical addition operation. For example, the equation 3 + 5 = 9, obviously does not satisfy the quantitative addition, but the analogous qualitative operation, `[positive]⊕[positive]≅ [positive]`, is correct.

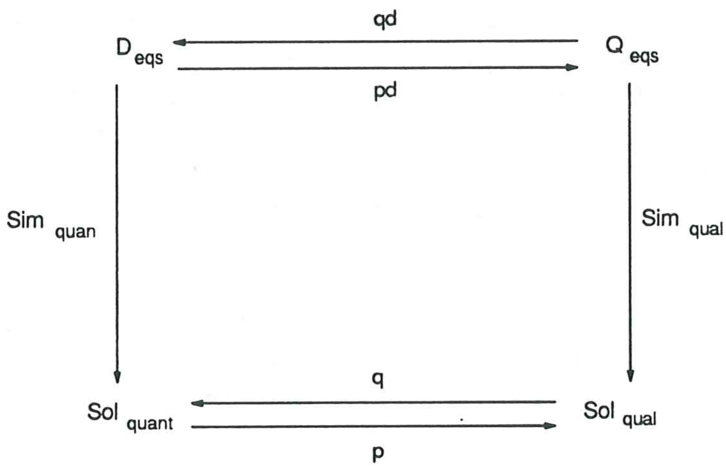



Figure 2.2 Qualitative and quantitative representations of a model.

Table 2.1 Extended addition operation

<table>
<tr><td colspan="2" rowspan="2">R≅X⊕Y</td><td colspan="3">X</td></tr>
<tr><td>−</td><td>0</td><td>+</td></tr>
<tr><td rowspan="3">Y</td><td>-</td><td>−</td><td>−</td><td>+, |X|>|Y|<br>0, |X|=|Y|<br>−, |X|<|Y|</td></tr>
<tr><td>0</td><td>−</td><td>0</td><td>+</td></tr>
<tr><td>+</td><td>+, |X|<|Y|<br>0, |X|=|Y|<br>−, |X|>|Y|</td><td>+</td><td>+</td></tr>
</table>

The extended addition operation is sound because it is possible to map each of the thirteen possible results from Table 2.1 into at least one quantitative operation. For example, the addition of a positive and a negative number, with the positive number greater in absolute value than the negative number, is a positive number. This result is mapped into the quantitative operation, 5 - 3 = 2, where |5| > |3|. The extended addition operation is both sound and complete.

### 2.2.2.- Inconsistent solutions

Qualitative solutions that do not satisfy the fundamental laws of the domain are termed inconsistent. There are only two causes for this behavior [Bredeweg et al. 90]: a lack of proper calculus, or incomplete information about the model. This chapter considers only the first cause of inconsistent behavior. The second cause occurs when the model is not properly described. For example, if the geometry constraints are not represented, the simulation can infer inconsistent solutions.

Methods to avoid, or at least to reduce, the prediction of inconsistent solutions are the subject of considerable research in qualitative reasoning, and various approaches have been proposed. A mixed algebra between the one for the real numbers and the one for qualitative quantities is one alternative [Williams 90]. The mixed algebra differs from standard qualitative algebra because equations may contain qualitative and quantitative operators at the same time. An equation is evaluated first by using the quantitative algebra and then by using the qualitative algebra. The mixed algebra reduces inconsistent solutions caused by the mapping of real values into qualitative values. For example, if the length of a structural component is two meters, the algebra does not map the length into the interval `positive` but it maintains the length as a real number.

Struss [90a, 90b] claims that inconsistent solutions are caused by the application of fundamental principles only at connections between components and he proposes global filters to eliminate inconsistent solutions. For example, for the qualitative simulation of a single degree-of-freedom vibrating mass, he proposes the incorporation of knowledge about the phase portrait. The phase portrait has inconsistent patterns, such as its intersection or bifurcation, and inconsistent solutions can be eliminated using this knowledge. A contribution of Struss' work is that it focused attention into the causes of inconsistent solutions and the necessity of global laws. Unfortunately, a general procedure for the derivation of global laws has not been developed.

An approach for generating global laws, in the context of component centered framework, was developed by Dormoy and Raiman [90]. They propose a qualitative Gauss elimination rule that reduces inconsistent solutions by deriving a global equation from two qualitative equations. According to the rule if x, y, z, a, b are qualitative quantities such as:

$$x \oplus y \cong a$$
$$-x \oplus z \cong b$$

and if the qualitative value of x is known, then a new global law is: $y \oplus z \cong a \oplus b$. The qualitative equations may come from different quantitative equations such as,

$$xL^2 + y = a, \quad -xL + z/H = b, \quad L,H>0 \qquad (1)$$

or

$$x + y = a, \qquad -x + z = b \qquad (2)$$

because under the mapping pd from Fig. 2.2, equations (1) or (2) are transformed into exactly the same qualitative equations. The qualitative Gauss rule is applicable if two conditions are satisfied. The sign of the quantity in common, x, must be known, otherwise it is not possible to know if the equations should be added or subtracted to eliminate x. The second condition is that the equations cannot contain more than one variable in common.

In the context of the process centered framework, an approach by D'Ambrosio [89] uses linguistic variables such as {SMALL,MEDIUM,LARGE}. The linguistic variables, represented by fuzzy sets, reduce the ambiguity caused by the addition of parameter with opposite signs. The approach is suitable for equations relating parameters with different orders of magnitude, such as the stiffness of a shear wall and a frame.

Finally, a more general approach which can eliminate inconsistent solutions is constant elimination. Constant elimination is not applied to the qualitative equations $Q_{eqs}$ but to the equations $D_{eqs}$ and, as with symbolic manipulation, constant elimination derives new equations by simplifying common terms [Simmons 90]. As with symbolic manipulation, the technique may generate equations that are not useful. To increase efficiency, constant elimination has been restricted to the manipulation of linear equations [Bredeweg et al. 90]. The approach is discussed in more detail in Section 4.4.3.

## 2.3.- GENERAL QUALITATIVE REASONING FRAMEWORKS

This section presents the four qualitative reasoning frameworks that have been developed by researchers. These frameworks are primarily intended for reasoning about systems that change over time. Since the geometry representation is simple, the frameworks are cumbersome to apply except for one-dimensional models. An example discussed in the chapter is a structure consisting of three springs in series and parallel. The geometry is simple, as with most examples reported in the qualitative reasoning literature.

Qualitative reasoning is defined by four concepts [Bredeweg et al. 90]:

(1) *Quantity space* is an abstraction of the values for a parameter into a small set of values and intervals that are relevant to the problem. For example, a bending moment parameter may be mapped from the infinite set of real numbers to the discrete set {negative, zero, positive}.

(2) *Qualitative calculus* is the set of rules for operating on qualitative values. These rules are analogous to the operations of addition, subtraction, multiplication, and division for real numbers.

(3) *Modeling primitives* are the fundamental tools provided by the reasoning framework to represent first principles. They are typically components, processes, views, or constraints. The primitives are a formal mechanism to describe fundamental principles such as equilibrium, compatibility, and force-deformation for structural evaluation.

(4) *Inference scheme* is the procedure that combines states of the components.

The four frameworks are discussed in terms of these basic concepts.

### 2.3.1.- The component centered framework

The component centered framework was proposed by De Kleer and Brown [84]. Physical behavior is modeled by the qualitative states of individual components, which are related by connections between the components. The qualitative states define the behavior of a component, and the first principles are indirectly represented by the valid states for the components.

Quantity space

The component centered approach uses the quantity space `{negative, zero, positive}`, which is common in most qualitative reasoning frameworks.

Qualitative calculus

This framework uses a very simple calculus based on the addition operation in Table 2.2. For example, the addition of two negative parameters, such as bending moments at a connection, results in a sum with a negative value. The addition of parameters with opposite signs results in an ambiguous result. Excluding the cases when the operands are zero, half of the operations produce ambiguous results. Other operations such as multiplication, division, and subtraction are defined similarly.

Table 2.2 Standard addition operation

| $R \equiv X \oplus Y$ | | X | | |
|---|---|---|---|---|
| | | – | 0 | + |
| Y | - | – | – | ? |
| | 0 | – | 0 | + |
| | + | ? | + | + |

Modeling primitives

The modeling primitives of the framework are components with a number of states that describe their behavior. The laws for components are represented by confluences, which are qualitative versions of differential equations. A confluence relates the sign of the derivatives of parameters with respect to time. For example, equilibrium between the two forces at the connections of a spring implies that the forces and their derivative with respect to time are equal and opposite. Following the component framework notation, the confluence for the equilibrium equation is $[\partial F_1] \equiv [\partial F_2]$ meaning that the sign of the increment of $F_1$ is always equal to the sign of the increment of $F_2$.

The force-deformation relationship for an elastic softening or hardening material can be represented as a monotonic relation between force and deformation. Expressed as a confluence this principle is $[\partial F_1] \equiv [\partial D]$, meaning that the sign of the increment of the internal force $F_1$ is equal to the sign of the increment in deformation $D$. The confluence is valid for an elastic softening or hardening materials. As a summary, the quantitative equations and confluences for a spring are:

| Equilibrium | Equilibrium confluence |
|---|---|
| $F_1 = F_2$ or | |
| $dF_1/dt = dF_2/dt$ | $[\partial F_1] \cong [\partial F_2]$ |

| Force-deformation | Force-deformation confluence |
|---|---|
| monotonic characteristics | $[\partial F_1] \cong [\partial D]$ |

For a quantity space {`negative, zero, positive`} the component spring has nine states, three for each of the following cases: The component is in compression, tension, or unloaded, and for each case there is a positive, negative, or null increment in the internal force and deformation.

The component states should follow the so-called "no function in structure" principle which says that the states of a component are independent of their context. In other words the components should behave independently of their surroundings or the way in which they are interconnected. As De Kleer and Brown [84] postulate, "the laws of the parts of the devices may not presume the functioning of the whole." For the spring example, if the states associated with a null force are eliminated, the resulting description violates the no function in structure principle because it presumes that a zero force is not a valid state. There are situations, however, in which a spring can have a zero force.

Inference scheme

The inference scheme, named envisionment, follows three steps to derive qualitative solutions:

(1) Combine each component state with another component state and generate $m^n$ initial solutions, where m is the number of states per component and n is the number of components. Each initial solution has a set of confluence values corresponding to the states of the components.
(2) Test each initial solution and eliminate those that have contradictory confluences. In the context of structural engineering, this eliminates solutions that do not satisfy fundamental principles of equilibrium and compatibility at the connections. Each parameter in the model is assigned a qualitative value.
(3) Identify the possible transitions in time between states and if the transition does not correspond to a termination rule, then return to the first step. Transition rules indicate how the system changes over time, as for example when a component reaches a yielding point and it is not able to transfer further load. Another example of a termination rule is when a structure develops a collapse mechanism and it cannot resist more load. Transitions and termination rules are illustrated for the constraint centered framework.

To illustrate the first and second steps in a structural engineering context, consider three springs in the structure shown in Fig. 2.3. As previously indicated, a spring with a monotonic force-deformation characteristic has nine states. To simplify the example, only consider the case when the internal force is null, so the component has three states: increasing in tension, increasing in compression, or constant null force. The connections are modeled as components but the example only includes solutions corresponding to the combination of the spring states. The initial combination generates twenty seven solutions ($3^3 = 27$). The confluences corresponding to equilibrium and compatibility at the connections eliminate inconsistent solutions reducing the set to only one qualitative solution.

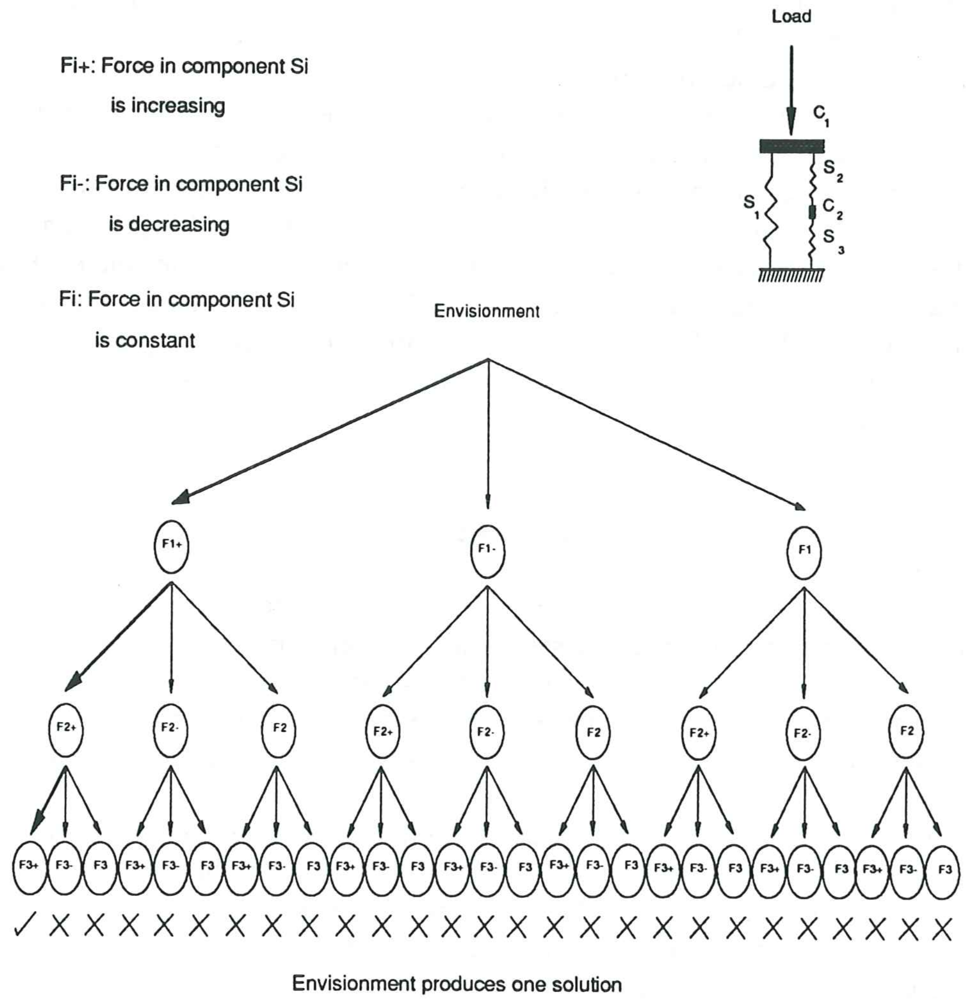



Figure 2.3 Inference tree for the component centered framework.

To illustrate the second step in the inference scheme consider the set of confluences generated by solution two, $\{\partial F_1+, \partial F_2+, \partial F_3-, \partial D_1+, \partial D_2+, \partial D_3-\}$, in Fig. 2.3. This contradicts the equilibrium confluence at connection $C_2$, $\partial F_2 \equiv \partial F_3$ because the increment of $F_2$ is positive but the increment of $F_3$ is negative. In a similar manner, twenty-five sets are eliminated, except for the first set, which is the only correct solution. The example illustrates a deficiency of the inference scheme: most of the solutions initially generated by envisionment are inconsistent and consequently the inference scheme is inefficient. The valid solution indicates that an increment in the external load causes an increment in the displacements and internal forces for all the springs. Since the structure is simple, neither inconsistent nor ambiguous solutions are predicted.

Critique of the component centered framework

The equilibrium of a set of components attached to a connection is represented by the qualitative states of the component modeling the connection. The number of components attached to the connection can vary, so a connection is defined (1) by a set of component connections, one for each two, three, four and so on, components attached to the connection, or (2) by an abstract component connection with an arbitrary number of structural components attached to it. The second alternative represents equilibrium at a connection by an abstract component, but equilibrium is better modeled as a process acting between components.

The framework can represent problems with geometry more complex than one-dimensional. A plane frame structure can be represented by horizontal frame member components, vertical frame components, and inclined frame member components. This is a poor representation, however, because each orientation requires a separate frame member component, but in reality they are the same type of component with different orientations.

The component centered approach was one of the first frameworks proposed and it was probably not intended for large problems. Envisionment is an inefficient algorithm because the combination of m components with n valid states produces a very large number of sets. A simple structure with five springs has 243 ($3^5$) initial solutions.

The inference scheme complicates the automated explanation of behavior because it does not keep track of the causal relations between parameters. The causal relations must be derived by a postprocessing of the solution [Bredeweg et al. 90].

### 2.3.2.- The process centered framework

The process centered framework, proposed by Forbus [84], represents first principles explicitly as primitives that induce changes in the qualitative states of the model. Compared with the component centered framework, the qualitative calculus reduces ambiguity and the inference scheme is more efficient.

Quantity space

The quantity space is defined by the user with the restriction that the value ZERO must be included, because ZERO differentiates the sign of a parameter. A quantity is defined by (1) the magnitude and its relation with other quantities in the model, (2) the sign, (3) the magnitude of its derivative, and (4) the sign of its derivative. Consider for example the cantilever beam shown in Fig. 2.4. The quantity space is defined as {-Mp,negative,zero,positive,Mp} where Mp is the plastic moment capacity of the beam. A relation in magnitude between the bending moment parameters is |My3|>|My2|.

Qualitative calculus

The relations between magnitudes of parameters reduce the ambiguity of qualitative calculus compared with the component framework. The framework uses the extended addition operation in Table 2.1, which avoids the ambiguity caused by the addition of parameters with opposite sign. The addition of two qualitative parameters has thirteen possible results. The sum depends on the sign of the parameters, and if they have opposite signs it also depends of the relationship between their magnitudes. For example, the addition of two parameters a,b where a is positive and b is negative, is positive, zero, or negative, according to the relations, $|a|>|b|$, $|a|=|b|$, or $|a|<|b|$, respectively. The extended addition operation incorporates parameters relations during the search for solutions, according to the new equations and the signs of the parameters. For example for a,b,c positive, the equation $a \oplus b \equiv c$, incorporates the parameter relations $|c|>|b|$ and $|c|>|a|$.

The calculus includes transitivity rules between relations to reduce ambiguity. The transitivity rules are:

$$A > B \text{ and } B > C \Rightarrow A > C$$
$$A = B \text{ and } B = C \Rightarrow A = C$$
$$A > B \text{ and } B = C \Rightarrow A > C$$

Consider for example the following set of relations:

$$\{|a|>|b|, |d|=|f|, |b|>|g|, |a|>|g|\}.$$

The sum of two parameters, [a] = positive, [b] = negative, such as $a \oplus b \equiv c$, is positive c because $|a|>|b|$. On the other hand, if a and b have the same sign, the magnitude of c is greater than the magnitude for a and b. Applying the transitivity rules to this second case, the new relationships are:
$\{|c|>|a|, |c|>|b|, |c|>|g|\}$.

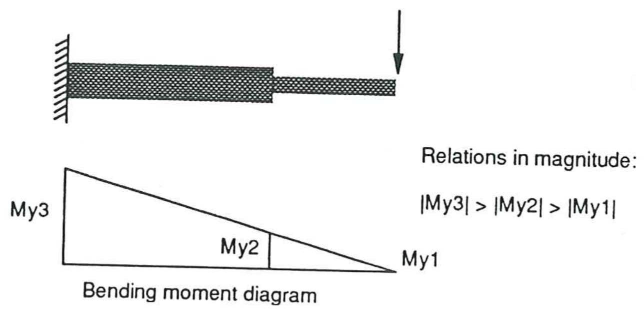



Figure 2.4 Quantity space and relations in magnitude for bending moment parameters.

Modeling primitives

The two main modeling primitives in the framework are views and processes. A view is a static description of an object or its behavior. A view is separated into four parts: (1) a set of individuals which are objects that must exist for the view to be applicable; (2) a set of quantity conditions which are relations between quantities that must hold in order for the view to be applicable; (3) a set of preconditions which are relations not included in the quantity conditions that must also hold for the view to be applicable; and (4) a set of relations or statements that hold because the view is applicable. Figure 2.5 illustrates a view instance for equilibrium of a horizontal frame member component without member loads. The preconditions indicate that the bar is oriented along the X axis. The relations for equilibrium are qualitative, so the length, which is always positive, does not appear in the relations for moment equilibrium.

`Instances:`
bar, `bar_1`

`Preconditions:`
Orientation of `bar_1` along the global X axis

`Quantity_conditions:`
No quantity conditions for this view

`Relations:`

$F^1_x \oplus F^2_x \equiv 0$

$F^1_z \oplus F^2_z \equiv 0$

$M^1_y \oplus M^2_y \oplus F^1_z \equiv 0$

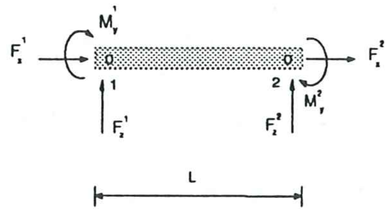


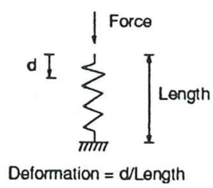

Figure 2.5 Representation for equilibrium of a bar using a view.

Consider next the view for an elastic object, taken from Forbus [84], page 140, in Fig. 2.6. This is a one-dimensional problem and therefore its representation is relatively simple. The internal force is directly proportional to the `Deformation` because an increment in `Deformation` causes an increment in the internal force of the spring. The view is similar to the spring qualitative states in the component centered framework, although the view represents the force-deformation law explicitly. In the component framework the qualitative states represent the laws of equilibrium, compatibility and force-deformation. Forbus [84] on page 141, defines four additional processes, `Relaxing-Minus`, `Relaxing-Plus`, `Stretching`, and `Compressing` to complete the representation of the behavior for a spring component.

`Individuals:`
Object B

`Preconditions:`
Made of an elastic material

`Quantity_conditions:`
No quantity conditions for this view




`Relations:`
Object B has a Length
Object B has a rest length
Object B has an internal force
The rest length does not change in time
The internal force is directly proportional to the Deformation
The internal force is `ZERO` when Length equals the rest length

Figure 2.6 View representation of an elastic object.

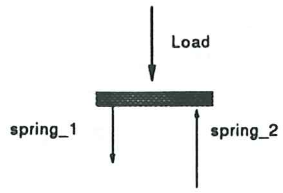
The second primitive in the framework is a process which represents the changes in the states of the objects defined in the model. A process has: (1) `instances`, (2) `quantity conditions`, (3) `preconditions`, (4) `relations`, and (5) the `influences` of the process. Influences are the agents or causes that change the qualitative values of a parameter. The influences between two parameters A and B are derived by evaluating the changes in parameter B caused by an increment in parameter A with the other parameters held constant. To illustrate a process consider the equilibrium of a connection between two springs and an external load. With a quantity space `{negative, zero, positive}` there are thirteen states for the connection. The thirteen states are selected by considering the twenty-seven combinations for the three components with three states ($3^3$) and eliminating combinations that violate the equilibrium equation $F_1 \oplus F_2 \oplus Load \cong 0$. According to the framework it is necessary to define thirteen processes, one for each state of the connection. Figure 2.7 shows one process for the connection state with the load and one internal force acting downwards and the other internal force acting upwards. Taking into account the signs, the equilibrium equation at the connection is: `|Load|+|Spring_1|=|Spring_2|`. If the `Load` increases but the force `Spring_2` remains constant, the force `Spring_1` must decrease. Similarly, if the `Load` increases and the force `Spring_1` remains constant, the force `Spring_2` must increase.

<u>Inference scheme</u>

The inference scheme has four stages:

(1) An elaboration stage retrieves the processes and views for the model. In a structural engineering domain, the elaboration retrieves the processes of equilibrium and compatibility for the connections and components in the model.
(2) A second stage recovers the parameters and influences that apply to the processes and views retrieved by the elaboration stage. The preconditions between primitives may be contradictory and in those cases this stage divides the processes into mutually exclusive processes.
(3) An influence resolution stage determines the sign of the derivative for the parameters. If there are influences acting in opposite directions without information about their relative magnitude, this stage generates three solutions: the parameter increases, decreases, or is constant.
(4) From the previous model state, transitions and termination rules and the parameters influences, a propagation stage generates a new model state. An example of a transition rule is: "if the bending moment for a bar is positive and increasing, then the bending moment reaches the positive yielding capacity and the bending moment cannot increase further." An example of a termination rule is the formation of a collapse mechanism in a structure.

Processes are followed in a depth-first procedure because the second stage of the inference scheme retrieves parameter values and influences that are consistent with known influences and qualitative values included in the description [Bredeweg et al. 90]. The depth-first procedure improves the inference efficiency compared with the component framework because it reduces the inconsistent solutions generated by the initial combination of states in the component centered framework. The depth-first procedure also simplifies the explanation of physical behavior because the inference scheme follows processes in the same causal order as they appear.




`Instances:`
A connection.
A load.
Two spring components, `spring_1` and `spring_2`.

`Preconditions:`
The load is applied at the connection.
The springs are attached to the connection.

