X International Conference on Textile Composites and Inflatable Structures
STRUCTURAL MEMBRANES 2021
K.-U.Bletzinger, E. Oñate, R. Wüchner and C. Lázaro (Eds)

APPROPRIATE DESIGN OF STRUCTURAL MEMBRANES

JOSEP LLORENS

Barcelona School of Architecture (ETSAB), Universitat Politècnica de Catalunya
Diagonal, 649 - 08028 Barcelona, Spain
E-mail: *ignasi.llorens@upc.edu*
web page: *https://wwwgat.webs.upc.edu/portada.html*

Key words: Structural membranes, Textile architecture, Structural efficiency.

Summary. Progress is being made in the understanding of the appropriateness of membrane structures that are much more efficient than bending solutions based on trusses and cantilevers. But some designs do not take it into account to the point that membrane structures frequently end up being covered conventional steel structures. Attention to the structural characteristics of membranes and feasible improvements can reverse this situation. Dissemination of best practices may also help.

1 FIND THE DIFFERENCES

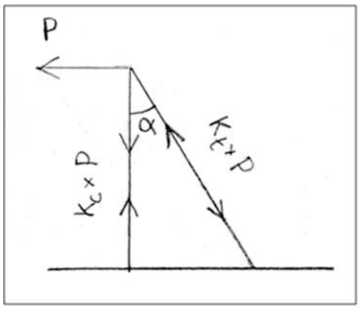
Figures 1 to 7 show different ways of approaching the design of structural membranes according to the material properties, material consumption, efficiency and visual expression, among many others.

Figures 1, 2: Different approaches to design the edges of a membrane. Figures 3, 4: Different approaches to design the support structure. Figures 1, 3, 5, 6, 7: Steel structures cladded with membranes. Figures 2, 4: Membrane structures.

Appropriate design of membrane structures should be based on the lightness and the ability to follow the load paths, provided they have the right combination of curvature and depth to minimize material and energy.

2 PRINCIPLES

Three basic principles govern structural membranes: only tension (funicularity and efficiency) , curvature (resisting through form) and pre-stress (*avoiding bending* and compression). Their load-carrying capacity results from the capability of the flexible membrane to develop an axial response to any pattern of loading by assuming a shape that is funicular for that pattern.



$$P = C \cdot \tan\alpha = k_c \cdot P \cdot \tan\alpha$$

$$k_c = 1 / \tan\alpha$$

$$P = T \cdot \sin\alpha = k_t \cdot P \cdot \sin\alpha$$

$$k_c = 1 / \sin\alpha$$

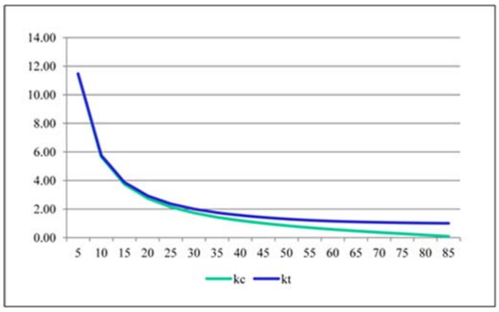
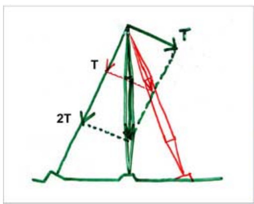


Figure 8: Being able to divide a force into two components is highly dependent upon the angle of projection.

Funicularity implies that the form follows the directions along which internal forces flow. When three or more forces come into play at a corner, equilibrium requires that each force be the resultant of the others. When derivations of forces are needed, the influence of angles is significant (fig.8). The best way to save energy and material is by directly following the path of the loads.



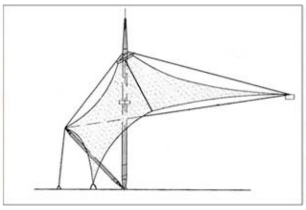




Figure 9: The inclination of the mast greatly influences the compression it has to withstand. Figure 10: The masts are balanced with the fabric because they are sloped outward bisecting the angle of the cone.
Figures 11,12: The masts are misaligned with the resultant of the fabric form.

The angle effect is relevant on top of the masts that support the membrane. Balancing the mast with the membrane expresses equilibrium and reduces loading (figs.9, 10). If the masts are misaligned, external and stabilizing guy cables must perform the additional function of resisting unbalanced forces, which are considerably increased. (figs.11,12).

Graphic statics also reveals the angle effect. In a Pratt flat truss, for example, by reducing the depth the angles decrease, thereby increasing the member forces. In addition, the mechanical work $\Sigma P^*\ell$ can also be calculated as an indicator of structural efficiency (fig.13).

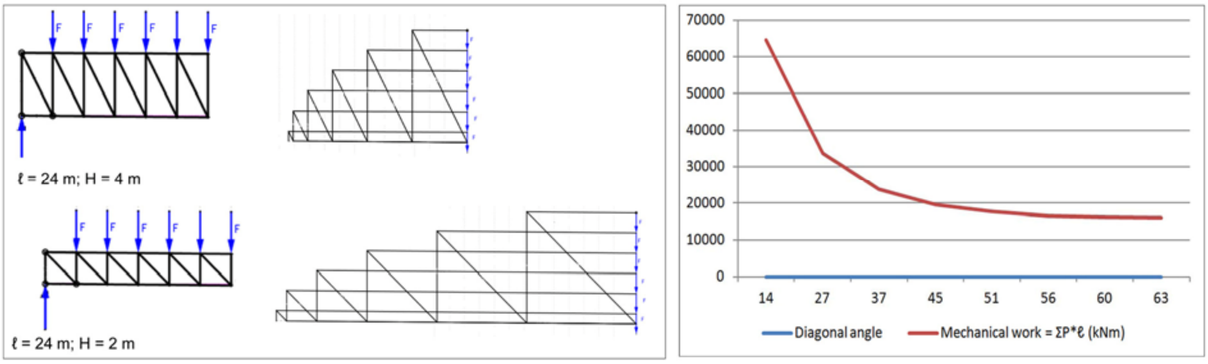


Figure 13: If the depth is reduced, diagonal angles are also reduced but the member forces increase and the mechanical work too.

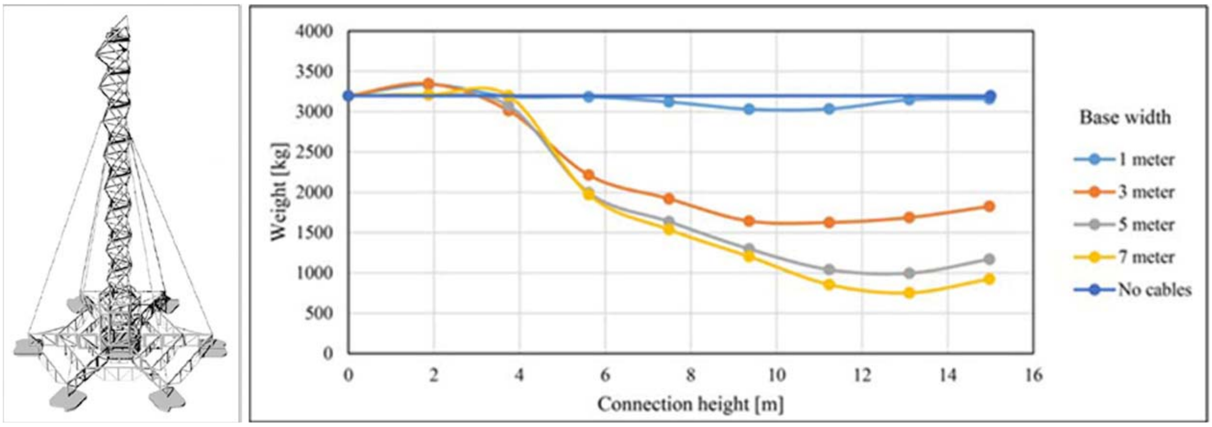


Figure 14: Green Energy Mill Tower. The geometry has been optimized to reduce the weight.

Considering the angle effect, a parametric study has been carried out to optimize the geometry of the Green Energy Mill Tower (E.Pujadas et al.2021) evaluating the influence of the stays on the total weight of the tower for a deformation limit of 250 mm under a 28 m/s wind. The better position corresponding to the lowest weight was the cable connection at 13 m in height and secured at 7 m from the structure (fig 14).