`Quantity_conditions:`
The sign of the load is positive.
The sign of the internal force for `spring_1` is positive.
The sign of the internal force for `spring_2` is negative.
The derivative of the load is positive.

`Relations:`
No relations are required for this process.

`Influences:`
An increment in the external load causes a decrease in the internal force of `spring_1`
An increment in the external load causes an increase in the internal force of `spring_2`

Figure 2.7 Process representation for equilibrium at one connection state.

To illustrate the second and third stages of the inference scheme, consider the structure in Fig. 2.8. The second stage begins by selecting an active process, such as `Compressing` for spring $S_1$, which in Fig. 2.8 corresponds to the node $\partial F_1+$ in the graph. If $S_1$ is in compression, by equilibrium and compatibility at $C_1$, the state for spring $S_2$ corresponds to the processes of `Compressing`, `Stretching`, or `Relaxing`. Forbus defines two processes, `Relaxing-plus` and `Relaxing-minus`, although the example considers one process `Relaxing`. The inference takes the first process for spring $S_2$ and selects the process `Compressing` for spring $S_3$ because of equilibrium at connection $C_2$. The other two processes for spring $S_2$ violate equilibrium at connection $C_2$ and compatibility at connection $C_1$. For example, the solution which indicates that spring $S_1$ is in compression while spring $S_2$ is in tension is not correct because the equilibrium at $C_2$ establishes that $S_3$ is in tension. Therefore the view `Elastic_object` implies that the deformations of springs $S_2$ and $S_3$ are negative. Consequently, the total deformation of springs $S_2$ and $S_3$ is negative and opposite to the deformation of spring $S_1$, violating compatibility at connection $C_1$.

The third stage of the inference scheme assigns qualitative values to each of the parameters. Assuming an unloaded initial state, this step assigns a positive value to each of the forces and displacements parameters because all quantities are increasing.

Critique of the process centered framework

The extended addition table and the transitivity relations define the qualitative calculus, but they do not guarantee that all the relations for a model are consistent. As a consequence, the framework can predict solutions that do not satisfy the laws of the domain. To illustrate this shortcoming, consider the equilibrium laws at the connections of the two bay frame shown in Fig. 2.9. The horizontal forces in each member and connections are in qualitative equilibrium because their addition using Table 2.1 is zero. This is, however, an inconsistent solution because the relations between parameters are contradictory. The parameter relations derived at each connection are presented in Table 2.3. The transitivity rules derive the following parameter relations,

$$\{|F_2|>|F_1|,\ |F_2|>|V_1|,\ |F_4|>|F_5|,\ |F_4|>|L|,\ |F_4|>|F_1|,\ |F_4|>|V_1|,\ |F_7|>|F_5|,\ |F_7|>|L|,\ |F_7|>|F_1|,\ |F_7|>|V_1|,\ \mathbf{|F_3|>|F_4|},\ |F_7|>|F_2|,\ |F_7|>|F_6|\}.$$

However, the relation $|F_3|>|F_4|$ is not correct, as shown in Fig. 2.9(b), because it contradicts the relation $|F_2|>|F_1|$. An algebraic simplifier called constant elimination derives equations such as in Fig. 2.9(b), which can eliminate this type of ambiguity. Constant elimination is presented in Section 4.4.3.

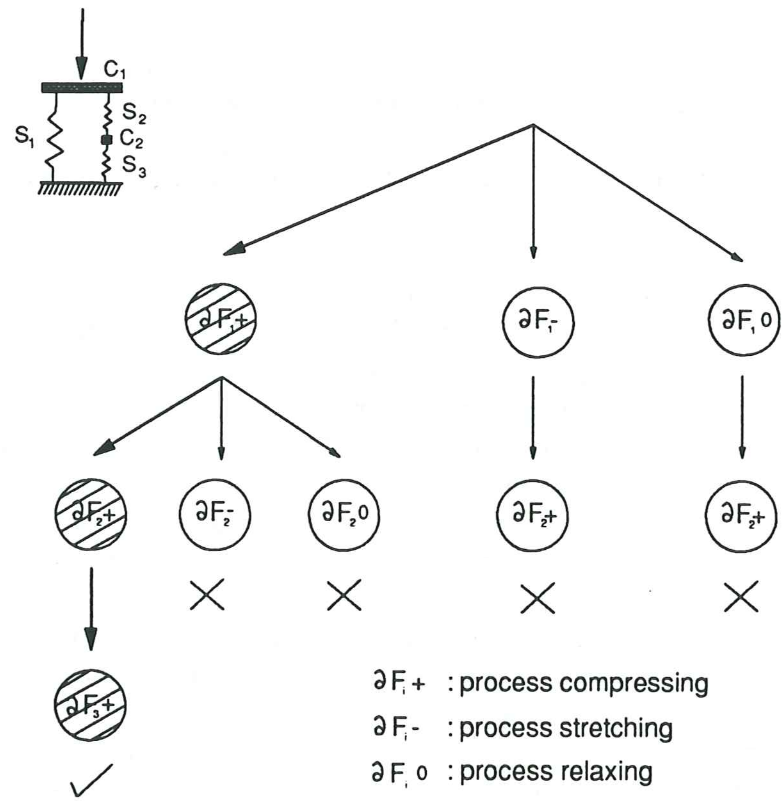



Figure 2.8 Second stage of the inference scheme for the process framework.

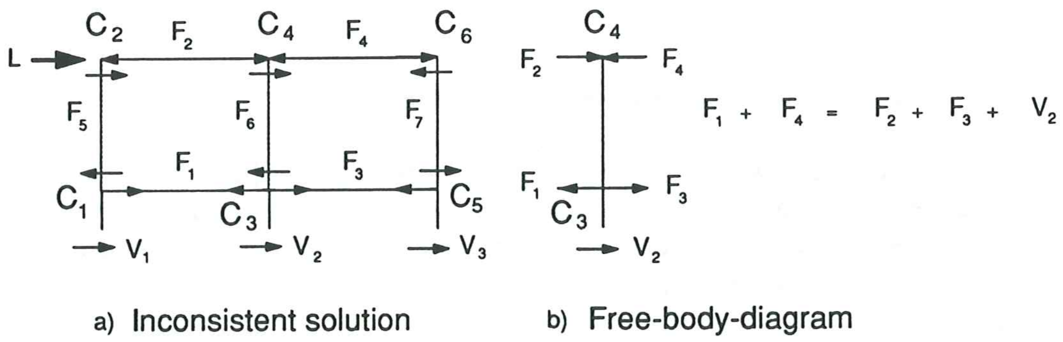



Figure 2.9 (a) Free-body-diagram showing an inconsistent solution; (b) Free-body-diagram showing an equilibrium equation that needs to be included in the reasoning.

Table 2.3 Parameter relations derived at connections for Figure 2.9(a)

| $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|
| $\lvert F_5\rvert > \lvert F_1\rvert$ | $\lvert F_2\rvert > \lvert F_5\rvert$ | | $\lvert F_4\rvert > \lvert F_2\rvert$ | $\lvert F_3\rvert > \lvert F_7\rvert$ | $\lvert F_4\rvert = \lvert F_7\rvert$ |
| $\lvert F_5\rvert > \lvert V_1\rvert$ | $\lvert F_2\rvert > \lvert L\rvert$ | | $\lvert F_4\rvert > \lvert F_6\rvert$ | $\lvert F_3\rvert > \lvert V_3\rvert$ | |

The inconsistency means that the intuitive concept "if all the components and all the connection in a structure are in equilibrium then any free body diagram must be also in equilibrium" does not hold for qualitative equilibrium. This observation suggests that either additional free body diagrams, besides components and connections, should be included or additional relations between parameters besides binary relations[1] should be incorporated.

The modeling primitives provided by the framework are cumbersome to apply in structural engineering because of the spatial and geometric relations. One view is required for each orientation of a member because there are no provisions for coordinate transformations. It is possible to define one static view for each possible transformation, but the number of views would be very large. It is cumbersome to model equilibrium and compatibility processes because the scope of the modeling primitives is limited. As described earlier, thirteen processes are necessary for a simple equilibrium relation. The modeling is even more complex for three-dimensional frame members. In structural engineering, as well as in many other problems in applied physics, a model is typically made up of several components of the same class, such as frame members, each with different locations in space. In the example of the equilibrium for the spring structure, if a third spring is added to the connection, a new large set of processes is required for the similar structure.

The process centered framework is not suitable for structural engineering, but the central idea is appealing: changes in the state of a model are caused by active processes. This has the advantage over the component centered framework of

[1] A binary constraint is defined between two parameters such as a > b, or a = b. Other types of constraints are a = b + c, or a = f * g.

eliminating the incorporation of qualitative states for abstract connection components and directly describing the fundamental laws as processes acting on components.

### 2.3.3.- The constraint centered framework

This framework was proposed by Kuipers [84, 86] to model physical behavior as a set of qualitative relations. The framework does not provide for the definition of processes, views, components, or other modeling primitives and the user specifies the constraints and the model. The constraints directly represent the fundamental laws. Geometry is not explicitly represented as with the other frameworks, so the application of the constraint centered framework for multi-dimensional problems is cumbersome.

<u>Quantity space</u>

The quantity space is selected by the user. For example the quantity space for a bending moment may be defined as `{-Mp, negative, zero, positive, Mp}` where `Mp` is the plastic moment capacity.

<u>Qualitative calculus</u>

The qualitative calculus is similar to the calculus for the component centered approach, such as the addition operation on Table 2.2. The framework does not define parameter relations, so the addition of parameters with opposite signs gives an ambiguous sum. The framework uses global filters that apply constraints to the solution as a whole to reduce ambiguity. The global filters are domain specific constraints, as for example the phase portrait for a vibrating mass on a spring [Lee and Kuipers 90]. The phase portrait has features, such as trajectories do not intersect and a trajectory does not bifurcate, which are used as global filters. Kuipers [86] proves that the constraint framework is complete. It is not sound, however, as is the case for the other qualitative reasoning frameworks.

<u>Modeling primitives</u>

This framework represents first principles as qualitative constraints and it does not define modeling primitives. Qualitative constraints are obtained from the governing equations [Kuipers 84]. For example, equilibrium, compatibility, and force-deformation laws for the springs in Fig. 2.3 are:

| <u>Equilibrium</u> | <u>Compatibility</u> | <u>Force-deformation</u> |
|---|---|---|
| $F_1 + F_2 = -L$ | $D_1 = D_2 + D_3$ | Elastic material |
| $F_2 = F_3$ | | |

Following the conventions of the program *QSIM* (a program that implements the constraint centered framework), the equations are transformed into a set of qualitative constraints such as:

| <u>Equilibrium</u> | <u>Compatibility</u> | <u>Force-deformation</u> |
|---|---|---|
| add($F_1$, $F_2$, -L) | add($D_2$, $D_3$, $D_1$) | m+($F_1$, $D_1$) |
| equal($F_2$, $F_3$) | | m+($F_2$, $D_2$) |
| | | m+($F_3$, $D_3$) |

Where `equal` is a constraint that indicates equality between parameters and `add` is a constraint that indicates that the addition of two parameter is equal to a third parameter. The constraint `m+` indicates a monotonic relation between a force and a deformation. The monotonic constraint does not indicate whether the material is elastic softening or hardening, so a range of material behaviors is represented by the constraint.

Inference scheme

The inference scheme propagates the values through the set of constraints. The algorithm has six stages:

(1) For all the parameters in the model, specify the initial qualitative values and their derivatives with respect to time.
(2) For each parameter select possible transitions to new qualitative values. For example, a bending moment may be increasing, so its next qualitative value would be either to continue increasing or remain constant at the plastic moment.
(3) For each constraint combine the new qualitative values for the parameters in the constraint and generate a set of tuples (pairs or triples). Filter each tuple for consistency with the constraint.
(4) Perform piecewise consistency checking of the tuples derived in the previous stage. For example, if a constraint indicates that an axial force is increasing, then another constraint on the same axial force should also indicate an increase.
(5) From the remaining tuples generate all possible tentative global solutions by assigning a qualitative value for each parameter and its derivative.
(6) Apply global filtering to each tentative global solution and for each of the remaining states go back to the second stage.

To illustrate the inference scheme, consider the spring structure in Fig. 2.3. The steps in the inference scheme are:

(1) The specified description corresponds to an unloaded structure and an increment in the external load:

$\text{qualitative\_state}(F_1, t_0) = \langle 0, \text{steady} \rangle$
$\text{qualitative\_state}(F_2, t_0) = \langle 0, \text{steady} \rangle$
$\text{qualitative\_state}(F_3, t_0) = \langle 0, \text{steady} \rangle$
$\text{qualitative\_state}(L, t_0) = \langle 0, \text{increasing} \rangle$
$\text{qualitative\_state}(D_1, t_0) = \langle 0, \text{steady} \rangle$
$\text{qualitative\_state}(D_2, t_0) = \langle 0, \text{steady} \rangle$
$\text{qualitative\_state}(D_3, t_0) = \langle 0, \text{steady} \rangle$

(2) For each parameter $F_1, F_2, F_3, D_1, D_2, D_3, L$ select the new qualitative state. In this case all the parameters except L, may increase, decrease or remain steady, such as:

$F_1$ $P_1$: $\langle 0, \text{steady} \rangle \Rightarrow \langle 0, \text{steady} \rangle$
$P_2$: $\langle 0, \text{steady} \rangle \Rightarrow \langle (0, \infty), \text{increasing} \rangle$
$P_3$: $\langle 0, \text{steady} \rangle \Rightarrow \langle (-\infty, 0), \text{decreasing} \rangle$

If an axial force were positive and increasing over time, a new state in the model would be when the axial load reaches the compression yielding point which is represented as a transition from an interval to a point .

(3) For each constraint generate a set of tuples and apply consistency filtering. For simplicity only consistent tuples are listed below, so tuples such as $(P_1, P_2)$ or $(P_3, P_2)$ corresponding to the constraint "equal" are not shown:

| add($F_1$,$F_2$,-L) | equal($F_2$,$F_3$) | add($D_2$,$D_3$,$D_1$) | m+($F_1$,$D_1$) | m+($F_2$,$D_2$) | m+($F_3$,$D_3$) |
|---|---|---|---|---|---|
| $(P_1,P_2,P_2)$ | $(P_1,P_1)$ | $(P_1,P_1,P_1)$ | $(P_1,P_1)$ | $(P_1,P_1)$ | $(P_1,P_1)$ |
| $(P_2,P_1,P_2)$ | $(P_2,P_2)$ | $(P_1,P_2,P_2)$ | $(P_2,P_2)$ | $(P_2,P_2)$ | $(P_2,P_2)$ |
| $(P_2,P_2,P_2)$ | $(P_3,P_3)$ | $(P_1,P_3,P_3)$ | $(P_3,P_3)$ | $(P_3,P_3)$ | $(P_3,P_3)$ |
| $(P_2,P_3,P_2)$ | | $(P_2,P_1,P_2)$ | | | |
| $(P_3,P_2,P_2)$ | | $(P_2,P_2,P_2)$ | | | |
| | | $(P_2,P_3,P_1)$ | | | |
| | | $(P_2,P_3,P_2)$ | | | |
| | | $(P_2,P_3,P_3)$ | | | |
| | | $(P_3,P_1,P_3)$ | | | |
| | | $(P_3,P_2,P_1)$ | | | |
| | | $(P_3,P_2,P_2)$ | | | |
| | | $(P_3,P_2,P_3)$ | | | |
| | | $(P_3,P_3,P_3)$ | | | |

(4) Apply piecewise filtering and eliminate mutually inconsistent tuples. For example, consistency between the constraints equal($F_2$,$F_3$), m+($F_2$,$D_2$), m+($F_3$,$D_3$) and add($D_2$,$D_3$,$D_1$) reduces the possible tuples for the constraint add($D_2$,$D_3$,$D_1$) to three:

add($P_1$,$P_1$,$P_1$), add($P_2$,$P_2$,$P_2$), add($P_3$,$P_3$,$P_3$).

The remaining tuples after piecewise filtering are:

| add($F_1$,$F_2$,L) | equal($F_2$,$F_3$) | add($D_2$,$D_3$,$D_1$) | m+($F_1$,$D_1$) | m+($F_2$,$D_2$) | m+($F_3$,$D_3$) |
|---|---|---|---|---|---|
| $(P_2,P_2,P_2)$ | $(P_2,P_2)$ | $(P_2,P_2,P_2)$ | $(P_2,P_2)$ | $(P_2,P_2)$ | $(P_2,P_2)$ |

(5) There is one tentative global solution which corresponds to the solution because there are no global filters in this example. The solution is:

$$(F_1, F_2, F_3, L, D_1, D_2, D_3) = (P_2, P_2, P_2, P_2, P_2, P_2, P_2)$$

The global solution indicates that the increment in the external load causes an increment in the internal forces and deformations for the springs.

(6) The new global solution is the initial condition for the second step. In the subsequent propagations a transition from an interval such as <(0,∞),increasing> to a point such as <yielding_force,steady> takes place first for the spring S1 and then for springs S2 and S3 simultaneously. These behaviors result in case 1 from Fig. 2.10.

The result of the inference scheme is a set of qualitative structural behaviors, such as the nine solutions illustrated in Fig. 2.10. To distinguish between springs in the yielding range and those not yielding, the yielded springs are shown by a dashed line. Case 1 is separated in three consecutive states: the increase in external load causes, an increase in the internal forces or state one; the increase in internal force continues until spring S1 yields in compression, or state two, and finally the increase in internal load continues until springs S2 and S3 yield in compression at the same time, or state three. The third state corresponds to a termination rule because the external load cannot be increased further. The simulation continues deriving the other eight behaviors.

<u>Critique of the constraint centered framework</u>

The constraint framework limits the modeling adequacy allowing a more efficient implementation. It is a convenient approach when the governing differential equations are known in advance. However, for structural engineering it has the same disadvantages as the other frameworks and its application is cumbersome except for simple one-dimensional structures.

### 2.3.4.- The unifying framework

The unifying framework was recently proposed by Bredeweg et al. [89, 90]. The fundamental principles of the domain are represented as components and processes in a common framework. Changes in the component states are caused by active processes as well as components.

<u>Quantity space</u>

The quantity space for a parameter is defined by the user, as for the process and constraint centered frameworks. The value ZERO may be excluded.

<u>Qualitative calculus</u>

The process framework introduced the relations in magnitude between parameters to reduce ambiguity resulting from operations on qualitative parameters. However, the transitivity relations do not guarantee consistency as discussed in Section 2.3.2. The unifying framework uses an algebraic simplifier, called linear constant elimination, to eliminate inconsistent solutions such as the one illustrated in Fig. 2.9. Constant elimination is expressed as a set of axioms summarized in Table 2.4. These axioms are a subset of the complete constant elimination set included in Simmons [90].

Table 2.4 Linear constant elimination

| | A = B | A >= B | A > B |
|---|---|---|---|
| C = D | A+C = B+D | A+C > = B+D | A+C > B+D |
| C >=D | A+C >= B+D | A+C > = B+D | A+C > B+D |
| C > D | A+C > B+D | A+C > B + D | A+C > B+D |

To prove the solution shown in Fig. 2.9(a) is inconsistent, consider the parameter relations, $|F_2|>|F_1|$ and $|F_3|>|F_4|$. By using the last column and last row from Table 2.4, constant elimination indicates that:

$$|F_2| + |F_3| > |F_1| + |F_4|$$

This result contradicts the equation, $|F_2|+|F_3|+|V_2|=|F_1|+|F_4|$, from Fig. 2.9, because if $|F_2|+|F_3|+|V_2|=|F_1|+|F_4|$ then $|F_2|+|F_3|<|F_1|+ |F_4|$.

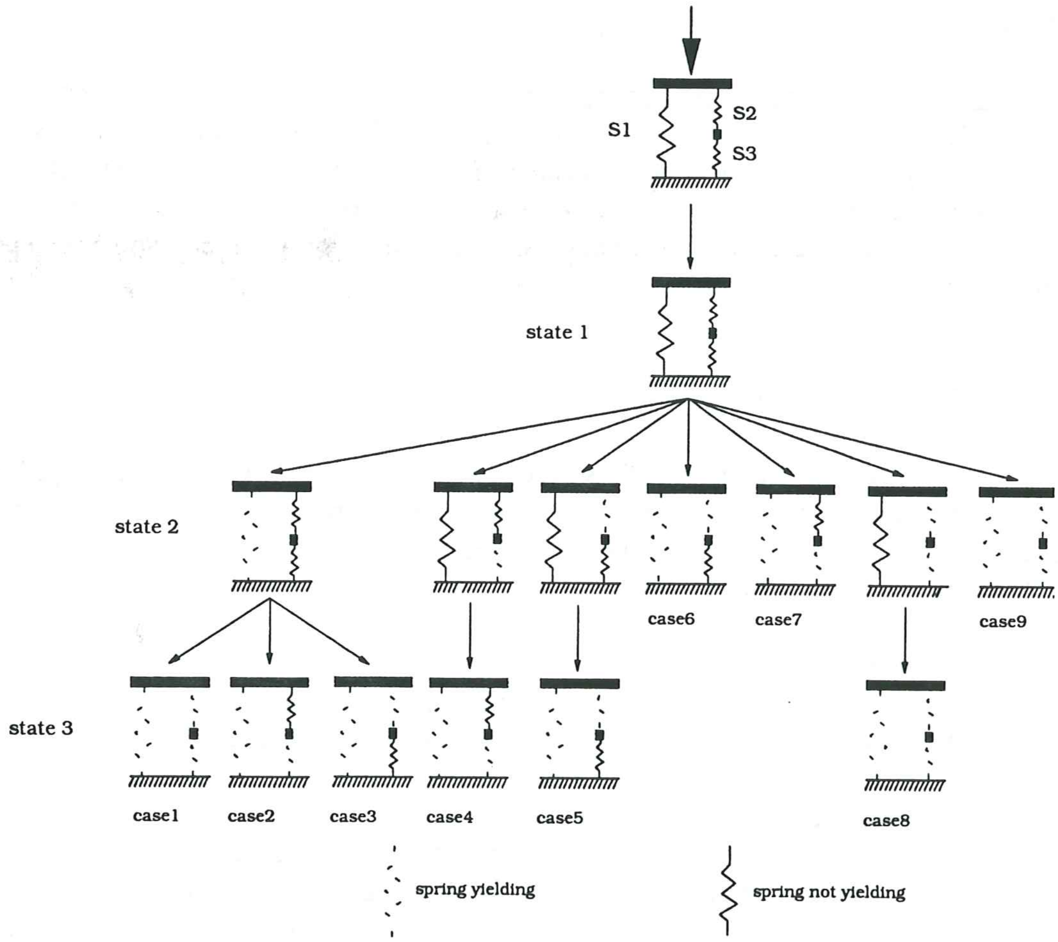



Figure 2.10 Tree of the solutions derived by the constraint framework for a spring structure.

Modeling primitives

The framework provides static descriptions, component descriptions, and process descriptions as the modeling primitives. The static descriptions are similar to views in the process centered framework. The states of a model are described by five classes: system elements, parameters, parameter values, parameter relations, and system structures. These classes describe the state of the model at any time. The class `system elements` describes instances of objects present in a particular state in time of the model. The class `parameters` describes parameters that represent the behavior. The class `parameter values` describes the quantity space and instance values for the parameters. In this framework there are a variety of `parameter relations` that indicate how the change of a parameter affects the change in other parameters. The `system structures` are process, components, or static views that apply to the system element's instances.

Inference scheme

The inference scheme is similar that for the process centered framework. Details are presented in Bredeweg [90], page 40.

Critique of the unifying framework

The unifying framework enhances the component and process frameworks by the following features: (1) the qualitative calculus reduces the ambiguity, at least for linear equations, (2) it uses components and processes to model the changes in states of the model, and (3) states of the model are represented by distinct classes such as object instances and parameter values.

On the negative side, a consequence of applying only a subset of the constant elimination axioms is that the inference scheme does not guarantee that all the solutions satisfy the governing equations. Furthermore, its application to multi-dimensional problems is cumbersome as with the other frameworks.

## 2.4.- PREVIOUS WORK IN QUALITATIVE REASONING APPLIED TO STRUCTURAL ENGINEERING

Qualitative reasoning derives the behavior of a device from a high-level description of the device and a representation of fundamental laws of the domain. Heuristic knowledge could be used by qualitative reasoning frameworks to reduce the search space, but a qualitative reasoning framework does not rely on experiential knowledge. This is a fundamental difference with knowledge-based-expert systems which focus on heuristic knowledge gained by engineers. Previous work in qualitative reasoning has focused on: (1) knowledge bases or heuristic structural evaluation [Ganguly et al. 90, Subramany et al. 89, Fenves and Ibarra-Anaya 89]; (2) qualitative reasoning using heuristics [Slater 86, Fruchter et al. 91]; (3) application of the constraint centered framework [Roddis 88, Schwartz and Chen 92]; and (4) application of the component centered framework [Grosso and Zucchini 91, Adorni et al. 88].