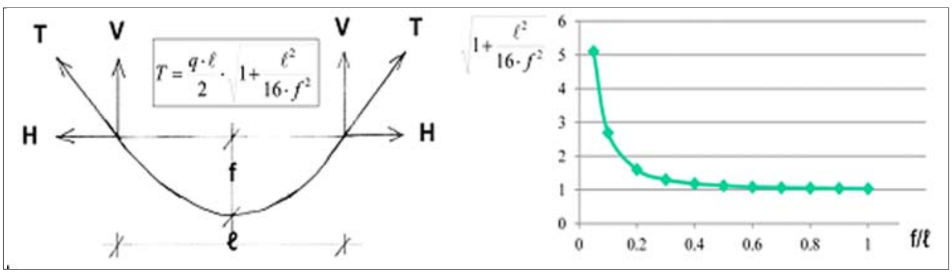


Figure 15: Multiplication factor due to the sag/span ratio in a parabola submitted to uniform load q.

The sag/span (curvature) is also relevant (fig.15). A sag being equivalent to 30% of the span amplifies the load by 1,30, but a sag equivalent to 5% of the span amplifies the load by 5,10. This effect is somewhat soften taking into account the deformation, that increases the length and curvature. It is why (almost) flat surfaces are feasible. Note also that unavoidable variations due to temperature, loads or manufacturing and installation tolerances have a much greater impact on small values of the sag/span ratio.

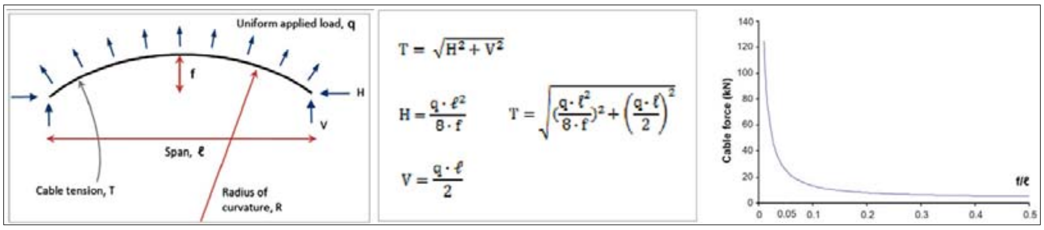


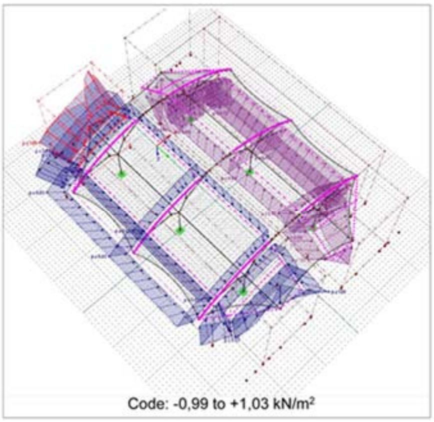
Figure 16: Tension T of a cable subjected to a uniform load q related to the sag/span ratio.

Similarly, relating the tension **T** with the ratio **f/ℓ** of an edge cable (fig.16), it can be observed that a dip to span ratio greater than 0.1 ensures an efficient transfer of fabric stress back to the supporting structure that will result in smaller diameter cables, smaller end fittings, and more elegant connection details and supporting steelwork. A dip to span ratio of 0.05 –0.1 may be desirable for architectural or functional reasons, for example to provide increased coverage, but it should be noted that over this range the cable force doubles. A dip to span ratio less than 0.05 should clearly be avoided as the cable force increases dramatically (B.Bridgens & M.Birchall, 2012).

3 DESIGN PROCESS

Figure 17: If equilibrium is not satisfied, a framework is required**.**

Some steps for the design of membrane structures frequently mentioned are form finding, structural analysis and cutting pattern generation (N.Stranghöner et al.2016). The shape of membrane structures is sometimes qualified as free, but it is not at all arbitrary nor free because it has to fulfil the three-dimensional equilibrium following the load paths, that is, it has to be funicular, curved and pre-stressed. The shape results from the boundary conditions and supports, the loads, material properties, curvatures and pre-tension. It can be predicted by physical or computer assisted models. Authentic free forms require a steel framework and may be cladded with fabric or foil (fig.17).