In the earthquake engineering field various knowledge-based expert systems have been recently developed. Expert systems represent a shallow level of knowledge, so a slight variation of a problem from the scope of the rules could give incorrect results. The knowledge base by Subramani et al. [89] has three levels: (1) building code information about period, base shear, and story drift; (2) recommendations based on the opinions of engineers; and (3) numerical analysis procedures. The first level supports the preliminary seismic design of buildings. For example, the vibration period is approximated using the 1988 Uniform Building Code provision:

$$T = C_t h_n^{3/4}$$

where $T$ is the fundamental period of vibration, $C_t$ depends on the type of lateral load resisting system, and $h_n$ is the height of the building. Obviously, code-type information only applies to regular buildings for which the response characteristics are fairly well established in advance.

The knowledge base by Ganguly et al. [90] evaluates preliminary designs using causal links between selected structural behaviors. The system provides expert advice based on heuristic knowledge and so-called qualitative reasoning capabilities. However, at the level of knowledge capture by `IF-THEN` rules there is no difference between heuristic knowledge and causal knowledge. Consider for example a causal link from this knowledge base:

**IF** there are reentrant corners
    **THEN** there is torsion

The rule does not provide information about which components sustain the torsion, or the relative magnitude of the torsional response. Furthermore, the rule is not complete because a symmetric building with reentrant corners does not develop torsional response. This methodology attempts to represent fundamental premises of the domain, but the level of representation is shallow and it is not very different from other knowledge based expert systems. Heuristic knowledge is useful at the very first stage of the design process, when there is not enough information even for a qualitative model. In the absence of all information except the number of stories, heuristic knowledge provide an estimate of the structural period.

Slater developed an application of qualitative evaluation that derives the direction of forces and displacements in continuous beams under gravity loads [Slater 86]. The approach reduces the degree of static indeterminacy by assuming the locations of inflection points. A parameter is represented by the intervals {zero, small, medium, large}. However, each interval is matched to a numeric value and operations are performed on these real numbers, therefore no qualitative calculus is defined. The aforementioned qualitative values correspond to the numerical values {0.0, 0.33, 0.67, 1.0}. This "quantity space" corresponds closer to the linguistic variables in fuzzy sets than to the quantity spaces used by qualitative reasoning frameworks [Yager et al. 87]. The solution strategy for this methodology can be summarized as follows:

(1) Consider individual gravity loads acting on the structure.
(2) For each load, conservation of energy says that the displacement at the centroid of the load is in the direction of the load. Heuristic knowledge derives the direction for forces and displacements for the bar where the load is applied. For example, the solution for the loaded span of a continuous beam corresponds to a bar with two inflection points close to the connections, a positive bending moment close to the middle, and negative bending moments at the connections.

(3) Propagate the known rotations, displacements, moments, and shears. Due to the simplicity of the beam structures this is accomplished in manner similar to a one-pass moment distribution procedure. This methodology is primarily based on a representation of the equilibrium and not compatibility or material characteristics.
(4) Combine the solutions for individual loads using superposition and resolve ambiguities by relative magnitude of the numerical values.

Heuristic knowledge about the location of inflection points is necessary for the inference and therefore the results are only as valid as the rules.

A recent methodology for the heuristic evaluation of continuous beams under gravity loads has been developed by Fruchter et al. [91]. The methodology uses kinematic assumptions similar to Slater, so it is suitable for structures whose response can be characterized in advance. It does not derive a range of possible structural behaviors, rather it develops one solution with possible ambiguous results. In contrast to Slater's methodology, this approach uses a quantity space and a qualitative calculus. The quantity space `{negative, zero, positive}` and the qualitative calculus correspond to the component centered framework. A program called *QLattice* is used to enhance the resolution of ambiguity in qualitative calculus by keeping track of ordering relations between quantities [Simmons 90].

The program *QLattice* reduces ambiguity in a different manner that the techniques presented in Section 2.3. *QLattice* finds the ordering relation between two parameters upon request. As new parameter relations are added, the program does not keep an incremental set of consistent relations; rather it verifies their consistency by request from the inference scheme. Other frameworks maintain a partial order between quantities, and every time a new relation is added its consistency with previous knowledge is verified incrementally. *QLattice* derives relations between quantities using five techniques which are tried in order. These techniques are: (1) transitivity based on graph search, (2) numeric constraint propagation, (3) interval arithmetic, (4) relational arithmetic, and (5) constant elimination.

Transitivity relations were described for the process centered framework. Numeric constraint propagation is a quantitative technique that infers relations based on the overlapping of intervals. For example, if `A=[-1,2]` and `B=[2,100]` then `|B|>=|A|`. Interval arithmetic is a technique that enables reasoning about expressions such as `A + 5`. Based on simple rules, the interval resulting from the addition, subtraction, multiplication, or division of intervals is evaluated. The major restriction of this technique is that it typically produces ever growing intervals and loses inference power. Another technique used by *QLattice* is relational arithmetic. It enables the reasoning about commonsense relations that are not detected by interval arithmetic. For example if `X = Y + 1` and `X >= 0`, interval arithmetic would conclude that `Y>=-1` but it would not derive that `X` is greater than `Y`. The final technique, constant elimination, has been discussed previously.

These techniques provide a powerful arithmetic reasoning system that is efficient for deriving the relations between quantities defined by intervals and numeric values. One limitation is that the reasoning is not performed incrementally and therefore consistency checking is delayed until it is requested. *QLattice* is convenient for the inference scheme used by Fruchter et al. [91] because the framework only generates one solution. Heuristic knowledge determines the solution for certain components of the structure and their qualitative values are propagated using equilibrium and compatibility at connections. If an ambiguity arises, the system calls *QLattice* and attempts to obtain the relation which could resolve the ambiguity. The final solution is a single structural behavior with possibly ambiguous parameters. The propagation of known values is performed keeping a stack of tasks. Each task has an associated default priority that indicates the order in which the tasks will be executed. Apparently this default priority is independent of the topology of the problem. The stack of tasks indicates how to proceed with the inference and which equations to use to infer qualitative values.

In another application of qualitative reasoning to structural engineering, the constraint framework has been applied to the problem of fatigue and fracture of steel bridges [Roddis 88]. The problem is modeled as a non-dimensional one which facilitates the use of the framework. The constraint approach has also been applied to the vibration of beams by Schwartz and Chen [92], although it is not clear how they reduce the ambiguity in qualitative calculus. The constraint, as well as all previous frameworks, can generate inconsistent solutions caused by the ambiguity in qualitative calculus.

The component framework has been applied to the analysis of planar elastic frames [Del Grosso and Zucchini 91, Adorni et al. 88]. Frame members are represented by various components such as a vertical component, a horizontal component, and other components with different orientations. The reasoning is divided into two stages, one that assembles the model and the second stage that propagates a parameter change at a connection. To derive the qualitative model, a quantitative model based on a matrix formulation of the displacement method in terms of network theory is formulated. A structure is represented by a directed graph with the nodes and branches of the graph representing connections and components, respectively. The parameters in the qualitative model are the internal forces, connection displacements, component deformations, and external loads.

The second stage propagates the change in one external load. The initial disturbance is propagated through a constraint network, but if local information is not sufficient, three heuristic rules indicate how to continue. A heuristic rule indicates that if the propagation stops at an equilibrium equation for a connection, and the value of one force is much larger than the value of the other unknown forces at the connection, then the displacement follows the direction of the greater force. The other two heuristics are included in Adorni et al. [88].

The limitations of this application are similar to those of the component centered framework, although there are two characteristics to consider. It is not clear how the approach performs the inference scheme, but it seems to avoid the envisionment step because it generates one qualitative solution. This probably enhances the inference efficiency compared to the component framework. The second characteristic is that the qualitative calculus does not use parameter relations which limits the usefulness of the information derived by the reasoning system. Furthermore, heuristic rules are applied during the propagation.

## 2.5.- SUMMARY

Previous work in qualitative reasoning attempts to automate the generation of causal behavior of physical systems. It provides formalisms for representing and understanding the behavior of models. Four approaches, the component, the process, the constraint, and the unifying framework, have been developed by artificial intelligence researchers.

A limitation of qualitative reasoning for structural engineering is that it is difficult to model boundary value problems for which geometry and spatial relations are important. The modeling of even one-dimensional boundary value problems typically requires the definition of many similar components or processes. Another limitation is that the frameworks are inefficient for problems that involve a large number of limited classes of components. A structure is typically formed by various components of the same class such as frame members, shear walls, and supports. Taking advantage of class behavior leads to a more expressive and efficient framework.

Previous work in qualitative reasoning applied to structural engineering has focused on the constraint and component centered frameworks, incorporating heuristics as part of the reasoning. This is a fundamental difference with respect to the approach taken in the current work, which assumes engineers derive load paths and deflected shapes based on a deep understanding of equilibrium, compatibility, and material characteristics. Previous heuristic qualitative reasoning methodologies are suitable for the evaluation of conceptual designs if the overall characteristics of the response can be described in advance. Heuristic structural evaluation and qualitative reasoning may complement each other. Heuristics can provide estimates of structural parameters with limited information, and heuristics can focus attention on important aspects of a conceptual design. However, qualitative reasoning provides information that is more specific and more useful for the designer than information derived from heuristics.

# Chapter 3

# KNOWLEDGE REPRESENTATION FOR THE EVALUATION OF CONCEPTUAL STRUCTURAL DESIGNS

## 3.1.- INTRODUCTION

Knowledge can be divided into procedural or dynamic knowledge and declarative or static knowledge. The first one refers to a specific operation such as Gauss elimination or, in the context of structural analysis, the direct stiffness method. In contrast, declarative knowledge refers to facts such as the description of the topology and geometry for a building. Extensive debate has focused on the relative importance of procedural and declarative knowledge. It is generally accepted, however, there are advantages to separating the representation of knowledge from operations on the representation. This chapter presents declarative representations appropriate for design of structural systems. Chapter 4 focuses on the operations on the knowledge to evaluate the structural behavior of the design.

Knowledge representation attempts to describe a domain in a suitable way for the computer to formally process this description and reach conclusions. There are three goals for a knowledge representation formalism. The first goal is to provide a formal notation to describe knowledge. This determines the expressive adequacy of the formalism. The second goal is to transform symbols representing facts into other facts not explicitly represented using an automated procedure. This determines the inference adequacy of the formalism. A representation may be powerful for deriving many conclusions, but it may be very inefficient and time consuming. The third goal is to provide a procedure for incorporating new knowledge. The most important issue in knowledge representation is that there is a tradeoff between inference efficiency and expressive adequacy. As the expressive adequacy of a representation increases, its inference efficiency decreases. Apparently simple changes in the expressive power may significantly affect the inference efficiency [Levesque and Brachman 85].

The representation of conceptual design knowledge includes the topology, geometry, structural function, structural behavior, and the fundamental principles of equilibrium, compatibility, and force-deformation, at a level of abstraction suitable for conceptual design. The representation for conceptual design knowledge must retain the relevant features of a design, without requiring parameters that may not be available. Design knowledge must be represented in a way suitable for the automated inference of useful structural engineering conclusions based on the load transfer characteristics of a conceptual design. The size of the design space is a function of the model representation and quantitative models are powerful representations regarding inference adequacy but weak for representational adequacy.

The model of a structure is an aggregation of components such as frame members, supports, shear walls, floor systems, and stories. There are two classes of components: individual structural components such as frame members and shear walls; and high-level components such as stories and lateral resisting systems. Individual structural components are objects that are not further decomposed into smaller components. High-level components are recursively formed by an aggregation of individual or other high-level components. According to the level of abstraction a building is a single higher level component `building` or it is an aggregation of stories and floors, among other representations. The monograph focuses on individual structural components, but the framework is suitable for representing high-level components.

## 3.2.- STRUCTURAL FUNCTION AND LOADS

For the purpose of evaluating conceptual designs, structural function is defined as the transfer of external loads through the structure to the supports. External loads are represented by static loads applied to a connections between components. Loads applied to a component require the definition of two components connected at the resultant of the load. For example, a frame member under a uniform load is represented as two frame members with load applied at the common connection. This coarse representation of loads is suitable for the conceptual design stage.

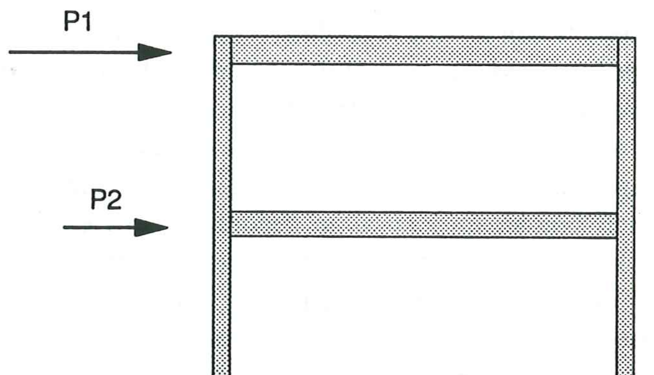



Figure 3.1 Example of qualitative representation of lateral earthquake loads on a building.

Loads are represented as qualitative parameters and they have a line of action, a direction, and a magnitude. A load is an external force or an external moment applied along a global axis `X`, `Y`, or `Z`. The load is positive or negative according to the direction of a global axis. The magnitude of a load can be related to the magnitude of other load or internal force in the same direction. The relations are `{greater_than, equal_to, less_than}`. The representation of loads must include the direction, but magnitude relations are not required. Figure 3.1 illustrates a definition for the lateral earthquake loads on a building, including a relationship between the magnitudes of the lateral loads.

## 3.3.- TOPOLOGY REPRESENTATION

Components are connected to each other by connections, which may be points, lines, or surfaces. There are two approaches for representing the topology of a structure such as shown in Fig. 3.2. The first approach represents the connections between components, as illustrated by the graph in Fig. 3.2(b). Nodes in the graph define connections and arcs define components. This graph is convenient for representing the components attached to a connection. The second approach represents the components between connections, as shown in Fig. 3.2(c). Nodes define components and arcs define connections. This description is good for representing knowledge about components. For example, this graph directly indicates which components are connected to a given component. To retrieve this information using the first representation it is necessary to retrieve the connection and then the components attached to the connection. Both representations describe the same topological information, and one can be expressed in terms of the other. The choice depends on the particular application. The implementation of the space centered framework uses both representations because it enables an efficient access of information about components and connections.

## 3.4.- GEOMETRY REPRESENTATION

Geometric modeling techniques represent physical form in a way that it can be processed by computers [Martini 90]. Geometric modeling has been primarily applied to drafting and graphics, although there have been recent developments to support the design process. However, geometric models primarily represent precise quantitative descriptions for the final stages of a design. They are not useful for the conceptual design where form must be qualitatively represented.

Shape grammars are another formalism for representing geometric knowledge and generating instances of designs. Architects have used them to generate facades and floor plans. In structural engineering shape grammars can be used for the automated generation of preliminary designs based on knowledge about geometry and structural function [Fenves and Baker 87, Fenves and Baker 90]. Shape grammars can complement qualitative reasoning by modifying design solutions according to a qualitative evaluation of structural behavior.

An approach for innovative design by Cagan [90] describes geometry and topology in the same graph-theoretic representation. Solid objects are divided into regions represented by nodes in a graph. Arcs between the nodes represent links or adjacent relationships between regions. Links may follow each direction of a coordinate system. For a XYZ coordinate system there are six directions for each arc (three positive and three negative). For example, a +X link indicates that a region connects to another region along the positive X axis. This representation is convenient for modeling three-dimensional volumes but not for objects that are aggregations of components such as framed structures. The approach does not represent components orientations which is an important geometrical parameter for the conceptual design of framed structures.

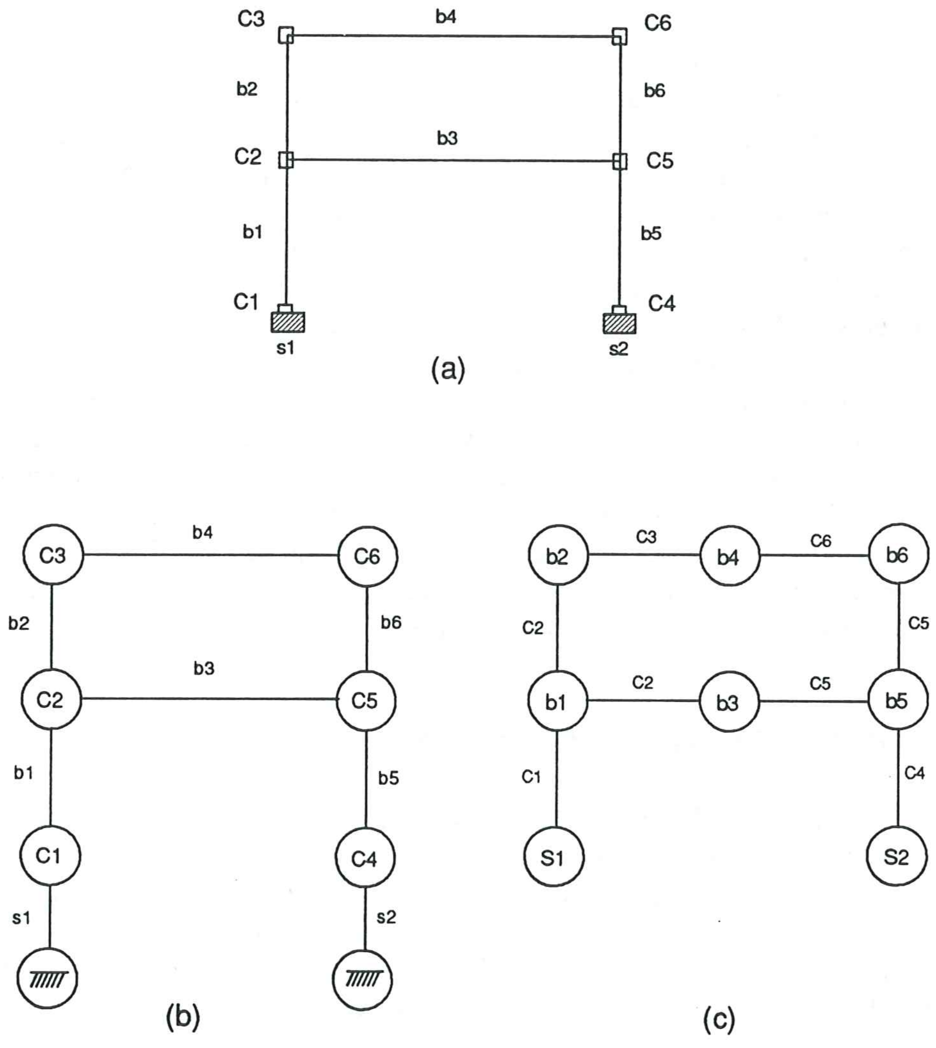



Figure 3.2 (a) Framed structure. (b) Topology representation that focuses on connections. (c) Topology representation that focuses on components.

The approach in the current work is to represent the geometry of a structure by attributes of the structural components. A component such as a frame member or a wall has two geometric attributes, length and orientation. The orientation is defined by the axis across the component connections as shown in Fig. 3.3. Cross section properties of components are not explicitly represented as geometric attributes because the combination of the material characteristics and cross section properties define the section behavior of the component. When reasoning about structural behavior, engineers often consider section properties and material characteristics as one parameter (for example, the stiffness `EI` for a beam). During conceptual design the section properties are usually unknown. The qualitative representation of section behavior increases the inference adequacy of the space centered framework because of the reduction in the number of parameters for the cross section geometry.

Length is always a positive number and consequently it is represented by the simple quantity space, `{positive}`. The parameter relations, `{greater_than, equal_to, less_than}`, relate the lengths of different components. For example, the relationship `greater_than(length1,length2)` states that the magnitude of `length1` is greater than the magnitude of `length2`.

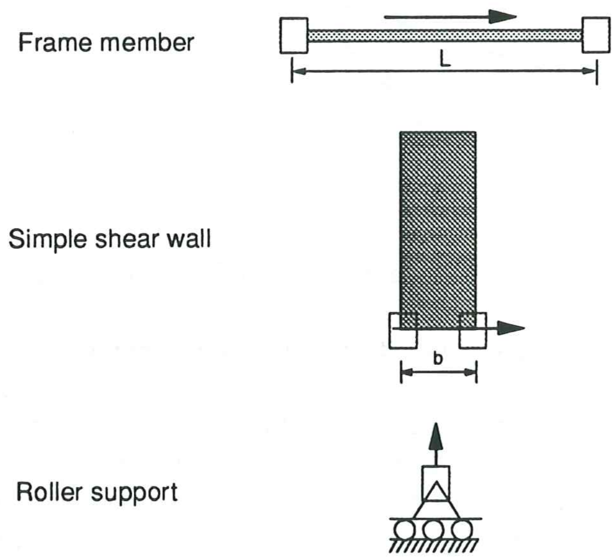



Figure 3.3 Geometric attributes for a frame member, a shear wall, and a roller support components.

Thirteen qualitative directions describe the orientation of a vector in space, as illustrated in Fig. 3.4. Three directions follow the `X,Y,Z` axes, one direction is in the `X,-Y` plane, one direction is in the `-X,-Y` plane, one direction is in the `X,Z` plane, one direction is in the `-X,Z` plane, one direction is in the `Y,Z` plane, one direction is in the `-Y,Z` plane, and the remaining four directions correspond to vectors in the four upper level quadrants. Orientations that can be expressed by an opposite orientation are not included. A vector along a negative coordinate axis, such as `-X` can be represented as the negative of the `X` axis and therefore a qualitative direction along the `-X` axis is avoided in the representation. The thirteen orientations define the minimum number of qualitative directions for representing any vector. The elimination of the redundant orientations does not reduce expressiveness of the geometric description but it does increase inference efficiency.

A qualitative orientation for a vector has three parameters, one for each of the direction cosines. As with other qualitative parameters, the parameter relations {greater_than, equal_to, less_than} relate the absolute value of the direction cosines for two vectors.

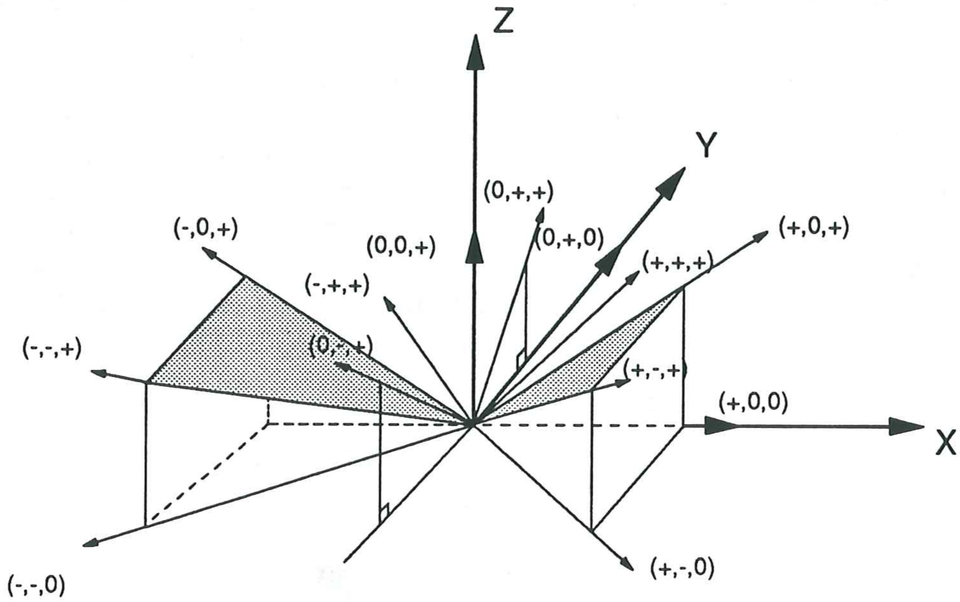


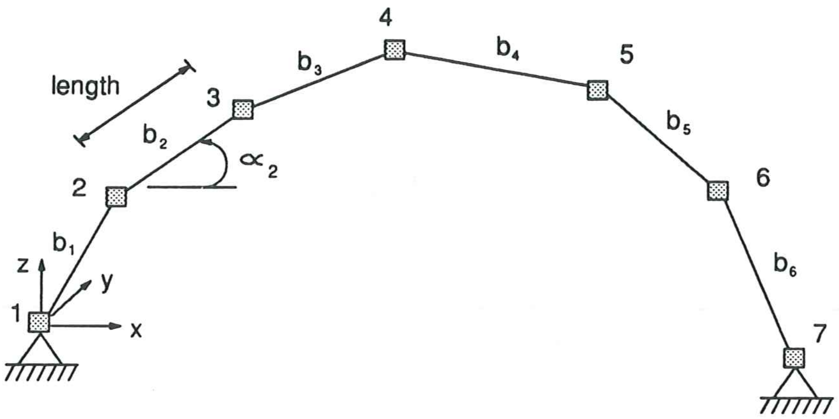

Figure 3.4 Qualitative directions for vectors.

Figure 3.5 presents a qualitative geometry representation for an arch that incorporates fundamental features of its geometry. Six frame member components and two pinned support components model the curved geometry of the arch. The description provides the sign and relative magnitude of the direction cosines for each frame member. For example, the direction cosine for component $b_2$ is negative and its magnitude is less than the magnitude of the direction cosine for component $b_3$. As previously indicated, the representation of conceptual designs should include a large design space. This representation for the arch geometry accomplishes this because it includes arches with different height to width ratios and supports at different elevations.

## 3.5.- FUNDAMENTAL PRINCIPLES FOR STRUCTURAL COMPONENTS

Structural behavior for a component is defined by the relationships between connections forces and displacements. The relationships are affected by the material characteristics, section properties, equilibrium, and compatibility. The qualitative section behavior is assumed constant over the length of the component. The material characteristics correspond to nonlinear softening elastic materials.

The forces and displacements at the connections define the qualitative states for the components. Their relationships must satisfy the laws of equilibrium, compatibility, and material characteristics. The component states include the *qualitative values* for the forces, moments, displacements, and rotations at the connections, and the *parameter relations* between forces, moments, displacements, and rotations. The qualitative values are points and intervals that describe relevant features of the parameters describing the component behavior. The parameter relations define relationships between parameters of the same type (e.g. displacements, moments, etc.) at the component connections. For example, a bending moment along the X axis may be related to another bending moment along the X axis but it is not related to other parameters such as a bending moment along a different axis.




Qualitative values: $\cos(\alpha_i)$ = positive, $i = 1, 2, 3$
$\cos(\alpha_i)$ = negative, $i = 4, 5, 6$

Parameter relations:

$$length_1=length_2=length_3=length_4=length_5=length_6$$
$$|\cos(\alpha_3)| > |\cos(\alpha_2)| > |\cos(\alpha_1)|$$
$$|\cos(\alpha_4)| > |\cos(\alpha_5)| > |\cos(\alpha_6)|$$

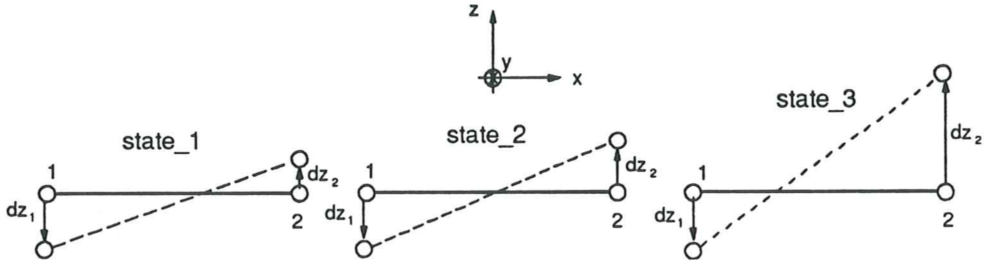
Figure 3.5 Qualitative description for the geometry of an arch.

A central contribution of the current work is the use of parameter relations to describe the structural behavior of components. When engineers reason about structural behavior, they typically use relations between the magnitudes of parameters. Limiting the representation of components to qualitative values seriously restricts the expressive adequacy. Parameter relations also reduce ambiguity in the reasoning process.

To illustrate parameter relations, consider the states for an axial rod. The rod has three qualitative states: tension, compression, or unloaded. Each state has the same three parameter relations: (1) the forces along the X axis at the two connections are equal, (2) the forces along the Y axis at the connections are equal, and (3) the forces along the Z axis at the connections are equal. The use of parameter relations for the laws of compatibility and material characteristics is more interesting. Figure 3.6 shows only three states for a frame member component for illustration. These three states have the same qualitative values for rotations about the Y axis and displacements along the Z axis. If parameter relations are not used the three states are identical. The parameter relations increase the number of component states because they describe the component behavior more precisely.

### 3.5.1.- Force-deformation relationships

The force-deformation relationship for the section of a frame member is represented by a monotonic function. The monotonic relationship includes a range of elastic behaviors such as linear, softening, or hardening materials, as illustrated in Fig. 3.7. This relationship represents section behaviors that are useful in structural engineering without requiring the details of specific material models. The representation does not include non-monotonic curves, such as the hysteretic behavior of frame members under cyclic loads, because they do not provide much further knowledge at conceptual stage of the design. Non-monotonic force-deformation relations also increase the ambiguity of the qualitative calculus.




| | | |
|---|---|---|
| $dz_1$ = negative | $dz_1$ = negative | $dz_1$ = negative |
| $dz_2$ = positive | $dz_2$ = positive | $dz_2$ = positive |
| $\theta_{y1}$ = negative | $\theta_{y1}$ = negative | $\theta_{y1}$ = negative |
| $\theta_{y2}$ = negative | $\theta_{y2}$ = negative | $\theta_{y2}$ = negative |
| $\lvert\theta_{y1}\rvert = \lvert\theta_{y2}\rvert$ | $\lvert\theta_{y1}\rvert = \lvert\theta_{y2}\rvert$ | $\lvert\theta_{y1}\rvert = \lvert\theta_{y2}\rvert$ |
| $\lvert \mathbf{d}_{z1}\rvert > \lvert \mathbf{d}_{z2}\rvert$ | $\lvert \mathbf{d}_{z1}\rvert = \lvert \mathbf{d}_{z2}\rvert$ | $\lvert \mathbf{d}_{z1}\rvert < \lvert \mathbf{d}_{z2}\rvert$ |

Figure 3.6 Example of component states provided by parameter relations for a frame member.

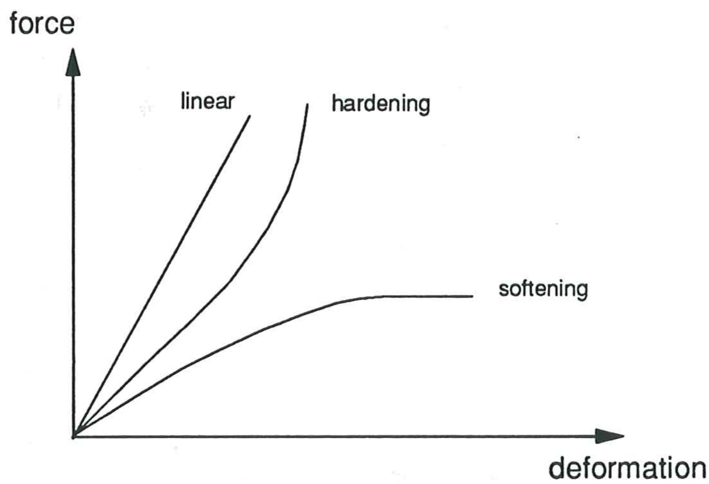



Figure 3.7 Monotonic force-deformation relationships for components.

In contrast to forces and displacements, which are represented by a single parameter, section behavior is represented by a monotonic function. A parameter relation between section behaviors should indicate a relationship over the range of values. There are three relations between section behaviors, `stiffer`, `less_stiff` or `equal_stiff`, which are relationships between deformations under the same force. For example, a relation `stiffer(component1, component2)` between two frame members indicates that for the same bending moment, $M_c$, the curvature for `component1` is less than the curvature for `component2`, as illustrated in Fig. 3.8. A relation `less_stiff(component1, component2)` means that for the same bending moment, $M_c$, the curvature of `component1` is greater than the curvature of `component2`. Similarly, a relation `equal_stiff(component1, component2)` describes two sections for which the curvatures of `component1` and `component2` are equal at the same bending moment.

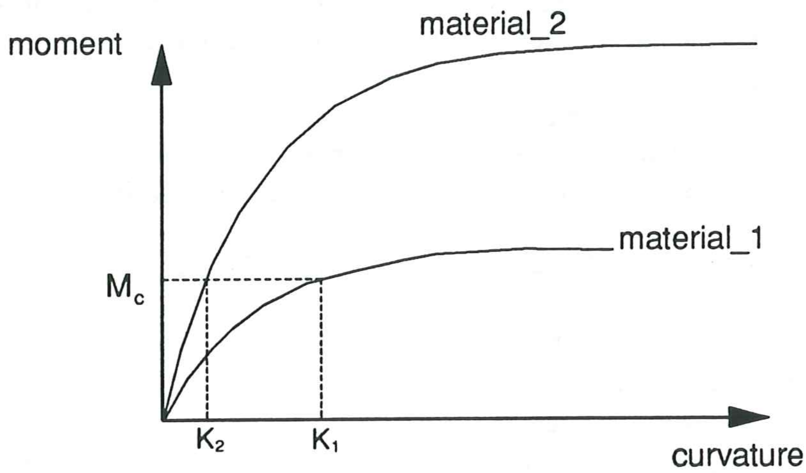


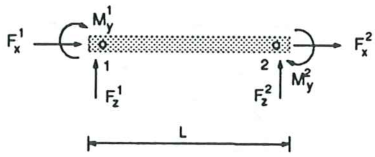

Figure 3.8 Parameter relation `stiffer(material_2, material_1)` between two section behaviors describes a relation for a range of curvatures.

### 3.5.2.- Frame member component

A frame member is a component with two connections at the end. The representation of the equilibrium, compatibility, and material characteristics laws for a frame member is presented in two subsections. The first subsection presents the equilibrium relationship between end forces and section forces. Using the force-deformation relationship and equilibrium, the second subsection presents the relationships between the end displacements and member deformations according to the compatibility laws.

<u>Equilibrium</u>

The equilibrium laws establish a relation between the forces and moments at the two connections. To illustrate the relations consider the equilibrium laws for a planar free- body diagram:




| Axial Forces | Bending Moments and Shear forces |
|---|---|
| $F_x^1 + F_x^2 = 0$ | $F_z^1 + F_{z_1}^2 = 0$ |
| | $M_y^1 + F_z^1 L + M_y^2 = 0$ |

With the quantity space {−, 0, +} for the forces, a frame member has seventeen states for equilibrium of moments and shear forces, as illustrated in Fig. 3.9. Each state in Fig. 3.9 defines the qualitative values of the forces and moments and the parameter relations that satisfy equilibrium for a frame member. In addition to the seventeen bending states, there are three states for axial forces (tension, compression and null force). The seventeen bending states are derived by using the quantity space, the parameter relations, `greater_than`, `equal_to`, and `less_than`, and the extended addition operation in Table 3.1.

The equilibrium relation, $F_z^1 + F_z^2 = 0$, indicates that the shear forces have the same magnitude but opposite direction; therefore there are three states for shear:

$$[F_z^1 \cong +,\ F_z^2 \cong -],\ [F_z^1 \cong -,\ F_z^2 \cong +],\ [F_z^1 \cong 0,\ F_z^2 \cong 0].$$

Length is a positive parameter, so moment equilibrium is represented by the extended addition operation as, $M_y^1 \oplus F_z^1 \cong -M_y^2$.
The table has thirteen results such as, `positive` ⊕ `positive` ≅ `positive`, which correspond to state 5 in Fig. 3.9, or `positive` ⊕ `zero` ≅ `positive`, which corresponds to state 4. For parameters of opposite signs the operation has six results and two of those results are `positive` ⊕ `negative` ≅ `negative`, and `negative` ⊕ `positive` ≅ `positive`. These two results do not prescribe the relationship between the end moments, so the relationships can be `greater_than`, `equal_to` or `less_than`. The seventeen states result from the thirteen cases in Table 3.1 and four cases from the ambiguous relationships between end moments.

Table 3.1 Extended addition operation

<table>
<tr><td colspan="2" rowspan="2">R ≅ X ⊕ Y</td><td colspan="3">X</td></tr>
<tr><td>−</td><td>0</td><td>+</td></tr>
<tr><td rowspan="3">Y</td><td>-</td><td>−</td><td>−</td><td>+, |X|>|Y|<br>0, |X|=|Y|<br>−, |X|<|Y|</td></tr>
<tr><td>0</td><td>−</td><td>0</td><td>+</td></tr>
<tr><td>+</td><td>+, |X|<|Y|<br>0, |X|=|Y|<br>−, |X|>|Y|</td><td>+</td><td>+</td></tr>
</table>

If the qualitative value zero is eliminated from the quantity space, the number of states is reduced from seventeen to ten because states 1, 2, 4, 9, 10, 12, and 17 are eliminated. This restriction increases the inference efficiency although it limits the expressive adequacy.

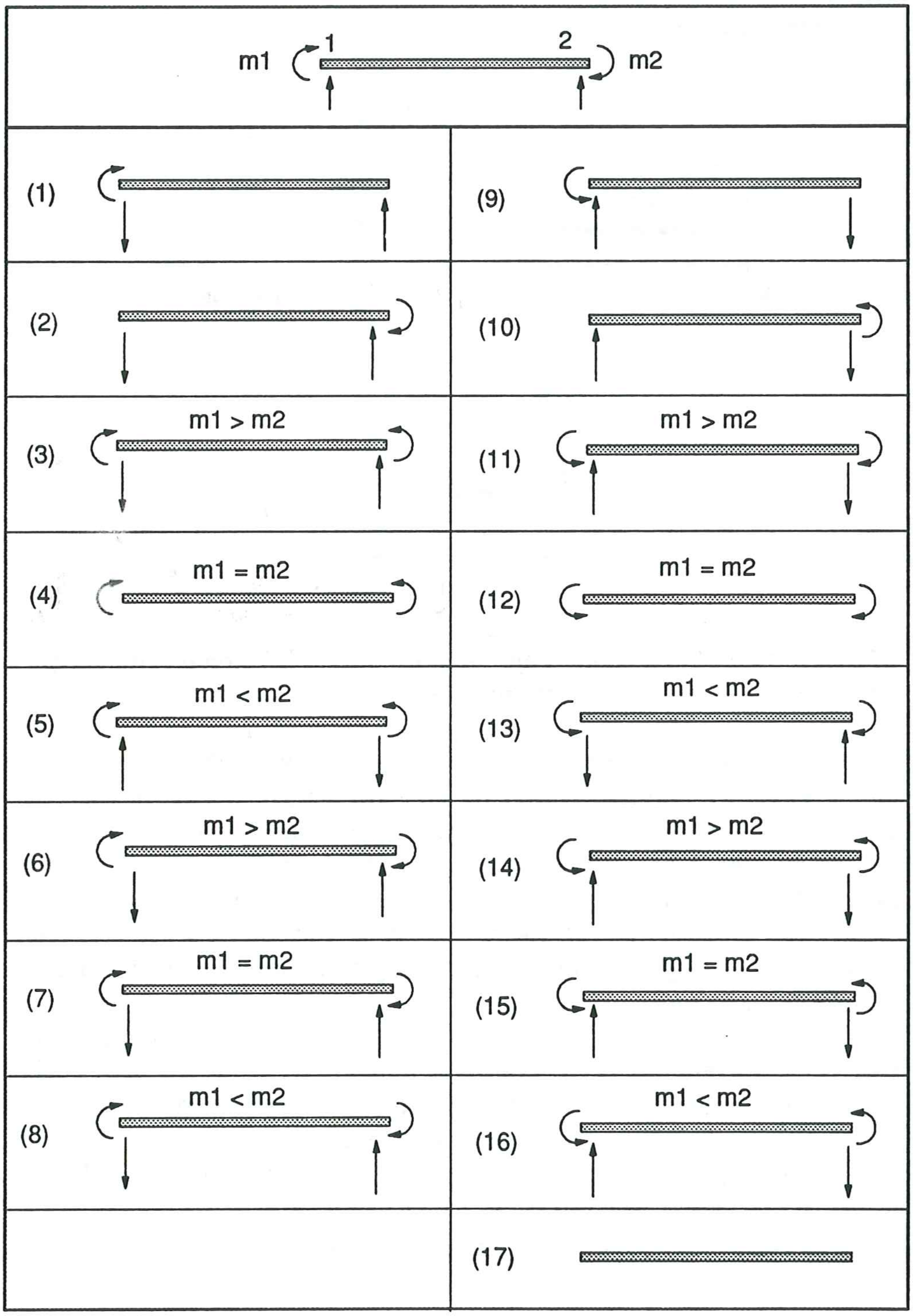



Figure 3.9 Equilibrium bending states for a frame member component.

Compatibility and force-deformation

The conjugate beam analogy recognizes the similarity between the equilibrium and compatibility laws [Oden 67, Bazant 66, Hjelmstad 86]. The analogy has been used to teach concepts of compatibility and deformations by using the familiar concepts of equilibrium and forces. The analogy provides insights into the qualitative structural behavior, and the representation of both equilibrium and compatibility laws can be implemented using similar predicates.

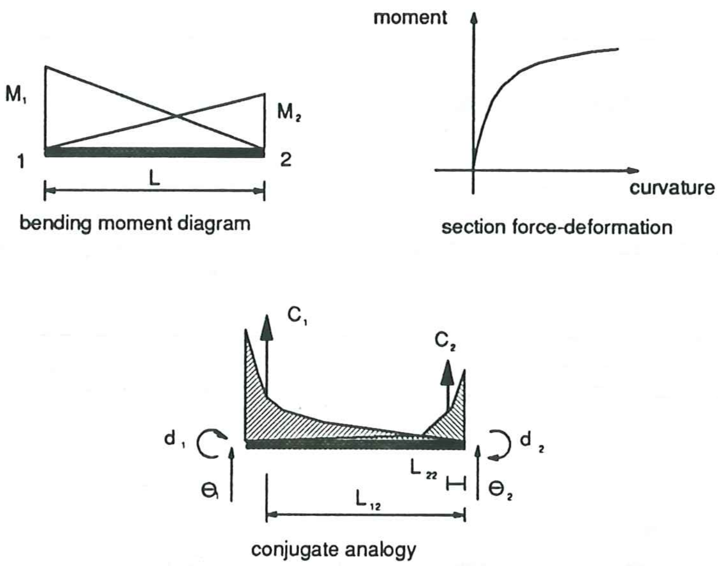



Figure 3.10 Conjugate beam analogy for the compatility and force-deformation component processes.

In qualitative terms there is no distinction between the force-deformation behavior of a beam constructed of a linear elastic material or an elastic softening material. This is illustrated in Fig. 3.10 by the conjugate beam analogy. If the moment $M_1$ is greater than the moment $M_2$, then the conjugate load $C_1$ is also greater than the conjugate load $C_2$ for a linear material. An elastic softening material accentuates the relation between conjugate loads. There is a similar relation between displacements because material softening translates the center of gravity of the conjugate loads closer to the ends of the member.

Presently there is no procedure that maps the nonlinear equations for compatibility and force-deformation into a set of predicates representing the qualitative states of a frame member. In the current work the governing laws are decomposed into simpler problems, for which each is represented by first order logic predicates[1]. The representation of equilibrium of conjugate forces is presented first and then the equilibrium of conjugate moments is presented.

Equilibrium of the conjugate forces representing rotations is expressed as:

$$\theta_j = \theta_i + C_i + C_j$$

1 See appendix A for an introduction about logic representation.

where $C_i$ and $C_j$ are conjugate loads which depend on the end moments at their respective connections. Length and section behavior do not enter the compatibility expressions because they are constant throughout the length, and softening accentuates the behavior. To derive the compatibility states the increment of the conjugate load $(C_i+C_j)$ is obtained and the extended addition operation is used to relate the rotations at the connections, such as:

$$\theta_j \equiv \theta_i \oplus \texttt{increment\_conjugate\_load}$$

The use of the extended addition operation is the same as for equilibrium. If the conjugate loads $C_i$ and $C_j$ are in the same direction (the frame member is in single curvature), the increment of the conjugate loads is in the common direction. If one load is zero, the increment follows the direction of the nonzero conjugate load. If the conjugate loads are in opposite directions (the frame member is in double curvature), the parameter relation between the end moments determines the direction of the conjugate load. For example, if $C_i$ is positive and $C_j$ is negative but $|M_i| > |M_j|$, then the increment $C_i+C_j$ is positive.

The increment in conjugate moments, however, is not represented as simply as for the equilibrium of conjugate forces. The increment in conjugate moment is a function of three parameters, the rotation at the left connection and the two end moments. Four simple cases are identified in Fig. 3.11: (1) the influences of the three parameters, $\theta_i$, $M_i$ and $M_j$ are in the same direction, so the increment follows the common direction; (2) the influence of the rotation is opposite to the influences of the moments; (3) the influence of moment $M_i$ is opposite to the influences of the rotation and moment $M_j$; and (4) the influence of moment $M_j$ is opposite to the influences of the rotation and moment $M_i$. To derive the direction in the increment of conjugate moment, each of the four cases is considered.

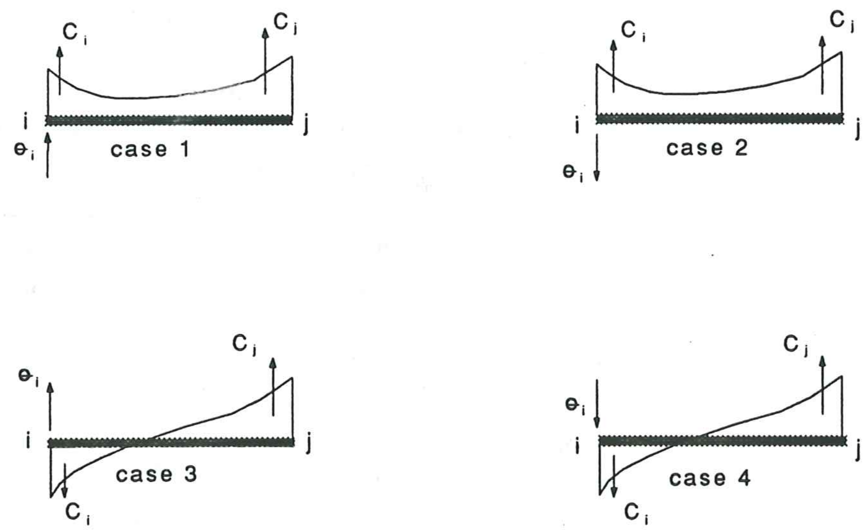



Figure 3.11 Four cases to represent the increment in conjugate moments.

To illustrate the representation consider the second case in Fig. 3.11, for which the rotation $\theta_i$ has an opposite direction with respect to the conjugate loads, $C_i$ and $C_j$. The parameter relations between the end moments and between the end rotations are known from the equilibrium of bending moments and conjugate loads, respectively. The parameter relations between end moments and end rotations are `greater_than` or `equal_to` or `less_than` and consequently there are nine cases ($3^2$). Consider the three cases when the bending moments have the same magnitude as do the conjugate loads $C_i$ and $C_j$. The bending moment influences have the same direction; therefore the resultant of the conjugate loads is located at the middle of the component. If the rotations have the same magnitude, then they must follow the same direction and their magnitude is one-half the magnitude of the resultant of the conjugate loads. Consequently, there is no increase in the conjugate moments. If the magnitude of the rotation at the left connection is greater than the magnitude of the rotation at the right connection, then the increase in conjugate moment must follow the direction of the rotation at the left connection. Similarly, if the magnitude of the rotation at the right connection is greater than the magnitude of the rotation at the left connection, then the increase in conjugate moment must follow the direction of the right rotation.

Consider the three cases when the magnitude of the bending moment $M_i$ is greater than the magnitude of the bending moment $M_j$, which implies the conjugate load is located at the left of the component. If the rotations have the same magnitude, then both must have a direction opposite to that of the conjugate load resultant and their magnitude is one-half the magnitude of the resultant. The increase in conjugate moment is in the direction of the conjugate load because the increment caused by the rotation $\theta_i$ is $L(C_i+C_j)/2$ which is less than the increment caused by the resultant, i.e. the load is at the left of the component. By similar considerations the increase in the conjugate moment is derived for the three cases when the magnitude of the bending moment $M_j$ is greater than the magnitude of the bending moment $M_i$, which implies that the conjugate load resultant is at the right of the component. The representation using first order logic[2] for the complete component behavior including the four cases illustrated in Fig. 3.11 is presented in Fig. 3.12.

Twenty-five qualitative states represent the behavior of a planar frame member that satisfies the equilibrium, compatibility, and force-deformation laws. These states are derived by the representation in Fig. 3.12 and they are illustrated in Fig. 3.13. The states are similar to the states introduced for the equilibrium laws in Fig. 3.9, but the latter set includes additional states to represent the end rotations and their relations. The additional states correspond to components in double curvature with unequal end moments. For example, if moment $M_1$ is greater than moment $M_2$ and both moments are clockwise, then rotation $\theta_1$ is greater than rotation $\theta_2$. However, rotation $\theta_2$ is either `{positive, zero, negative}` as illustrated by states 6, 9 and 10. If the zero value is eliminated from the quantity space, the number of states is reduced to fourteen, because the states 1, 2, 4, 9, 11, 13, 14, 16, 21, 23, and 25 are not be included. States 9, 11, 21, and 23 have one rotation with zero value.