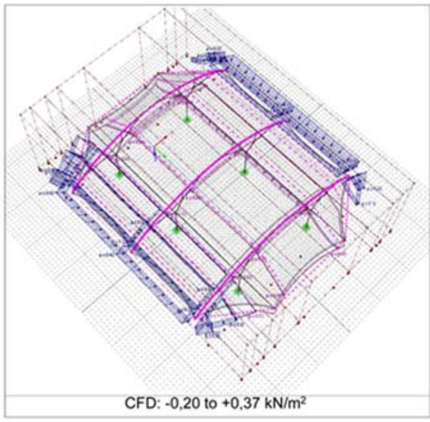
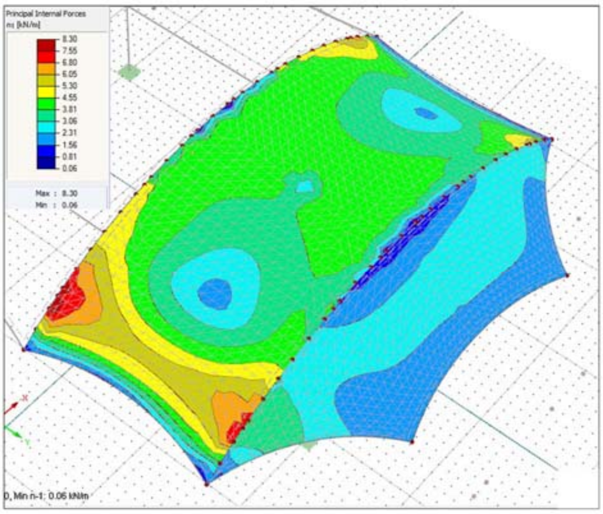
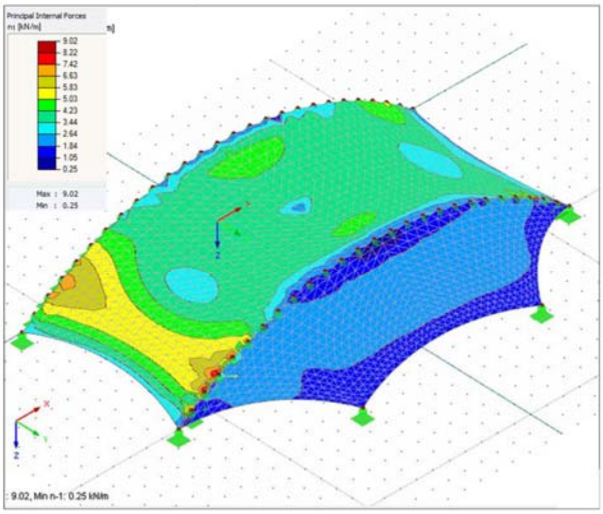


Figure 18: Almuñécar Aquarium canopy. Figure 19: Code wind loads: -0,99 to +1,03 kN/m². Figure 20: CFD wind loads: -0,20 to +0,37 kN/m².

Regarding the structural analysis, there is a concern for the wind loading because it is highly dependent on the method used to determine it. Codes are unfavorable and lead to over sizing and cost increase. Physical or simulated wind tunnel tests could be more reliable. When deformations are significant, additional tests may determine the deflected shape or theoretical estimations and numerical calculations may be performed. For the roof represented in fig.18, the application of the Spanish code, based on the Eurocode (fig.19), was compared with the RWIND digital tunnel test simulation, (fig.20). The difference is considerable.



Subsystems together: 0,06 to 8,30 kN/m



Separated subsystems 0,25 to 9,02 kN/m

Figure 21: Internal forces provided by the hybrid model. Figure 22: The membrane and the supporting structure have been analyzed separately (RFEM Structural Analysis System).

On the other hand, structural membranes are usually supported or complemented by rigid elements such as masts, beams or arches made of steel, timber and even concrete or masonry. The membrane itself carries the external loads to the supporting structure. So, the design model has to be hybrid and the subsystems should be analyzed together, but they are often analyzed separately. The comparison of the results obtained with both procedures demonstrates that the separation is expensive because the interactions, especially the deformations, are favourable.