---

2 The example uses the notation by Genesereth and Nilsson [88].

```
( same_direction_influences(θ_i,M_i,M_j) ⇒
   ( θ_i= 0 ∧ M_i= 0 ∧ op_dir(M_j,Increment)
   ∨ θ_i= 0 ∧ ¬M_i= 0 ∧ Increment = M_i
   ∨ ¬θ_i= 0 ∧ Increment = θ_i)
∨ rotation1_oppose_others(θ_i,M_i,M_j) ⇒
   (|M_j|=|M_i| ∧ (|θ_j|=|θ_i| ∧  Increment = 0
                 ∨ |θ_j|>|θ_i| ∧  op_dir(θ_i,Increment)
                 ∨ |θ_j|<|θ_i| ∧  Increment = θ_i)
   ∨ |M_j|<|M_i|∧ (|θ_j|=|θ_i| ∧  op_dir(θ_i,Increment)
                 ∨ |θ_j|>|θ_i| ∧  op_dir(θ_i,Increment)
                 ∨ |θ_j|<|θ_i| ∧ (op_dir(θ_i,θ_j) ∧ Increment= θ_i
                      ∨ ¬op_dir(θ_i,θ_j)∧ θ_j =0 ∧ Increment= θ_i
                      ∨ ¬op_dir(θ_i,θ_j)∧¬θ_j=0 ∧ Increment=0
                      ∨ ¬op_dir(θ_i,θ_j)∧¬θ_j=0 ∧ Increment= θ_i
                      ∨ ¬op_dir(θ_i,θ_j)∧¬θ_j=0 ∧ op_dir(Increment,θ_i)
                            ))
   ∨ |M_j|>|M_i|∧ (|θ_j|=|θ_i|∧ Increment= θ_i
                 ∨|θ_j|<|θ_i|∧ Increment= θ_i
                  ∨|θ_j|>|θ_i|∧(Increment = pos
                               ∨Increment = neg
                               ∨Increment = 0 )))
∨ moment1_oppose_others(θ_i,M_i,M_j) ⇒
    (θ_i =θ_j ∧ (|θ_i|=|θ_j|∧ Increment = M_i
               ∨|θ_i|<|θ_j|∧ Increment = M_i
               ∨|θ_i|>|θ_j | ∧ (Increment = pos
                               ∨Increment = neg
                               ∨Increment = 0 ))
    ∨¬θ_i = θ_j ∧ (Increment = pos
                 ∨Increment = neg
                  ∨ Increment = 0 ))

∨ moment2_oppose_others(θ_i,M_i,M_j) ⇒
     (|M_j|=|M_i| ∧ Increment = θ_i
     ∨|M_j|<|M_i| ∧ Increment = θ_i
     ∨|M_j|>|M_i| ∧ ( op_dir(θ_i,θ_j) ∧ Increment = θ_i
                   ∨¬op_dir(θ_i,θ_j) ∧ θ_j=0 Increment = θ_i
                   ∨¬op_dir(θ_i,θ_j) ∧ ¬θ_j=0 ∧ (|θ_i|=|θ_j|∧ Increment= θ_i
                                              ∨ |θ_i|>|θ_j|∧ Increment= θ_i
                                              ∨ |θ_i|<|θ_j| ∧(Increment=pos
                                                            ∨Increment=neg
                                                          ∨Increment=0)))))
```

Figure 3.12 Logic representation of displacement compatibility for frame members.

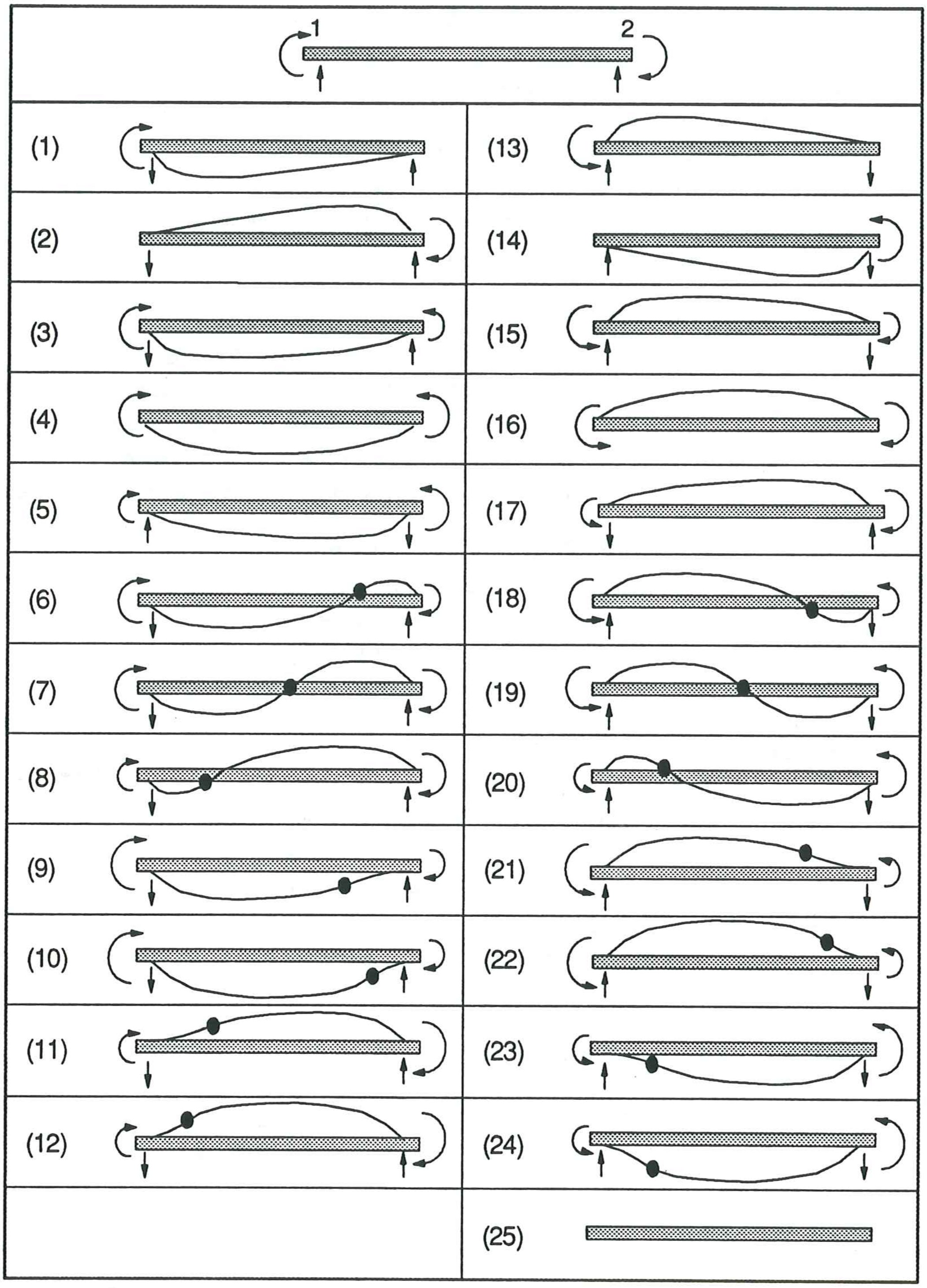



Figure 3.13 Complete set of states for bending of a frame member.

### 3.5.3.- Supports

Supports are components described by the qualitative forces, moments, rotations, and displacements at one connection. Currently, there are four types of supports, pinned, fixed, roller, free, and boundary conditions for planes of symmetry. Figure 3.14 presents the qualitative states for the supports. The forces and rotations are defined by the quantity space { − , 0 , + }. A pinned support transfers only forces, so the qualitative value for the moments is { 0 }. The qualitative value for the displacements is also { 0 }.

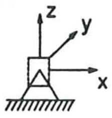
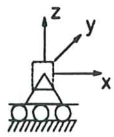
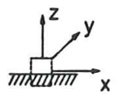
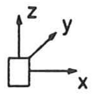
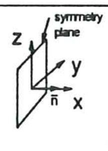
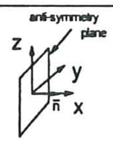
| SUPPORT | | QUALITATIVE VALUES |
|---|---|---|
| simple |  | Fx = {-,0,+} Fy = {-,0,+}<br>Fz = {-,0,+} Mx = My = Mz = {0}<br>Rx = {-,0,+} Ry = {-,0,+}<br>Rz = {-,0,+} dx = dy = dz = {0} |
| roller |  | Fx = Fy = {0} Fz = {-,0,+}<br>Mx = My = Mz = {0}<br>Rx = Ry = Rz = dz = {0}<br>dx = {-,0,+} dy = {-,0,+} |
| clamp |  | Fx = {-,0,+} Fy = {-,0,+}<br>Fz = {-,0,+} Mx = {-,0,+}<br>My = {-,0,+} Mz = {-,0,+}<br>Rx = Ry = Rz = dx = dy = dz = {0} |
| free |  | Fx = Fy = Fz = Mx = My = Mz = {0}<br>Rx = {-,0,+} Ry = {-,0,+}<br>Rz = {-,0,+} dx = {-,0,+}<br>dy = {-,0,+} dz = {-,0,+} |
| symmetry |   | Fx = {-,0,+} Fy = Fz = Mx = {0}<br>My = {-,0,+} Mz = {-,0,+}<br>Rx = {-,0,+} Ry = Rz = dx = {0}<br>dy = {-,0,+} dz = {-,0,+} |
| anti-symmetry |   | Fx = My = Mz = {0} Fy = {-,0,+}<br>Fz = {-,0,+} Mx = {-,0,+}<br>Rx = dy = dz = {0} Ry = {-,0,+}<br>Rz = {-,0,+} dx = {-,0,+} |

Figure 3.14 Qualitative values for forces and displacements for various supports, including planes of symmetry.

Other components such as roller and fixed supports are similarly defined by their traditional structural engineering meaning. A free support is a component with zero forces and moments that displaces in any direction. Symmetric and anti-symmetric boundary conditions facilitate the modeling of symmetric structures with symmetric or anti-symmetric loading, respectively. Non-zero values for supports displacements or loads can be specified.

### 3.5.4.- Shear wall

The current work includes the representation of a shear wall with two connections, as illustrated in Fig. 3.15. The equilibrium equations for the planar shear wall are:

$$F_x^1 + F_x^2 + P = 0$$
$$F_z^1 + F_z^2 = 0$$
$$M_y^1 + M_y^2 + F_z^1 * b + P * L = 0$$

The compatibility relations satisfy the condition that plane sections remain plane:

$$\theta_1 = \theta_2 \qquad d_1 - \theta_1 * b = d_2$$

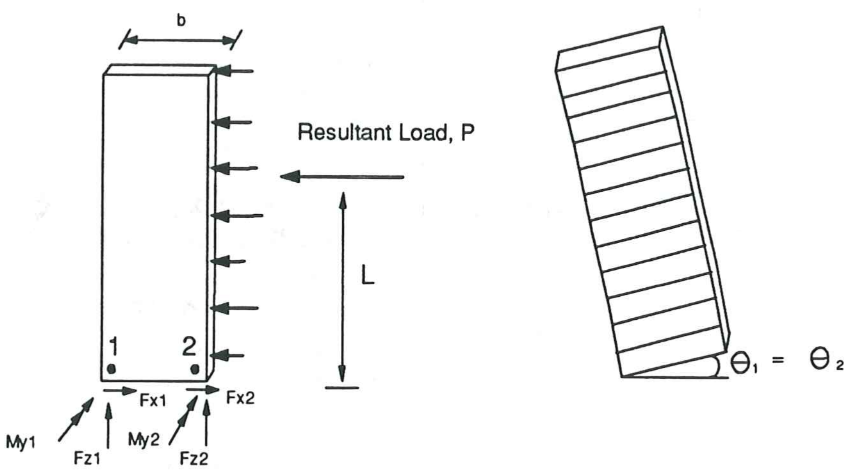



Figure 3.15 Simple wall structural component.

### 3.5.5.- Abstract components

The behavior of complex structural systems can be described by overall characteristics of their behavior. In the previous section the behavior of the shear wall component was abstracted using the hypothesis that plane sections remaining plane. Similarly, a story in a building can be abstracted as a two-tuple component with one connection at the upper level and a second connection at the lower level, as illustrated in Fig. 3.16. The forces and moments at the two levels are related in a manner similar to that of a frame member. Compatibility can be described assuming the story deforms in a shear mode, i.e. the rotations at the connections are equal.

In general a component can be defined by n-tuples and the qualitative states do not have to be complete. This provides the flexibility of including heuristic knowledge about the structural behavior of the component without limiting the capability to derive qualitative behavior. For example, a kinematics assumption similar to the one used by Fruchter et al. [91] can be used in the representation of a frame member with one state, in contrast with the twenty five states derived without heuristics.

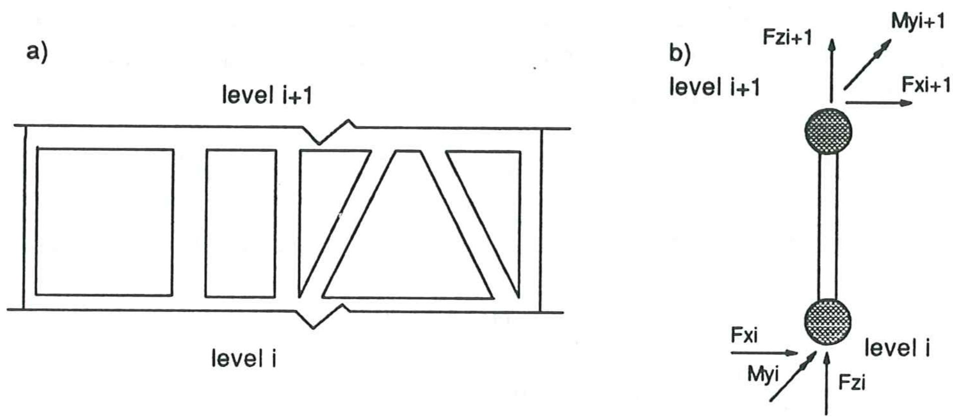



Figure 3.16 An aggregation of frame member components defined as a story in a building.

## 3.6.- FUNDAMENTAL PRINCIPLES FOR STRUCTURAL CONNECTIONS

Equilibrium and compatibility at a connection are represented by processes. Connections are not represented as a separate class of components because the component framework requires defining one component for each class of connection. For example Fig. 3.17 represents four different connection components using the component framework. If a component is added to a connection, a new connection component must be defined. The representation of connections follows the process centered framework. Changes in the states of a component attached to a connection are affected by processes of equilibrium and compatibility. The approach allows a direct definition of the equilibrium and compatibility relationships at the connections without having to define connection components.

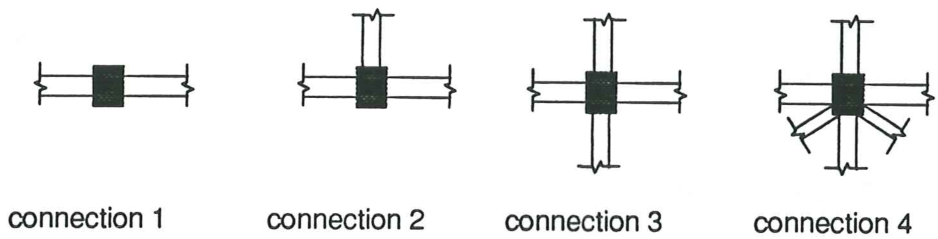



Figure 3.17 Connection components using the component centered framework.

### 3.6.1.- Equilibrium

Equilibrium at a connection requires that the sum of qualitative forces or moments applied at the connection is zero. The forces and moments include the loads applied at the connection and the forces at the ends of the components. The addition of three or fewer forces or moments is represented directly by the extended addition operation. The equilibrium for more than three parameters is described by recursion of the extended addition operation. By using first order logic, for example, the conjunction of the following predicates represents the equilibrium of moments, at a connection of four frame members:

$$\texttt{add}(M_1,\ M_2,\ M_{12}) \wedge \texttt{add}(M_{12},\ M_3,\ M_{4\_op}) \wedge \texttt{equal\_op}(M_{4\_op},\ M_4)\ .$$

where `add` is the extended addition operation and `equal_op` is the operation:

$$M_4 = -M_{4\_op}.$$

The result of applying the equilibrium process at a connection is a number of states for the connection. For example, Fig. 3.18 illustrates the moment equilibrium process at a connection of four frame members with two known bending moments ($M_1$, $M_2$). The quantity space for the moments is `{negative, positive}` and as a result there are four connection states. This representation has been generalized to connections for three- dimensional members.

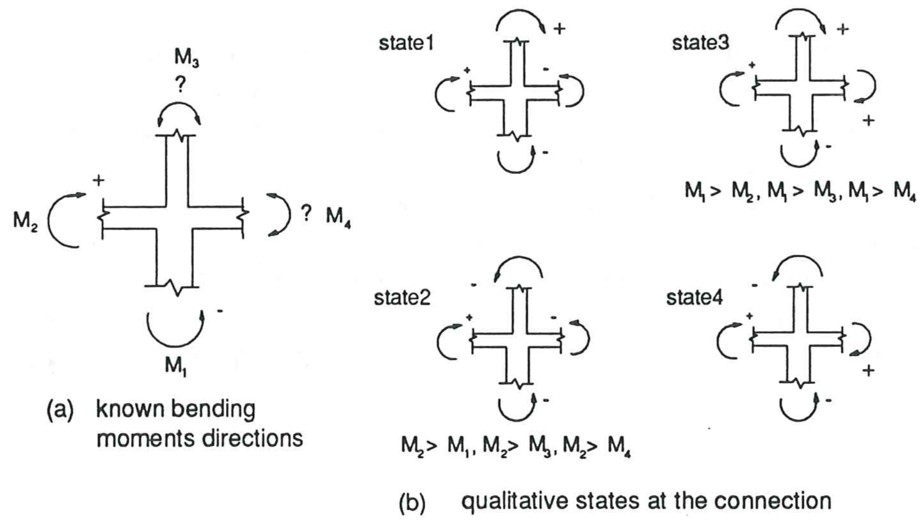
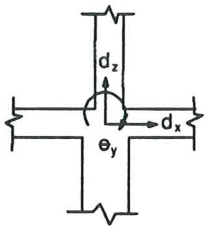



Figure 3.18 Known and unknown moment directions for a connection and related qualitative states.

### 3.6.2.- Compatibility

Compatibility at the connections requires that all the components attached to a connection have the same qualitative rotations and displacements. This is directly achieved in the space centered approach by defining the displacements of the connections as the parameters of the model. In this way, the displacements and rotations for any component attached to a connection are those of the connection. Figure 3.19 illustrates eight qualitative states for the displacements at a connection of a planar member. The representation has been generalized for three-dimensional members.




| state | $\theta_y$ | $d_x$ | $d_z$ |
|---|---|---|---|
| 1 | neg | neg | pos |
| 2 | neg | pos | pos |
| 3 | neg | neg | neg |
| 4 | neg | pos | neg |
| 5 | pos | neg | pos |
| 6 | pos | pos | pos |
| 7 | pos | neg | neg |
| 8 | pos | pos | neg |

Figure 3.19 Qualitative states for a connection.

### 3.7.- SUMMARY

At the conceptual stage of the design process the designer is concerned with overall and possibly undesired behavior of a structure, and the information needed for the formulation of a numerical model is often scarce. Numeric results from a quantitative simulation are useful at later stages of the design process, but it is difficult to reason about structural behavior from numerical results. The design space is usually very large, and therefore it is not practical to consider each design. A description of a conceptual design should incorporate the relevant features of the proposed design without being so specific as to require parameters that are not available.

To provide a framework for reasoning, this chapter has presented a declarative representation for topology, geometry, structural function, and the fundamental laws of equilibrium, compatibility, and material deformation, suitable for the evaluation of conceptual structural designs. The representation is based on the abstraction of a parameter into (1) the sign of the parameter, `positive`, `zero` or `negative`, and (2) the relations in magnitude, `greater_than`, `equal_to`, or `less_than`, between similar parameters. The section behavior is represented by a monotonic relation, such as elastic softening, hardening, or linear, between forces and deformations. This characterization enables the representation of a range of behaviors without requiring precise mathematical definition. A relationship between section behaviors is defined over a range of values. For example, a section is `stiffer` than another section, if for any moment the curvature for the first section is less than the curvature for the second section. Equilibrium, compatibility, and force-deformation laws for components are represented as qualitative states for the component. Equilibrium and compatibility at connections are represented by processes.

# Chapter 4

## THE SPACE CENTERED FRAMEWORK

"But the real frontier is now partial differential equations, especially quantities that vary by space instead of time. Formalizing these *spatial quantities* will allow us to describe a vastly wider range of phenomena that at present. These phenomena include the flow over an airplane wing, the distribution of electric fields due to a distribution of charges, and the stresses on different parts of a bridge."

by Kenneth D. Forbus [90a] page 35.

### 4.1.- INTRODUCTION

The chapter presents a qualitative reasoning system called the space centered framework. The framework is applied to the evaluation of conceptual structural designs using the high-level representation of the principles of equilibrium, compatibility, and force-deformation presented in Chapter 3. The existing qualitative reasoning frameworks are suitable for one-dimensional initial value problems, as indicated in Chapter 2, but they have various deficiencies for representing complicated geometry and spatial relationships. There are many physical systems, such as structures, for which it is more important to represent changes over space than over time. Object instances do not appear or disappear over time but change their qualitative states throughout space. With previous frameworks it is difficult to model physical systems made up of several components that are of the same class but with different locations in space. The need for a qualitative reasoning framework more suitable for spatial systems has been stated by Forbus [90a] and more precisely by Cohn [87] on page 85. These observations motivate the development of the space centered framework for static boundary value problems.

Another deficiency of existing frameworks, when applied to structural engineering, is that they are inefficient even for simple structures. The computation time required for the qualitative simulation of simple one-dimensional fluid flow problems takes minutes using an efficient process centered framework implementation. Considering that a qualitative reasoning methodology for spatial systems requires many more parameters, it is evident that existing frameworks are computationally not viable.

Before presenting the space centered framework it is useful to discuss quantitative methods and symbolic manipulation as problem solving techniques in contrast with qualitative methods. To illustrate the different solution, consider the one bay frame structure as shown in Fig. 4.1. The forces at each connection and the conjugate loads for each component are shown in Figs. 4.1(b) and (c), respectively. Tables 4.1 and 4.2 present the fundamental principles of equilibrium, compatibility, and force-deformation for the components and the connections, assuming a linear elastic material and neglecting axial deformations in the frame members. The bending moment distribution for a frame member is divided in two linear distributions, corresponding to the bending moments at each end. The conjugate loads for the component are defined from each linear bending moment distribution,

$$C^j_i = \int_0^L \frac{M^j_{yi} X}{LEI_i} dx \,.$$

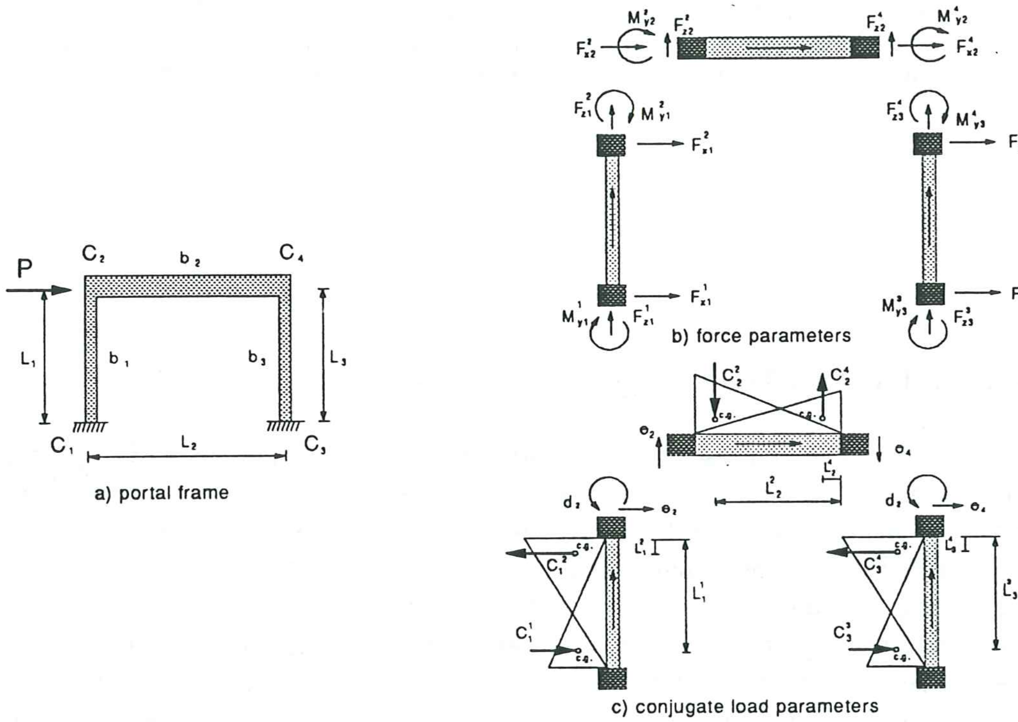



Figure 4.1 Portal frame including definitions of forces and displacements quantities.

## Quantitative solutions

A quantitative solution requires a precise knowledge of the parameters such as section characteristics (EI), connection coordinates, and external loads (P). A structural analysis procedure, such as the stiffness method, solves the governing equations efficiently to give a unique solution. A quantitative solution requires precise values for the parameters, which may not be available at the conceptual design stage. The solution is valid only for the specific values of the parameters. Quantitative solution procedures cannot explain why the behavior occurs in fundamental terms.

Table 4.1 Component laws

| Component | Equilibrium | Compatibility and Force-Deformation |
|---|---|---|
| $b_1$ | $F^1_{x1} + F^2_{x1} = 0$ | $\theta_{y2} = C_1^2 - C_1^1$ |
| | $F^1_{z1} + F^2_{z1} = 0$ | $d_{x2} = C_1^2 L_1^2 - C_1^1 L_1^1$ |
| | $M^1_{y1} + M^2_{y1} = F^1_{x1}\ L_1$ | |
| $b_2$ | $F^2_{x2} + F^4_{x2} = 0$ | $\theta_{y4} = \theta_{y2} + C_2^4 - C_2^2$ |
| | $F^2_{z2} + F^4_{z2} = 0$ | $\theta_{y2} L_2 + C_2^4 L_2^4 - C_2^2 L_2^2 = 0$ |
| | $M^2_{y2} + M^4_{y2} = F^4_{z2}\ L_2$ | |
| $b_3$ | $F^3_{x3} + F^4_{x3} = 0$ | $\theta_{y4} = C_3^4 - C_3^3$ |
| | $F^3_{z3} + F^4_{z3} = 0$ | $d_{x2} = C_3^4 L_3^4 - C_3^3 L_3^3$ |
| | $M^3_{y3} + M^4_{y3} = F^3_{x3}\ L_3$ | |

Symbolic manipulation

Symbolic problem solvers, such as Maxima and Mathematica [Wolfram 91], transform a set of equations into explicit relations for unknown parameters. For the structural analysis problem the governing equations are transformed to explicit relations for the displacements and rotations as functions of the sections characteristics (EI), lengths (L), and external loads (P). The disadvantages of symbolic manipulation are that it requires a precise knowledge of the equations and it cannot explain physical behavior. For example if there is not an explicit function to represent the material characteristics, then symbolic manipulation cannot be used. Symbolic manipulation can only solve problems of limited complexity.

Table 4.2 Connestion laws

| Connection | Equilibrium |
|---|---|
| $c_2$ | $F^2_{x1} + F^2_{x2} = P$ |
| | $F^2_{z1} + F^2_{z2} = 0$ |
| | $M^2_{y1} + M^2_{y2} = 0$ |
| $c_4$ | $F^4_{x2} + F^4_{x3} = 0$ |
| | $F^4_{z2} + F^4_{z3} = 0$ |
| | $M^4_{y2} + M^4_{y3} = 0$ |

Qualitative reasoning

The goal of qualitative reasoning is to search for qualitative values for each unknown parameter that satisfy the fundamental laws of the domain. In qualitative analysis there is not distinction between specified and unknown parameters as exists in traditional structural analysis. For example in Fig. 4.1 the expressions in Tables 4.1 and 4.2 include twenty-one parameters for forces and displacements, not including section characteristics, lengths, and the external load. Considering that the shear and axial forces are constant for each member, the number of parameters reduces to fifteen:

$$\{F_{x1}, F_{z1}, M^1_{y1}, M^2_{y1}, F_{x2}, F_{z2}, M^2_{y2}, M^4_{y2}, F_{x3}, F_{z3}, M^3_{y3}, M^4_{y3}, \theta_{y2}, \theta_{y3}, d_{x2}\}.$$

An exhaustive search for the qualitative solutions is inefficient because with a quantity space of {negative,positive}, there are $2^{15}$ possible combinations of parameters, most of which do not satisfy the governing equations. Thus, the goal of qualitative reasoning is to find a subset from the $2^{15}$ combinations that satisfies the laws of equilibrium, compatibility, and force-deformation. The space centered framework provides an efficient framework to derive the qualitative values; however the objective at this point is to illustrate the problem and not the actual search procedure.

To illustrate the deficiencies in calculus for the frameworks described in Chapter 2, consider one combination of parameters listed in Table 4.3 and illustrated in Fig. 4.2. This solution is used to transform the laws of equilibrium, compatibility, and force-deformation into relations between magnitudes of the parameters since the algebraic signs of the parameters are abstracted by the quantity space. Thus the solution transforms the equations in Table 4.1 and Table 4.2 into the equations in Table 4.4 and Table 4.5, respectively.

Using the calculus for the component framework (see Section 2.3.1), the solution is correct because no equation in Tables 4.4 and 4.5 contradicts the addition operation defined in Table 2.2. The process framework includes parameter relations but their consistency is only maintained by transitivity relations and it would not detect that this solution is inconsistent. The limited calculus of existing qualitative reasoning frameworks predicts that the proposed solution satisfies the governing equations, when in fact it does not.

Table 4.3 A proposed qualitative solution

| Component | $F_x, F_z, M_y$ end 1 | $F_x, F_z, M_y$ end 2 | $d_x, d_z, \theta_y$ end 1 | $d_x, d_z, \theta_y$ end 2 |
|---|---|---|---|---|
| $b_1$ | (neg,neg,neg) | (pos,pos,neg) | (zero,zero,zero) | (pos,zero,neg) |
| $b_2$ | (pos,neg,pos) | (neg,pos,pos) | (pos,zero,neg) | (pos,zero,neg) |
| $b_3$ | (neg,pos,neg) | (pos,neg,neg) | (zero,zero,zero) | (pos,zero,neg) |

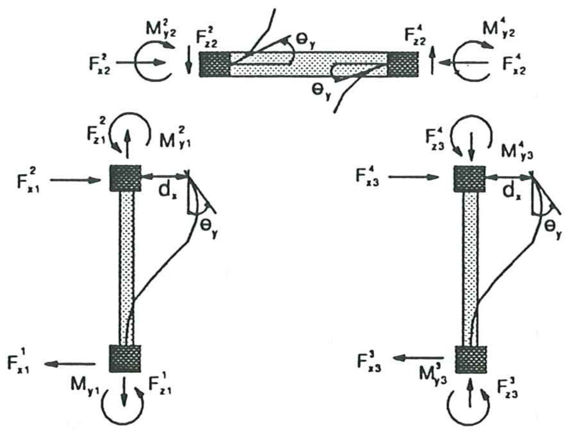



Figure 4.2 Proposed solution presented in Table 4.3.

Table 4.4 Component laws under proposed solution

| Component | Equilibrium | Compatibility and Force-Deformation |
|---|---|---|
| $b_1$ | $\lvert F^1_{x1} \rvert = \lvert F^2_{x1} \rvert$<br>$\lvert F^1_{z1} \rvert = \lvert F^2_{z1} \rvert$<br>$\lvert M^1_{y1} \rvert + \lvert M^2_{y1} \rvert = \lvert F^1_{x1} \rvert L_1$ | $\lvert \theta_{y2} \rvert = \lvert C^2_1 \rvert - \lvert C^1_1 \rvert$<br>$\lvert d_{x2} \rvert = \lvert C^1_1 \rvert L^1_1 - \lvert C^2_1 \rvert L^2_1$ |
| $b_2$ | $\lvert F^2_{x2} \rvert = \lvert F^4_{x2} \rvert$<br>$\lvert F^2_{z2} \rvert = \lvert F^4_{z2} \rvert$<br>$\lvert M^2_{y2} \rvert + \lvert M^4_{y2} \rvert = \lvert F^4_{z2} \rvert L_2$ | $\lvert \theta_{y4} \rvert = \lvert \theta_{y2} \rvert - \lvert C^4_2 \rvert + \lvert C^2_2 \rvert$<br>$\lvert \theta_{y2} \rvert L_2 + \lvert C^2_2 \rvert L^2_2 = \lvert C^4_2 \rvert L^4_2$ |
| $b_3$ | $\lvert F^3_{x3} \rvert = \lvert F^4_{x3} \rvert$<br>$\lvert F^3_{z3} \rvert = \lvert F^4_{z3} \rvert$<br>$\lvert M^3_{y3} \rvert + \lvert M^4_{y3} \rvert = \lvert F^3_{x3} \rvert L_3$ | $\lvert \theta_{y4} \rvert = \lvert C^4_3 \rvert - \lvert C^3_3 \rvert$<br>$\lvert d_{x2} \rvert = \lvert C^3_3 \rvert L^3_3 - \lvert C^4_3 \rvert L^4_3$ |

Table 4.5 Connection laws under proposed solution

| Connection | Equilibrium |
|---|---|
| $C_2$ | $\lvert F^2_{x1}\rvert + \lvert F^2_{x2}\rvert = \lvert P\rvert$<br>$\lvert F^2_{z1}\rvert = \lvert F^2_{z2}\rvert$<br>$\lvert M^2_{y1}\rvert = \lvert M^2_{y2}\rvert$ |
| $C_4$ | $\lvert F^4_{x2}\rvert = \lvert F^4_{x3}\rvert$<br>$\lvert F^4_{z2}\rvert = \lvert F^4_{z3}\rvert$<br>$\lvert M^4_{y2}\rvert = \lvert M^4_{y3}\rvert$ |

To prove that the combination of parameter values does not satisfy the governing equations, consider the compatibility and force-deformation equations for component $b_2$ from Table 4.4:

$$\lvert\theta_{y4}\rvert + \lvert C^4_2\rvert = \lvert\theta_{y2}\rvert + \lvert C^2_2\rvert, \quad \text{Equilibrium of conjugate forces}$$
$$\lvert\theta_{y2}\rvert L_2 + \lvert C^2_2\rvert L^2_2 = \lvert C^4_2\rvert L^4_2, \quad \text{Equilibrium of conjugate moments}$$

From the equilibrium of conjugate forces, the conjugate load at connection $C_4$ is less than the sum of the rotation and conjugate force at connection $C_2$:

$$\lvert C^4_2\rvert < \lvert\theta_{y2}\rvert + \lvert C^2_2\rvert \tag{1}$$

For linear elastic systems the distance of each conjugate load, $C^2_2$, $C^4_2$, from connection $C_4$ are:

$$L^2_2 = \tfrac{2}{3}L_2$$
$$L^4_2 = \tfrac{1}{3}L_2$$

Dividing the equilibrium equation for conjugate moments by $L^4_2$ and taking into account that, $L^2_2 > L^4_2$ and $L_2 > L^4_2$, the equation is rewritten as:

$$\lvert C^4_2\rvert = a\lvert\theta_{y2}\rvert + b\lvert C^2_2\rvert$$

where $a, b$ are greater than one. Finally the equation is transformed to the following relation:

$$\lvert C^4_2\rvert > \lvert\theta_{y2}\rvert + \lvert C^2_2\rvert$$

which contradicts the previously derived equation (1). Hence, the proposed solution is not valid. The example illustrates that: (1) qualitative frameworks may give solutions which do not satisfy the laws of the domain, and (2) the qualitative reasoning problem is to find an assignment of qualitative values with consistent relationships that satisfy the fundamental laws of the domain.

The proposed space centered framework is a qualitative reasoning methodology suitable for boundary value problems. In the current work the framework is presented in the context of structural engineering. As with the component centered framework, components (such as frame members or supports) have qualitative states, consistent with the requirements of equilibrium, compatibility, and material behavior. As with the process centered framework, equilibrium and compatibility at connections are represented as processes, and parameters are represented by qualitative values and parameter relations. As with the unifying framework, constant elimination enhances the qualitative calculus by eliminating solutions that do not satisfy the equilibrium laws.

## 4.2.- QUANTITY SPACE

The quantity space used by the component framework consists of three qualitative values: `negative`, `zero`, and `positive`. The space centered framework uses the same quantity space and the absolute value of parameters is ordered by the relations: `greater_than`, `equal_to`, and `less_than`. The quantity space along with the parameter relations provide the important characteristics of the forces and displacements in structural systems for the purpose of evaluating conceptual designs.

The quantity space is further restricted by eliminating the value zero, except if a quantity is defined to be zero, to improve the inference efficiency. This reduces the expressive adequacy by avoiding solutions in which a particular geometry and stiffness results in a parameter with exactly a zero value. For example in the linear elastic portal frame in Fig. 4.3, the rotation at the right end of the beam is negative as shown in Fig. 4.3.(b) for a particular length and stiffness. A symmetric frame, such as in Fig. 4.3(a), has positive rotations at both ends. It is possible to conclude that the rotation at the ends of the beam can be exactly zero with specific values of stiffness and lengths. Eliminating the value zero from the quantity space does not obscure the structural behavior and it considerably improves the inference efficiency by reducing the search space for solutions.

Eliminating the zero value from the quantity space is too restrictive for symmetric structures in which some parameters are exactly zero, as illustrated for the two story frame in Fig. 4.4. If the shear forces in the second story columns are not specified as zero, the space centered framework would conclude that there is no solution. It is advantageous to use symmetry where possible to model structures because it reduces the number of components and parameters.

## 4.3.- MODELING PRIMITIVES

The modeling primitives represent the fundamental laws of the domain. They have been presented in Chapter 3 for structural engineering problems and are only briefly discussed here. The modeling primitives include individual component qualitative states and connection processes. The component states enumerate the alternative behaviors for the component that satisfy the fundamental laws of the domain. The components have a number of connections and, at each connection, tuples represent the vectorial parameters for the component. For example a frame member component has two connections and at each connection a six-tuple represents the qualitative values for the force and displacement parameters.

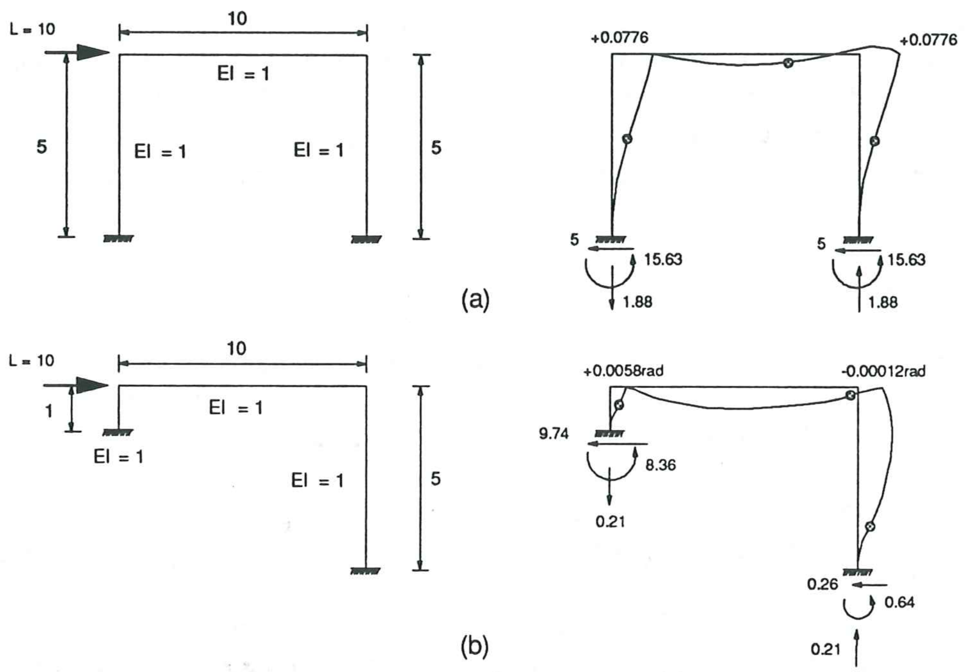



Figure 4.3 (a) Example of symmetric structure with positive rotations at both ends of the beam. (b) Example of stiffness and lengths distributions that induce beam rotations of opposite signs.

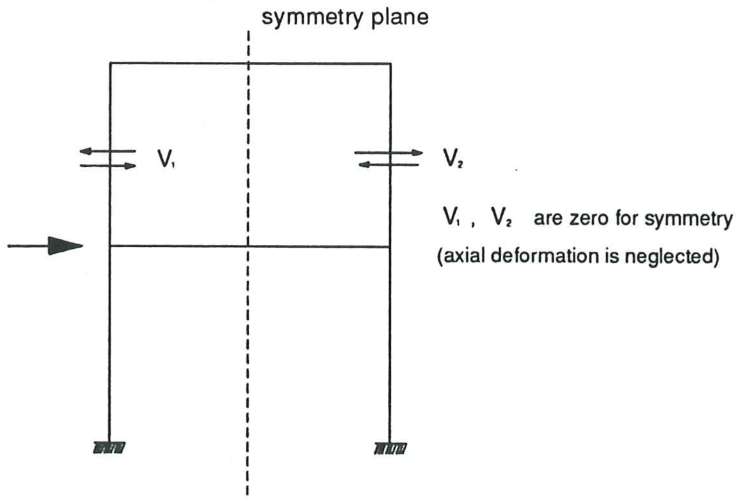



Figure 4.4 Two story frame with zero value for the shear forces at the second story.

The connections processes are relations between qualitative values of the components attached at a connection. The processes are independent of the number of components attached to the connection and they are defined by the vectorial parameters describing component behavior. The result of applying a process to a connection is an enumeration of connection states that satisfy the laws of the domain at the connection. For example, a compatibility process at a connection requires that all components attached to the connection have the same rotations and displacements.

## 4.4.- QUALITATIVE CALCULUS

A qualitative solution satisfies the fundamental laws of the domain if the relations between the parameters are consistent. Qualitative reasoning procedures search for qualitative values and parameter relations that satisfy the fundamental laws. The qualitative calculus defines the operations between qualitative values and verifies the consistency of the relations between the parameters in the model. The space centered framework uses four techniques to verify the consistency of a set of qualitative values and parameter relations:

(1) Basic qualitative calculus operations such as addition or subtraction. These operations are similar to the analogous operations for the real numbers.
(2) Transitivity rules between parameter relations. This is an efficient technique to derive relations such as: if `A>B` and `B>C` then `A>C`.
(3) Constant elimination for linear equations. Linear equations are expressed as the addition of parameters and do not have multiplication or division between parameters. Constant elimination is an algebraic simplifier that derives new equations based on existing ones.
(4) Consistency checking for nonlinear equations. Nonlinear equations are those expressed as the multiplication, or division between parameters. Consistency checking is a technique that verifies the consistency between relations in magnitude for the laws of two components.

These techniques are described in the following subsections.

### 4.4.1.- Basic qualitative calculus operations

The basic qualitative calculus operations are addition, subtraction, multiplication, and division. The space centered framework adds qualitative values using Table 4.6, which is similar to the extended addition table for the process centered framework but incorporates additional relations between parameters. Using this table to perform operations of the form, `R ≅ X⊕Y`, the relations in magnitude between `R` and `X` and between `R` and `Y` are maintained. Consequently, the addition does not introduce ambiguity between the parameter for the sum `R` and the parameters `X, Y`. To illustrate the use of the table, consider the addition of two parameters, `X⊕Y ≅ R`, such that `X ≅[positive]`, and `Y ≅[negative]`. The operation has seven possible results according to the relations between X and Y. For $|X|>|Y|$ there are three results, $|Y|>|R|$, $|Y|=|R|$, $|Y|<|R|$, and for all of them $|X|>|R|$. Subtraction, multiplication, and division operations are performed using Table 4.6 and the opposite sign operation in Table 4.7. The subtraction operation $X \ominus Y \cong R$ is represented by the operation $X \oplus [\ominus Y] \cong R$ where $[\ominus Y]$ is the equal and opposite operation.

The addition of three or more parameters cannot provide parameter relations in general. For example, an equation between four parameters is:

$$|M_{x1}| + |M_{x2}| = |M_{x3}| + |M_{x4}|$$

which does not incorporates additional relations. However, an equation such as:

$$|M_{x1}| + |M_{x2}| + |M_{x3}| = |M_{x4}|$$

incorporates the following parameter relations:

$$|M_{x4}|>|M_{x1}|,\ |M_{x4}|>|M_{x2}|,\ |M_{x4}|>|M_{x3}|.$$

Table 4.6 Addition operation

| Addition | | X | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R≅X⊕Y | | value - | rel(X,Y) | rel(X,R) | rel(Y,R) | value 0 | rel(X,Y) | rel(X,R) | rel(Y,R) | value + | rel(X,Y) | rel(X,R) | rel(Y,R) |
| | - | | | | | | | | | + | > | > | > |
| | | | | | | | | | | + | > | > | = |
| | | | | | | | | | | + | > | > | < |
| | | - | ? | < | < | - | < | < | = | 0 | = | > | > |
| | | | | | | | | | | - | < | > | > |
| | | | | | | | | | | - | < | = | > |
| | | | | | | | | | | - | < | < | > |
| Y | 0 | - | > | = | < | 0 | = | = | = | + | > | = | < |
| | + | - | > | > | > | | | | | | | | |
| | | - | > | > | = | | | | | | | | |
| | | - | > | > | < | | | | | | | | |
| | | 0 | = | > | > | + | < | < | = | + | ? | < | < |
| | | + | < | > | > | | | | | | | | |
| | | + | < | = | > | | | | | | | | |
| | | + | < | < | > | | | | | | | | |

Table 4.7 Equal and opposite operation

| $X \cong \ominus Y$ | X | Y | rel(X,Y) |
|---|---|---|---|
| | + | - | = |
| | 0 | 0 | = |
| | - | + | = |

### 4.4.2.- Transitivity between parameter relations

The basic qualitative calculus operations are limited because they fail to detect inconsistencies between relations. Consider for example the following three equations:

$$|X| = |Z| + |W|$$

$$|W| = |Y| + |T|$$

$$|Y| = |X| + |G|$$

These equations are consistent, with respect to Table 4.6, because each equation individually satisfies the addition operation. However, the equations are not consistent because from the first equation $|X|>|W|$, and from the second equation $|W|>|Y|$. Therefore $|X|>|Y|$, which contradicts the third equation.

An efficient technique, called graph search, derives transitivity relations and detects this type of inconsistency between equations [Forbus 84, Simmons 90]. Transitivity relations are defined by three axioms:

$$A > B, B > C \Rightarrow A > C$$

$$A = B, B > C \Rightarrow A > C$$

$$A = B, B = C \Rightarrow A = C$$

Transitivity relations are efficient to implement. However, they do not eliminate all inconsistent solutions, such as illustrated by the example shown in Fig. 2.9 for the equilibrium laws.

### 4.4.3.- Constant elimination for linear equations

Constant elimination is an algebraic simplifier that derives new equations based on existing ones [Simmons 90, Bredeweg et al. 90]. Constant elimination is defined by the following set of axioms:

For rel : { >, <, = }

$$x \text{ rel } y \Rightarrow (x + z) \text{ rel } (y + z)$$

$$x \text{ rel } y \Rightarrow (x - z) \text{ rel } (y - z)$$

$$x \text{ rel } y \Rightarrow (z - y) \text{ rel } (z - x)$$

$$z > 0 \text{ and } x \text{ rel } y \Rightarrow (x * z) \text{ rel } (y * z)$$
$$z < 0 \text{ and } x \text{ rel } y \Rightarrow (y * z) \text{ rel } (x * z)$$
$$z > 0 \text{ and } x \text{ rel } y \Rightarrow (x / z) \text{ rel } (y / z)$$
$$z < 0 \text{ and } x \text{ rel } y \Rightarrow (y / z) \text{ rel } (x / z)$$

Transitivity relations are special inferences performed by constant elimination. In the space centered framework the transitivity relations are implemented separately for efficiency reasons.

Constant elimination would detect any inconsistency in the relations if it applied to the set of relations obtained from a qualitative solution and all the possible new relations are derived. Unfortunately, this approach is similar to symbolic manipulation techniques and it is computationally too demanding, generating many useless equations.

The approach taken in the current work is to use constant elimination for linear equations in which it is efficient. Linear equations are those expressed as the addition of parameters without multiplication or division. Examples of linear equations are equilibrium relations in which the summation of forces at a connection is equal to zero. Constant elimination for linear equations is a subset of the axioms:

For rel : { >, <, = }

$$x \text{ rel } y \Rightarrow (x + z) \text{ rel } (y + z)$$
$$x \text{ rel } y \Rightarrow (x - z) \text{ rel } (y - z)$$
$$x \text{ rel } y \Rightarrow (z - y) \text{ rel } (z - x)$$

Each time a new equilibrium relation is considered, linear constant elimination derives new relations based on the existing ones. Consider for example an equilibrium relation:

$$|M_{x1}| + |M_{x2}| = |M_{x3}|$$

Each parameter has a set of known equilibrium relations such as the following set for $M_{x1}$:

$$|M_{x1}| = |M_{x4}| + |M_{x6}| + |M_{x2}|$$
$$|M_{x1}| = |M_{x7}| + |M_{x9}|$$

Linear constant elimination derives two new equations such as:

$$|M_{x3}| = |M_{x2}| + |M_{x2}| + |M_{x4}| + |M_{x6}|$$
$$|M_{x3}| = |M_{x2}| + |M_{x7}| + |M_{x9}|$$

From these two equations the following parameter relations are derived,

$|M_{x3}| > |M_{x2}|$, $|M_{x3}| > |M_{x4}|$, $|M_{x3}| > |M_{x6}|$, $|M_{x3}| > |M_{x7}|$, $|M_{x3}| > |M_{x9}|$.

### 4.4.4.- Consistency checking for nonlinear laws

The last technique used by the space centered framework applies to nonlinear equations such as:

| | |
|---|---|
| $V_i \; L_j = M_{ij} + M_{ki}$ | shear/moment equilibrium |
| $\theta_j = \theta_i + \int_0^L \kappa_x dx$ | equilibrium of conjugate forces |
| $d_j = d_i + \theta_i L_i + L^c_i \int_0^L \kappa_x dx$ | equilibrium of conjugate moments |

In the first equation, if the numeric value for the parameter $L_j$ is known the equation is linear, but in qualitative reasoning this is not generally the case. To illustrate the necessity for consistency checking between nonlinear laws, consider the frame in Fig. 4.5. The displaced shape does not satisfy the compatibility relationship for the beam because the displacements at both ends of the beam are not equal, as they should be if axial deformations are neglected. It is not possible to derive that the solution is inconsistent using the basic qualitative operations, transitivity relations, and linear constant elimination. To prove the solution does not satisfy compatibility for the beam, the nonlinear equations for the equilibrium of conjugate forces and moments for the columns need to be considered, as shown later in this subsection.

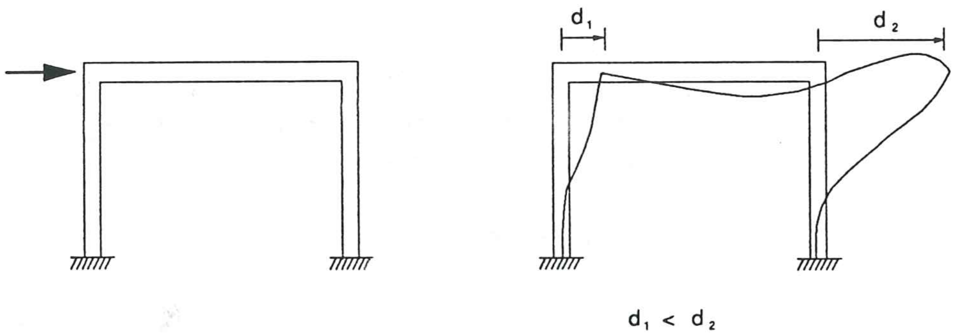



Figure 4.5 Solution which does not satisfy the compatibility law for the beam.

Consistency checking is invoked when a parameter relation is added to the solution. Each component has a small number of nonlinear equations describing its behavior. Checking verifies the consistency of the new parameter relation with respect to the nonlinear equations and existing parameter relations. Consequently, consistency checking is applied between two components at a time. The consistency checking procedure must incorporate all the parameter relations that follow from the equations and existing relationships, otherwise ambiguity causes the inference of inconsistent solutions. The equations of equilibrium for moments and the equilibrium for conjugate forces and conjugate moments are the three nonlinear equations associated with a frame member component. A constraint satisfaction technique verifies the consistency between the equations and derives new parameter relations.

To derive the equations for a frame member, two behaviors are identified: the component is in single curvature or the component is in double curvature. Consistency checking is performed between two components, so there are three cases: both members are in double curvature, both members are in single curvature and one is in double curvature and the other one is in single curvature. To derive the equations for a frame member component with an elastic softening material in single curvature consider Fig. 4.6. The conjugate load, C, is defined as a function of curvature, $\kappa_x$, along the length of the component as:

$$C = \int_0^L \kappa_x dx$$

The end moments vary linearly along the length of the component,

$$M_x = M_1 + (M_2 - M_1)\frac{x}{L}$$

and consequently the conjugate load is expressed by the complementary energy as:

$$C = \frac{L}{M_2 - M_1}\int_{M_1}^{M_2} \kappa_M dM = \frac{L}{M_2 - M_1} E_{comp}$$

where the integral is over the moment-curvature relationship. The complementary energy is directly proportional to the difference in bending moment, so the expression is not convenient because the difference in moments appears in the numerator and denominator producing ambiguity in the qualitative calculus. An expression which avoids the ambiguity is:

$$C = L\kappa_c$$

where $\kappa_c$ is the curvature at a moment $M_c$ that is between the end moments $M_1$ and $M_2$. Similarly the equilibrium in conjugate moments is represented by:

$$C_{load} L_{cg} = L\kappa_c L_{cg}$$

where $L_{cg}$ is the distance from the center of gravity of the conjugate loads to the end connection of the frame member.

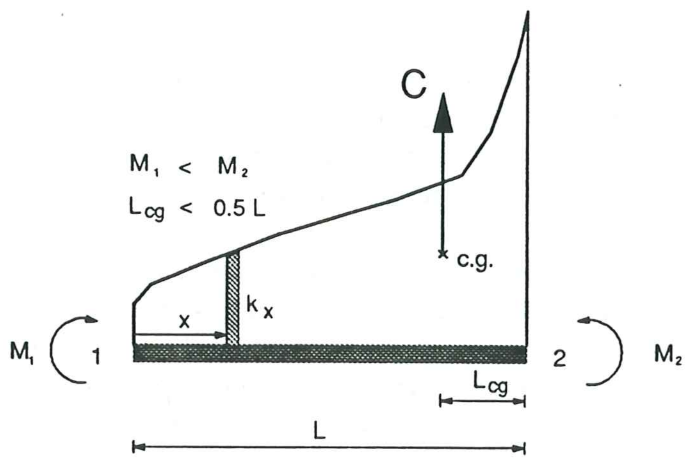



Figure 4.6. Conjugate load for a frame member component.

To derive the equations for a frame member in double curvature, consider a curvature variation along the frame member such as:

$$\kappa_x = \kappa_1 + (\kappa_2 - \kappa_1)(\tfrac{x}{L})^{\frac{1}{n}}$$

where n is an arbitrary positive number and $\kappa_1$, $\kappa_2$ are the curvatures at the ends of a member. The conjugate load and the conjugate moment equilibrium are expressed as:

$$C = (\tfrac{1}{n+1}\kappa_1 + \tfrac{n}{n+1}\kappa_2)L$$

or

$$\kappa_2 = \frac{C(n+1)}{nL} - \frac{\kappa_1}{n}$$

$$C_{moment} = (\tfrac{1}{2(1+2n)}\kappa_1 + \tfrac{n}{1+2n}\kappa_2)L^2$$

From the last two equations, the conjugate moment is represented as a function of the conjugate load as:

$$C_{moment} = C\,\tfrac{n+1}{1+2n}L - \tfrac{1}{2(1+2n)}\kappa_1 L^2 \qquad (2)$$

To illustrate the application of equation (2), consider the inconsistent solution for the frame illustrated in Fig. 4.5, and assume the rotation at the top of the right column is greater than that for the left column. At a particular stage in the search, the parameter relation $eq(d_1, d_2)$ is added into the solution, which initializes the consistency checking procedure between the two columns. The conjugate equilibrium of forces implies that the conjugate load for the right column must be greater than the conjugate load for the left column. In the beam, the right rotation is greater than the left rotation and consequently the moment at the right end is greater than the moment at the left end. By using equilibrium at the connections, the bending moment at the top of the right column is greater than the bending moment at the top of the left column. At this stage in the reasoning process the solution is as indicated in Fig. 4.7. The conjugate loads have opposite signs and the magnitude of $\kappa_2$ is greater than the magnitude of $\kappa_1$, so taking into account signs expression (2) is transformed to:

$$|C_{moment}| = |C|\tfrac{n+1}{1+2n}L + \tfrac{1}{2(1+2n)}|\kappa_1|$$

Furthermore the conjugate load and the curvature at the top of the right column are greater than the ones for the left column, therefore it follows that the conjugate moment is also greater. This result confirms that the solution shown in Fig. 4.5 is inconsistent because by compatibility relationships the conjugate moments must be equal.

The constraint satisfaction technique called redundant views verifies the consistency between equations for components in single or double curvature and known parameter relations in the solution. Redundant views verifies the consistency between the equations by supplying similar additional equations [Leler 88]. Redundant views can be explained by an example for the equilibrium equations of conjugate forces between two components in single curvature:

$$\theta_p = \theta_q + L_i\kappa_{ci} \qquad \text{and} \qquad \theta_m = \theta_n + L_j\kappa_{cj}$$

Using redundant views, the constraint satisfaction problem is represented by two equations as:

$$\theta_p + \theta_m = \theta_q + \theta_n + L_i\kappa_{ci} + L_j\kappa_{cj} \tag{3}$$

$$\theta_p - \theta_m = \theta_q - \theta_n + L_i\kappa_{ci} - L_j\kappa_{cj} \tag{4}$$

Consider two frame member components with the following relationships:

$\theta_p = \theta_m$, $\theta_q = 0, \theta_n = 0$, $L_i = L_j$, $M_p > M_q$, $M_m > M_n$, $M_q > M_m$

From equation (4) it follows that:

$$0 = 0 + \kappa_{ci} - \kappa_{cj} \qquad \text{or} \qquad \kappa_{ci} = \kappa_{cj}$$

but taking into consideration the relationships between the bending moments it follows that $\kappa_{ci} > \kappa_{cj}$, because $M_p + M_q > M_m + M_n$. Therefore the relationships between the two frame member components are inconsistent.

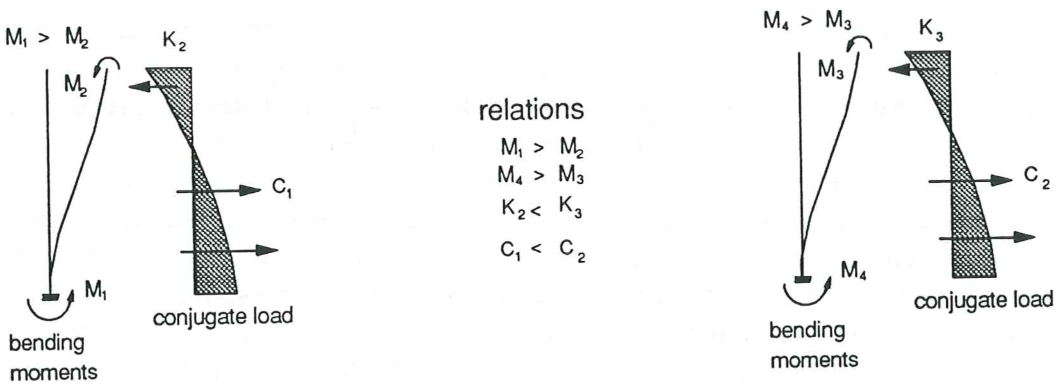



Figure 4.7 Partial information of the behavior for the columns for the frame in Figure 4.5.

Taking moments at each frame member connection, the equilibrium of conjugate moments is expressed by two equations, so for two components there are four combinations. The redundant views procedure adds and subtracts each of these four equations and the constraint satisfaction technique verifies the consistency of the resulting eight equations.

## 4.5.- INFERENCE SCHEME

The inference scheme is the procedure that combines the parameter values for the component states and the connection processes in the search for valid solutions. The inference scheme is related to the qualitative calculus because the calculus verifies the consistency between qualitative values and parameter relations. The inference scheme for the space centered framework consists of the two steps illustrated in Fig. 4.8:

(1) The elaboration step augments an initial model description by adding qualitative values derived without ambiguity.
(2) The solution propagation step completes the initial description by deriving values for unknown parameters in the model.

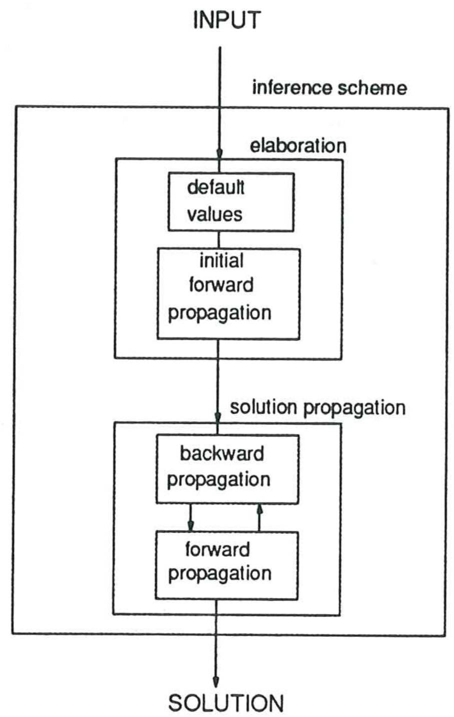



Figure 4.8 Overview of the inference process by the space centered framework.

### 4.5.1.- Elaboration step

The elaboration step is performed once at the beginning of the inference process. The motivation for the elaboration step is that an intelligent problem solver should not spend much time on simple problems. Elaboration enhances an initial model description adding default values and values derived without ambiguity. The first step in the elaboration is to incorporate default parameter values for the components in the model. For example, the out-of-plane forces and displacements for a planar frame are zero. Another example of default values are the zero displacements and rotations for a fixed support.

The second step during the elaboration is the initial forward propagation. In this step each component in the model is used as a starting point for the forward propagation procedure. The forward propagation infers values that are consistent with fundamental laws of the domain without ambiguity. The result of the initial forward propagation is: (1) it does not derive any values, (2) it derives some qualitative values, or (3) it detects a contradiction. Whether the procedure derives values or not, the propagation finishes satisfactorily. If the propagation detects a contradiction, the inference stops and so does the simulation. Consider the example in Fig. 4.9(a) with a spring and a downward force applied at one connection. From the equilibrium law, it follows that the direction of the force in the other connection is upward without ambiguity. Similarly, consider a connection between two frame members, as illustrated in Fig. 4.9(b). One frame member has a positive bending moment and it follows without ambiguity, also by equilibrium, that the other frame member must have a negative bending moment. Compatibility infers without ambiguity that all the component connected at a rigid connection have the same displacements, as illustrated in Fig. 4.9(c). As a final example, Fig. 4.9(d) illustrates a connection between two frame members and a horizontal external load that is not in equilibrium. The elaboration stops the inference and indicates that there is no solution for the model description.

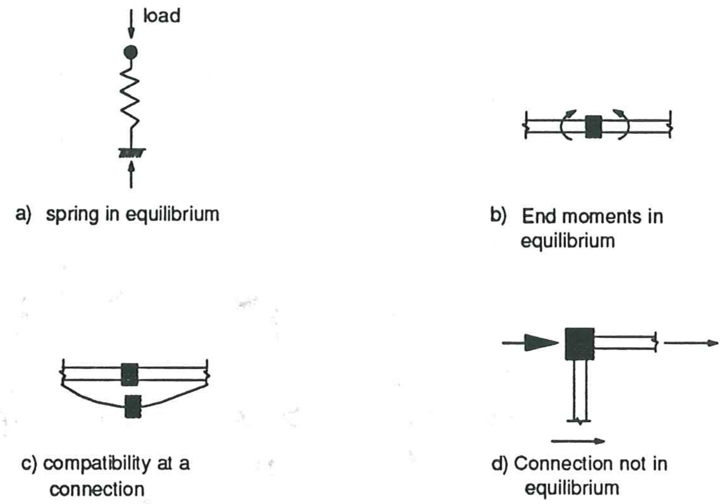



Figure 4.9 Examples of inferences derived without ambiguity.

As another example of the initial forward propagation during elaboration, consider the planar frame in Fig. 4.10. The first step of elaboration derives default values such as the out-of-plane forces equal to zero. The initial forward propagation applies equilibrium, compatibility, and force-deformation to each component in the structure and enhances the initial description as illustrated in Fig. 4.10(b) and Table 4.8. The methodology for the forward propagation is described in Section 4.5.2

Table 4.8 Values inferred by the elaboration for the frame in Figure 4.10

| | forces(before elaboration) | | forces(after elaboration) | |
|---|---|---|---|---|
| | end 1 $(F_x, F_z, M_y)$ | end 2 $(F_x, F_z, M_y)$ | end 1 $(F_x, F_z, M_y)$ | end 2 $(F_x, F_z, M_y)$ |
| $b_1$ | (?,?,?) | (?,?,?) | (neg,neg,0) | (pos,pos,neg) |
| $b_2$ | (?,?,?) | (?,?,?) | (0,neg,pos) | (0,pos,0) |
| $s_1$ | (?,?,?) | | (pos,pos,0) | |
| $s_2$ | (?,?,?) | | (0,neg,0) | |


Figure 4.10 Frame structure illustrating elaboration: (a) initial description, (b) qualitative values derived without ambiguity.

The elaboration step has two important features: it detects structural instabilities because of the lack of an equilibrium solution; and the reasoning process for many statically determinate structures is accomplished in the elaboration step. An instability due to the lack of a load path to transfer the external loads is detected during the initial forward propagation. For example, Fig. 4.11 illustrates a framed structure in which the external lateral load cannot be transferred to the foundation because the structure is unstable. The columns cannot develop shear forces because of the hinges at the ends. Connection equilibrium at the bottom of the columns infers without ambiguity that the support does not have an horizontal reaction. Connection equilibrium at the top of the columns infers that the load is equilibrated by the beam. Equilibrium of the beam indicates that the force at the left end is equal to the force at the right end. By connection equilibrium the right column must transfer an horizontal force. This is a contradiction, so the elaboration concludes that there is no equilibrium solution.

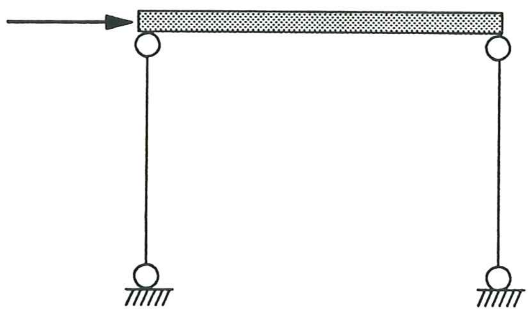


Figure 4. 11 Unstable structure detected during elaboration.

The planar truss in Fig. 4.12 illustrates a structure for which the elaboration cannot detect an instability. The forward propagation of values stops leaving an apparently statically indeterminate substructure, as shown in Fig. 4.12(b), because no additional values can be inferred without ambiguity. The elaboration fails to detect that the structure in unstable because equilibrium of the forces and the moments at the connections is not enough to detect the instability. Instead, equilibrium of moments for a free body diagram such as in Fig. 4.12(b) is necessary to detect the instability. The inference process will conclude that there is no solution, but the result is obtained from the backward-forward propagation procedure.

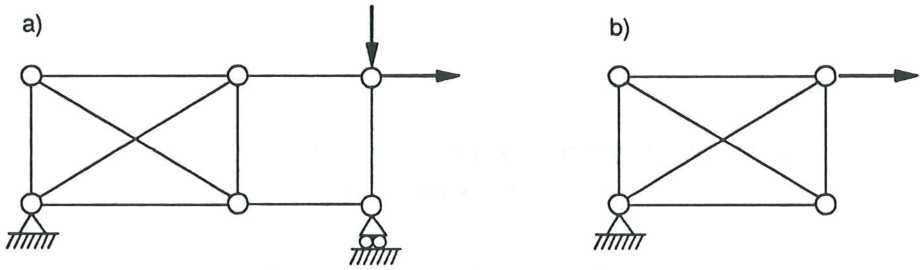



Figure 4.12 (a) Unstable structure detected by the inference process but not by the elaboration. (b) Free body diagram not included in the elaboration step.

An unstable structure either cannot reach an equilibrium state or has an equilibrium state, but under small changes in displacements it does not remain in equilibrium. The instabilities detected by the elaboration and inference scheme correspond to the first class of instability. The second class of instabilities, caused by bifurcation, are not detected in the inference process. For example, the pendulum shown in Fig. 4.13 is in equilibrium, although it is unstable because a small rotation would increase the external work and therefore it would decrease the potential energy. Presently, the inference scheme incorporates equilibrium, compatibility of small displacements, and force-deformation, so it does not detect bifurcation instabilities.

Figure 4.13 Unstable structure in equilibrium.

Finally, the elaboration procedure derives the solution for many statically determinate structures. Consider for example the determinate frame in Fig. 4.14. The elaboration step derives the qualitative values of forces shown in Fig. 4.14(b). The elaboration does not infer displacements and rotations of the structure because information about length or material characteristics is not specified.

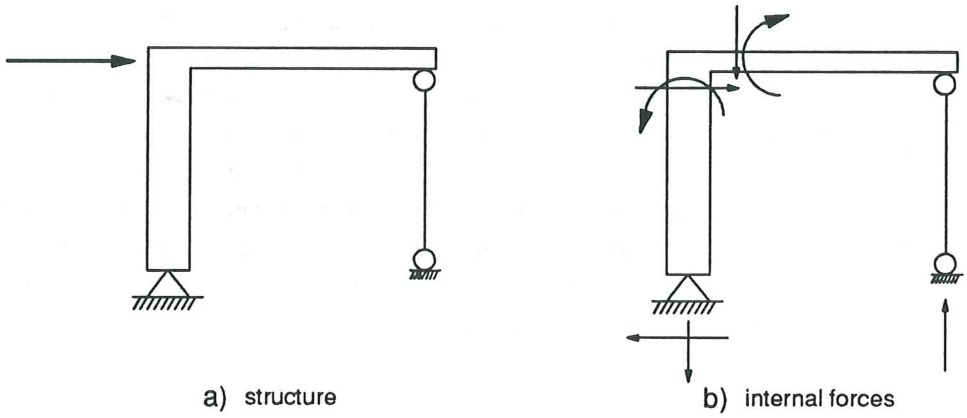



Figure 4.14 Elaboration step for a statically determinate structure.

#### 4.5.2.- Solution propagation

The solution propagation is invoked when the elaboration step does not provide a full description of the parameter values, which is typically the case for a statically indeterminate structure. The solution propagation uses backward and forward chaining. The goal of the solution propagation is to infer qualitative values for the parameters in the model that are consistent with the laws of the domain. The solution propagation starts by assuming qualitative values for a component in a backward chaining propagation. With the new information, the forward chaining propagation derives additional values.

The solution propagation is similar in concept to the flexibility method for structural analysis. The backward chaining procedure "cuts" a component at its connections and assumes values for the component parameters. The forward chaining procedure considers the structure without the component and applies the assumed forces and displacements at the component connections as prescribed forces and displacements. If the "cut" transforms the structure into a statically determinate free-body diagram, the forward chaining procedure can infer values for all the parameters. If the forward chaining procedure does not derive values for all the parameters, the backward chaining procedure makes another cut. The solution propagation proceeds until it derives a complete description of the parameters in the model.

Backward propagation

The backward or goal driven procedure starts with a list of components with parameter values that are given by the user, derived by the elaboration step, or are ambiguous. It is called a backward or goal driven procedure because it derives qualitative solutions by assuming component states. The backward propagation has two steps: to derive a valid qualitative state for a component and to derive a connection state that satisfy the connection processes. A component has a number of states which satisfy the laws of the domain, as illustrated in Fig. 3.13 for a planar frame member. The first step during the backward propagation is to select the component states which are consistent with the known values for the parameters. For example, consider the frame member with parameter values shown in Fig. 4.15. The known values reduce the frame member states from the possible twenty-five states in Fig. 3.13 to only two possible states in Fig. 4.15. One state corresponds to a positive bending moment at the right end and the second state corresponds to a negative bending moment. The state corresponding to zero bending moment is not considered because the quantity space only includes zero if the user defines a parameter as zero. Using the available information to reduce the number of component states, the efficiency of the inference process is considerably increased.

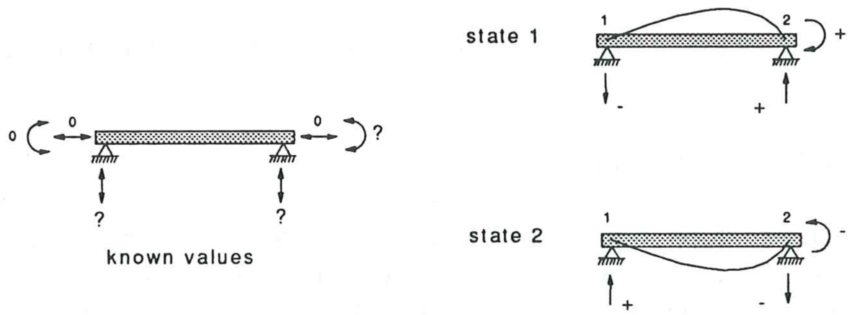



Figure 4.15 Known parameter values and derived component states for a frame member.

Table 4.9 presents the initially known values and the two component states for the beam in Fig. 4.15, in a representation which is useful to explain subsequent examples. At each connection a six-tuple represents forces and moments according to the following convention, $(F_x, F_y, F_z, M_x, M_y, M_z)$, and another six-tuple represents displacements and rotation according to the following convention $(\theta_x, \theta_y, \theta_z, d_x, d_y, d_z)$. The parameter relations between forces are not included in Table 4.9 because forces along a frame member are represented by the parameters directly. For clarity, the relations between displacements, which all have zero values, are also not included in the table. The backward chaining procedure selects a valid component state in Table 4.9. Afterwards, if the connection processes or the qualitative calculus detects an inconsistency, then the backward chaining procedure disregards a previous state and selects the another valid component state.

Table 4.9 Qualitative states for the frame member in Figure 4.15

| | forces | | displacements | | relations | |
|---|---|---|---|---|---|---|
| | end 1 | end 2 | end 1 | end 2 | displacements | forces |
| known vals. | (0,0,?,0,0,0) | (0,0,?,0,?,0) | (0,?,0,0,0,0) | (0,?,0,0,0,0) | | |
| state 1 | (0,0,-,0,0,0) | (0,0,+,0,+,0) | (0,-,0,0,0,0) | (0,+,0,0,0,0) | $\lvert\theta_y^2\rvert > \lvert\theta_y^1\rvert$ | $\lvert M_y^2\rvert > \lvert M_y^1\rvert$ |
| state 2 | (0,0,+,0,0,0) | (0,0,-,0,-,0) | (0,+,0,0,0,0) | (0,-,0,0,0,0) | $\lvert\theta_y^2\rvert < \lvert\theta_y^1\rvert$ | $\lvert M_y^2\rvert < \lvert M_y^1\rvert$ |

The second objective of the backward propagation is to select qualitative values that are in agreement with the processes at the connections. In the context of structural engineering, backward propagation derives values for forces and displacements at connections that are consistent with equilibrium and compatibility processes. To illustrate this goal, consider the continuous beam shown in Fig. 4.16(a). The beam consists of two frame members, each with the states presented in Table 4.9. The backward propagation for the components assumes the first valid state, or state 1 in Table 4.9, for the left component. With this information the backward propagation for connections derives three possible states for the forces and displacements at the connection 2, as illustrated in Fig. 4.16(b) and Table 4.10.

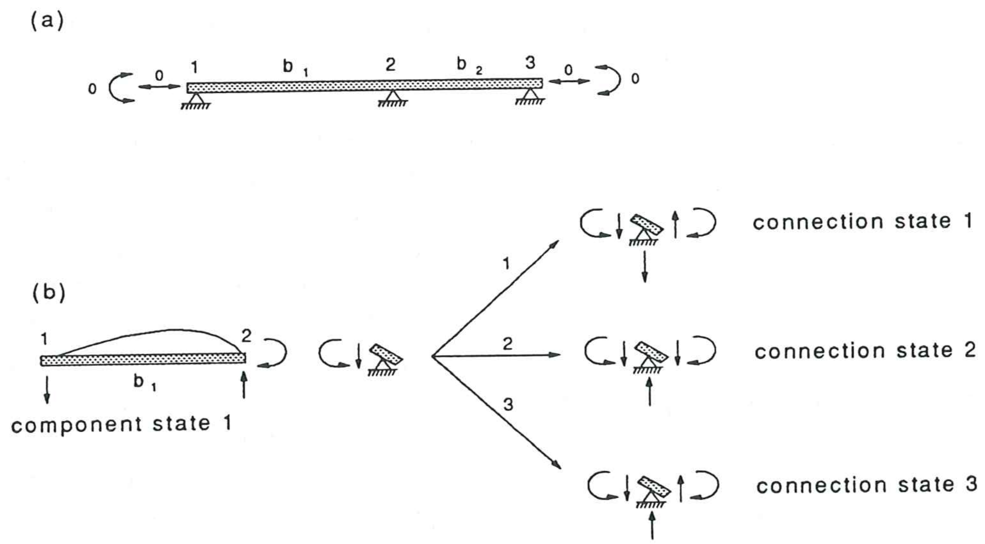



Figure 4.16 (a) Continuous beam. (b) States inferred by the backward propagation for compatibility and equilibrium at connection 2.

Table 4.10 Connection states resulting from state one for component $b_1$ in Figure 4.16

| conn. state | $b_1$ | | | | $b_2$ | | | |
|---|---|---|---|---|---|---|---|---|
| | forces | | displacements | | forces | | displacements | |
| | end 1 | end 2 | end 1 | end 2 | end 2 | end 3 | end 2 | end 3 |
| 1 | (0,0,-,0,0,0) | (0,0,+,0,+,0) | (0,-,0,0,0,0) | (0,+,0,0,0,0) | (0,0,-,0,-,0) | (0,0,?,0,?,0) | (0,+,0,0,0,0) | (0,?,0,0,0,0) |
| 2 | | | | | (0,0,+,0,-,0) | (0,0,?,0,?,0) | (0,+,0,0,0,0) | (0,?,0,0,0,0) |
| 3 | | | | | (0,0,-,0,-,0) | (0,0,?,0,?,0) | (0,+,0,0,0,0) | (0,?,0,0,0,0) |

Forward propagation

The forward propagation procedure is executed after the backward chaining procedure. The propagation is termed data driven because it uses the new qualitative values from the backward procedure to infer unambiguous values. For structural engineering problems, the forward propagation infers values according to component states and the processes of equilibrium and compatibility. In contrast with backward chaining, forward chaining infers unambiguous values and the goals are executed following paths along the structure.

A structure is represented as a graph formed by components and connections (see Section 3.3). Each time a connection process adds values for forces or displacements, a procedure similar to depth-first search traverses the paths through the structure starting at the connection. The difference with depth-first search is that a node in the graph (a component or a connection) may be visited more than once during the propagation. A node is revisited if the component or connection adds new qualitative values to the solution. The result of a propagation is:

(1) The path leads to ambiguity and therefore it is not pursued further.
(2) The path leads to the resolution of qualitative values that are consistent with previous information and those values are accepted into the solution.
(3) The path leads to the resolution of qualitative values that are not consistent with previous information and the assumed state of the component is rejected.

If the latter occurs, the forward propagation terminates and the backward chaining procedure resumes. The procedure terminates when all qualitative values have been determined. The propagation always terminates and infinite cycling is not possible because a component or connection is revisited only if it adds new qualitative values.

As an example consider the continuous beam and the three solutions generated by the backward propagation illustrated in Fig. 4.16(b) and Table 4.10. The forward propagation takes the first state for connection $c_2$ and attempts to derive a valid qualitative state for component $b_2$. However, there is no valid state because the moment and the shear in component $b_2$ are not in equilibrium. With this contradiction, the forward propagation terminates and the inference scheme returns to the backward chaining propagation which derives the second state for connection $c_2$. Forward propagation resumes with the second state and attempts to derive a valid qualitative state for component $b_2$. Again there is not a valid state because the middle connection has a positive rotation but the left end moment for component $b_2$ is negative. Similarly, state 3 for connection $c_2$ is not correct because the shear and moment for component $b_2$ are not in equilibrium. The backward propagation resumes and derives the second state for component $b_1$. This state corresponds to the real solution and consequently the forward propagation does not detect a contradiction.

As a second example, consider the forward propagation procedure for the equilibrium process in Fig. 4.17. For this two story frame a component state is assumed for the left first level column, as shown in Fig. 4.17(b). The forward propagation traverses the path "c2-c5-c6-c3" until no further inferences can be made because an ambiguity is reached at connection c3, as shown in Fig. 4.17(c). Then the forward propagation traverses the second path "c2-c3-c4" until no further inferences can be made, as shown in Fig. 4.17(d). The first path stops at connection c3 because it is not possible to infer qualitative values for the components connected at c3 with the information available at that stage. However, the second path can infer qualitative values for the components connected to c3.

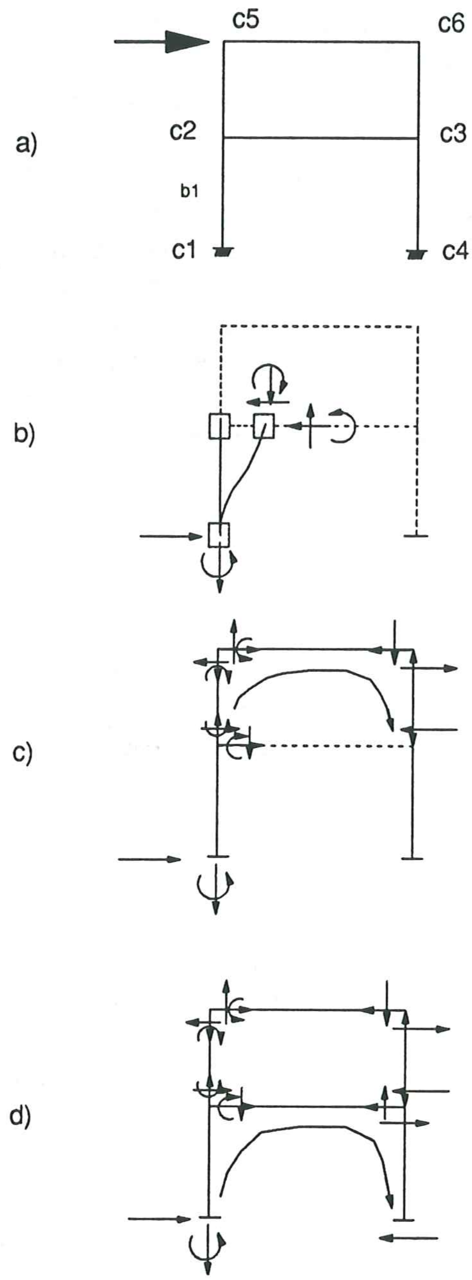



Figure 4.17 Forward propagation of the equilibrium solution for a planar frame.

### 4.5.3.- Example of inference scheme

The planar frame in Fig. 4.18 is used to demonstrate the efficiency of the inference scheme compared with an exhaustive search procedure. The frame has thirty-six parameters: twelve displacements and rotations and twenty-four forces and moments. By using the quantity space `{negative, positive}` an exhaustive search would have to test $2^{36}$ combinations of values for this simple problem. However, the inference scheme substantially reduces the search space, so the inference process is very efficient.

Each column has ten bending states illustrated in Fig. 4.18. Recognizing a column can resist axial compression or tension there are twenty states for a column. With the aforementioned quantity space, component $b_2$ has fourteen bending states, as illustrated in Fig. 3.13. The inference process starts with the elaboration which determines that the out-of-plane forces and displacements are zero, as shown in Fig. 4.19. The initial forward propagation process establishes values for the components attached to supports. Axial deformations are neglected and therefore this procedure asserts zero values for the vertical displacements.

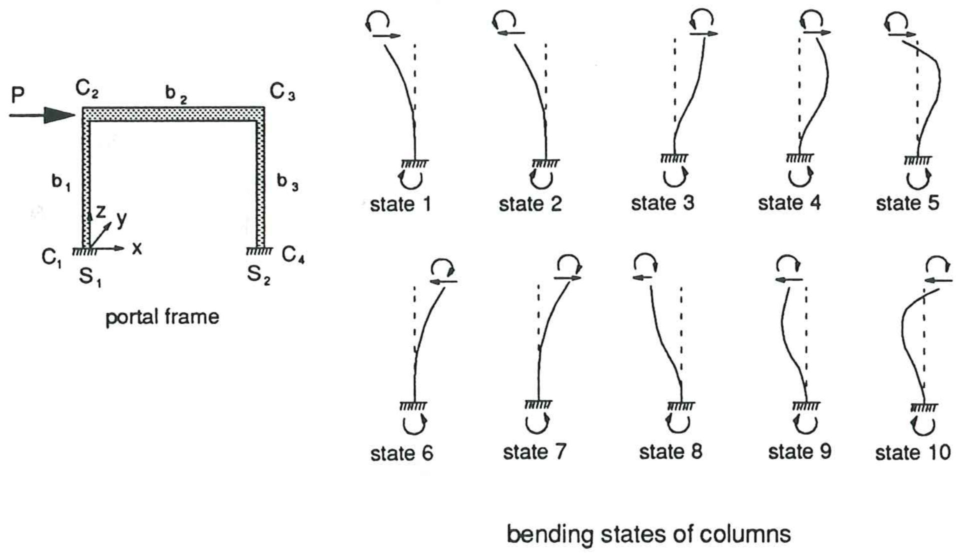



Figure 4.18 Example to illustrate inference of bending states for the columns in a portal frame.

The elaboration procedure does not infer values for all the parameters, so the backward-forward propagation procedure is invoked. The inference begins by assigning the first qualitative state for component $b_1$. The equilibrium and compatibility processes at the connections $C_1$, $C_2$ assign values for the components $b_2$ and support $S_1$. The forward propagation detects that the qualitative values for component $b_2$ are not consistent because there is not a valid component state, as presented in the table at the bottom of Fig. 4.19. There is no valid state for component $b_2$ because there is no compatibility in vertical displacements. Backward chaining resumes with the equilibrium and compatibility processes at connection $C_2$. Both processes, however, have one state, so the backward chaining derives the second state for component $b_1$. The inference scheme is efficient because the forward propagation detects that the first qualitative state for component $b_1$ is inconsistent and that it should not be pursued.

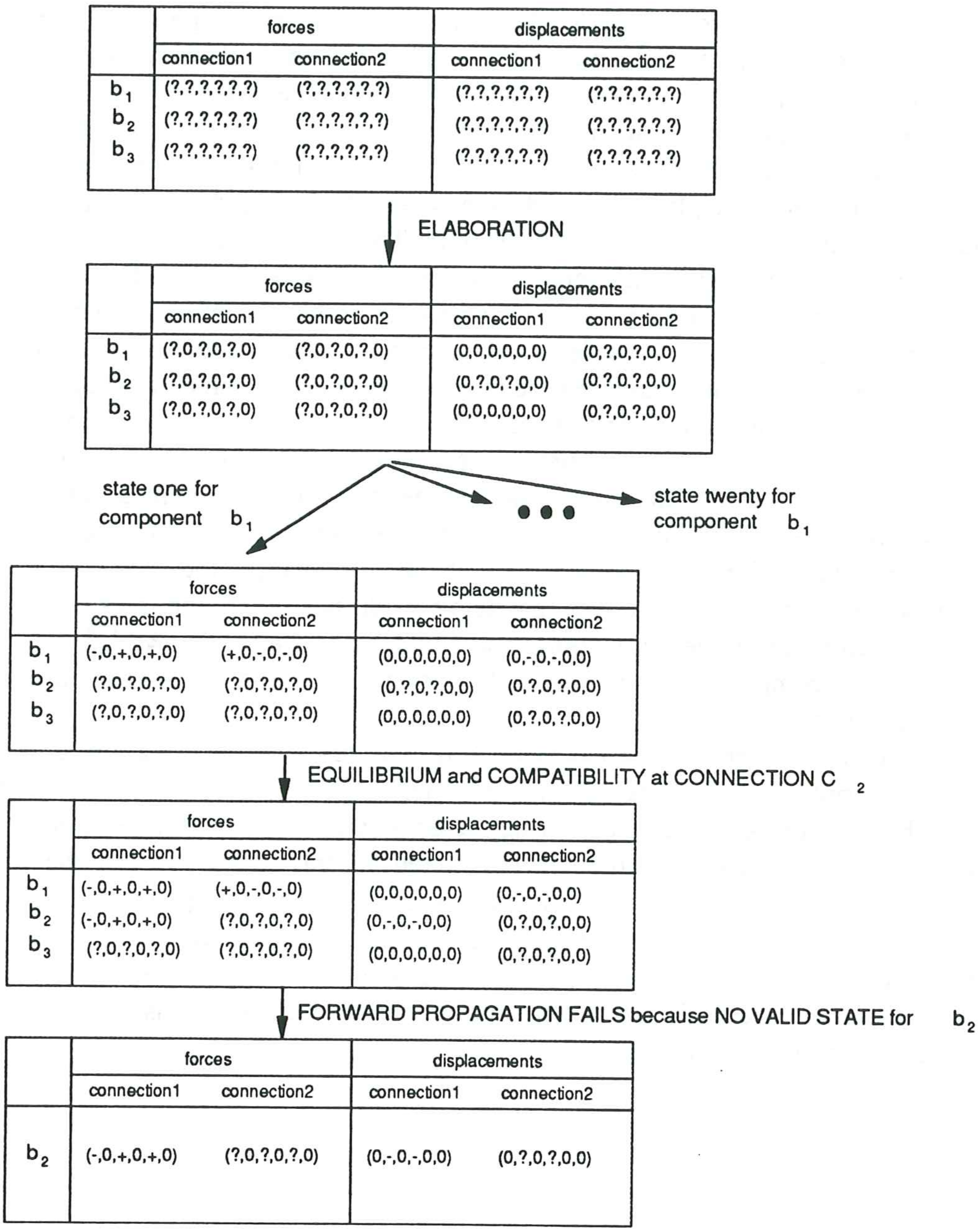



Figure 4.19 Inference scheme example for the portal frame in Figure 4.18. Forces and displacements are in global coordinate axes, X, Y, Z.

Continuing with the inference scheme, the frame has one valid solution for the qualitative value of forces and moments and five valid solutions for rotations and displacements. An application in Chapter 6 presents these solutions. The solutions for displacements are reduced by supplying information about the lengths and section characteristics of the members.

## 4.6.- SUMMARY

The space centered framework is a qualitative reasoning framework suitable for static boundary value problems because it incorporates geometry and spatial relationships. In contrast to previous qualitative reasoning frameworks, it represents three-dimensional geometry and spatial relationships. The space centered framework is suitable for the evaluation of the load transfer characteristics of conceptual structural designs. From a high-level description of the conceptual design and a representation of the fundamental principles of equilibrium, compatibility, and force-deformation, the framework infers a set of structural behaviors. The behaviors include a variety of cases that are possible with incomplete information about geometry and material characteristics. The solutions may include undesirable structural behavior and this information is available to the designer early in the design process. The validity of a solution can be explained based on the representation of fundamental principles.

The space framework uses the quantity space `{negative,zero, positive}` and the parameter relations `{greater_than,equal_to, less_than}`. The qualitative calculus defines four techniques, basic addition operation, transitivity relations, linear constant elimination, and consistency checking. Constant elimination and consistency checking enhance the qualitative calculus eliminating the combinations of parameters that do not satisfy the laws of the domain. The modeling primitives are component qualitative states that satisfy the laws of equilibrium, compatibility and material characteristics and connection processes of equilibrium and compatibility.

The inference scheme has two steps. The first step, elaboration, enhances the initial description by deriving qualitative values that follow from the initial description. Unstable structures are detected during the elaboration stage and the solution for many statically determinate structures is completed in this step. The second step is a backward-forward propagation that starts by assuming a qualitative state for a component. The new information added by this component state is propagated through the structure using the topology of the structure. This inference scheme detects early in the reasoning process if a combination of components states is not valid, so the inference scheme is very efficient.

# Chapter 5

## IMPLEMENTATION OF THE SPACE CENTERED FRAMEWORK

### 5.1.- INTRODUCTION

In the context of structural engineering, the space centered framework has been implemented in a computer program named *Agrippa*[1] using the computer language Prolog. As discussed in Section 4.1, an exhaustive search for the qualitative solutions is inefficient because of the large search space. The inference scheme for *Agrippa* considerably reduces the space, making the reasoning process very efficient. This chapter describes the implementation of *Agrippa* for the evaluation of complex planar structures and simple three-dimensional structures. An introduction to Prolog is included in the Appendix B.

A session with *Agrippa* is divided into four stages (1) model specification, (2) inference scheme, (3) qualitative solutions and post-processing, and (4) qualitative improvement or modification of the design solution. This chapter focuses on the implementation of the inference scheme and qualitative calculus because they are the central and most complex stages of *Agrippa.* Figure 5.1 presents an overview of the program and the twelve out of the fourteen modules that constitute the program. The two additional modules define general predicates that are used throughout the code. Figure 5.1 also indicates that the inference scheme and the qualitative calculus are related. In Chapter 4, the qualitative calculus is presented before the inference scheme, but in this chapter the qualitative calculus is presented after the inference scheme. The reason for the different order in presentation is that the inference scheme uses qualitative calculus operations, which are best illustrated by specifying how they are used to infer solutions.

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1 The name comes after the largest shell of the antiquity built by the Romans in 124 A.D.

## 5.2.- MODEL SPECIFICATION

The model specification for *Agrippa* consists of a precise representation about object instances and topology, but an imprecise representation about geometry and the parameters values for the model. The model name, object instances, and topological attributes are the minimal information that must be specified. The geometry, qualitative values, and parameter relations may be partially or totally unknown.

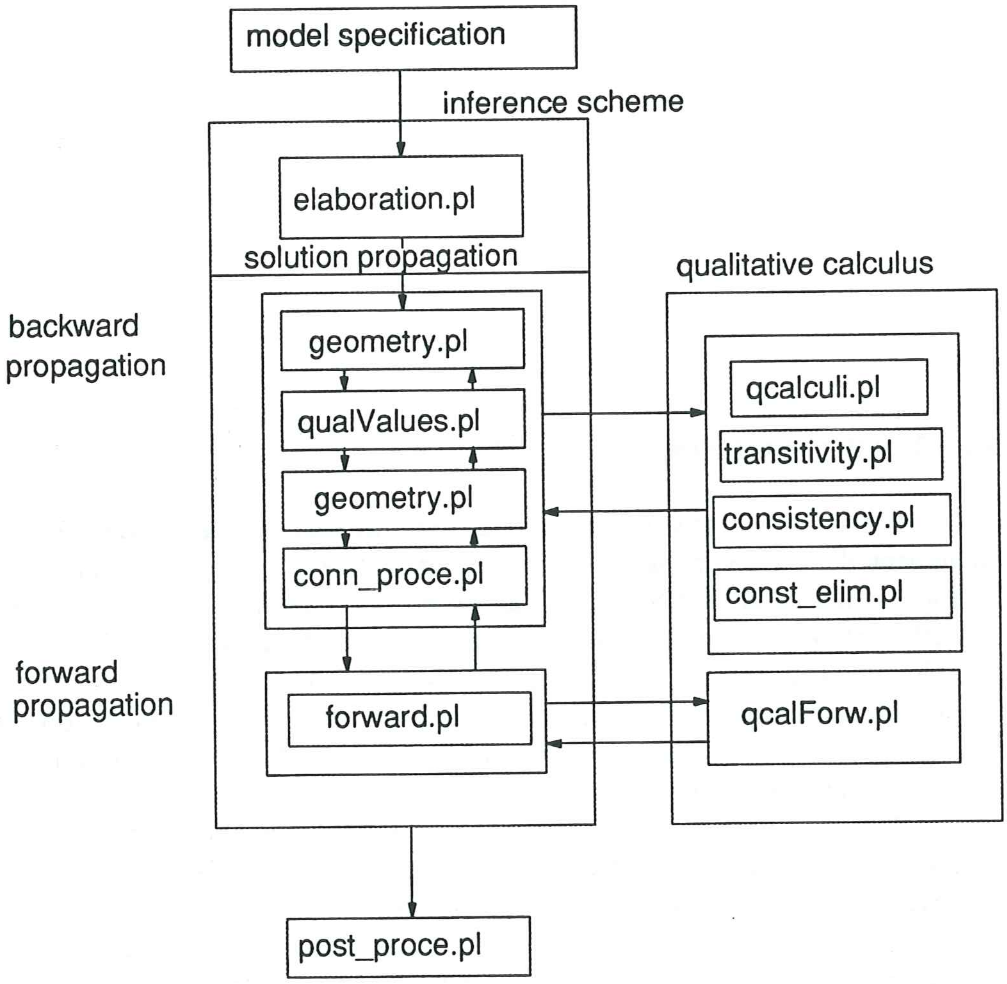


Figure 5.1 Overview of the modules in *Agrippa*.

The model specification is represented by an arity six compound term, `scenario`, illustrated in Fig. 5.2. The arguments for the term are: (1) model name, (2) object instances, (3) object attributes such as topology and geometry, (4) object qualitative states or structural behavior, (5) parameter relations, and (6) system structures.