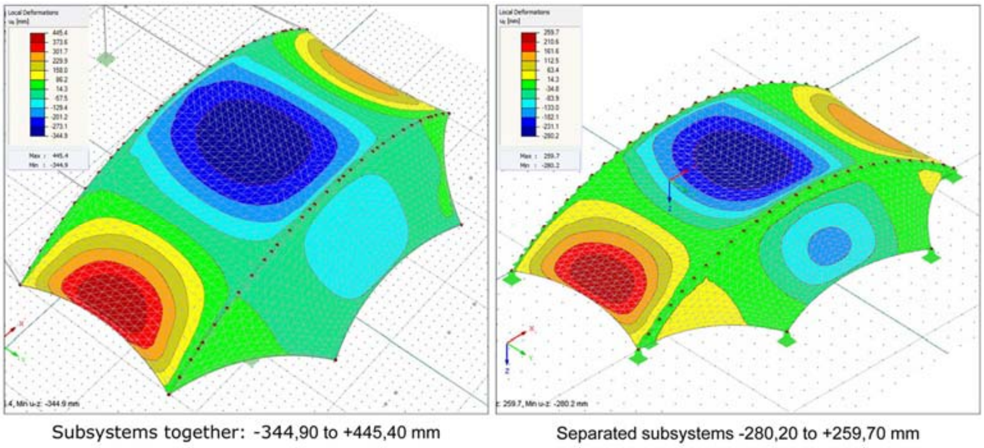


Figure 23: Local deformations provided by the hybrid model. Figure 24: The membrane and the supporting structure have been analyzed separately (RFEM Structural Analysis System).

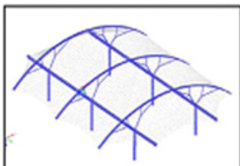
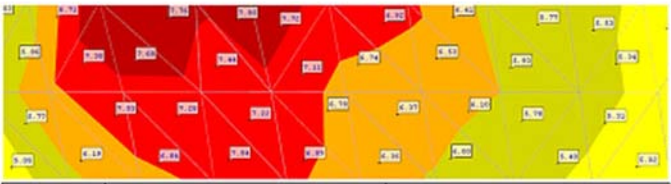
	(1) Side arch of the complete system	Side arch alone with the loads transmitted by the membrane		
		(2) Same sections as (1)	(3) Stress ratio ≤ 1	(2) Same deformation as (1)
WEIGTH (kp)	1.659	1.658	3.272	9.967
MAXIMUM STRESS RATIO	0,94	3,86	0,98	0,27
MAXIMUM DEFORMATION (mm)	61,6	795,3	318,1	62

Table 1: Different simulations for the same model. (1) The side arch is analyzed together with the membrane. All stress ratios have been adjusted to ≤ 1. It results in a weight of 1.659 kp and a maximum deformation of 61,6 mm. (2) Same sections as (1) but the arch has been analyzed separately. The stress ratio goes at 3,86 and maximum deformation at 795,3 mm. (3) The arch is analyzed separately but the stress ratio is kept ≤ 1. The stress ratio remains ≤ 1 but the weight goes to 3.272 kp and the maximum deformation to 318,1 mm. (4) The arch is analyzed separately but the maximum deformation is kept as (1). The stress decreases to 0,27, the maximum deformation to 62 mm but the weight shoots up to 9.967 kp.

As an example, the textile roof supported on steel arches shown in figure 18 has been analyzed with both procedures. The internal forces and local deformations are represented in figure 21 and 23 for the hybrid model. Membrane, arches, branches and columns interact. Curvatures are increased and internal forces relaxed. Figures 22 and 24 represent the internal forces and local deformations for the membrane and the steel analyzed separately. The membrane does not interact. As a result, deformations and curvatures decrease, contrary to internal forces that increase.

Regarding the efficiency, the steel has been measured through its weight, that goes from 1.659 kp to 9.967 kp (table 1). However, as the weight of the membrane does not vary, its efficiency has been measured adding the products of the internal forces by the area on which they act $\Sigma n_i \cdot A_i$ as an indicator of how stressed is the membrane (fig.25). Observe that if the panels are considered alone, the inner one gets worse ($\Sigma n_i \cdot A_i$ increases) while the side panel improves ($\Sigma n_i \cdot A_i$ decreases), so the sum varies very little. Therefore, the significant difference for the membrane is limited to deformations, because there is not much variation in stresses and the efficiency indicators are compensated.



	Inner and side panel of the complete system considered **together**			Inner and side panel considered **alone**		
	PRINCIPAL INTERNAL FORCES n_i (kN/m)	DEFOR-MATIONS (mm)	EFFICIENCY $\Sigma n_i \cdot A_i$ (kN·m)	PRINCIPAL INTERNAL FORCES n_i (kN/m)	DEFOR-MATIONS (mm)	EFFICIENCY $\Sigma n_i \cdot A_i$ (kN·m)
	0,06 to 8,30	-128 to +73	8.191	0,25 to 9,02	-280 to +260	8.620
	0,46 to 3,90	-345 to +445	2.724	0,71 to 3,47	-149 to +58,4	2.327
	0,06 to 8,39	-345 to +445	10.915	0,25 to 9,02	-280 to +260	10.947

Figure 25: Efficiency of the membrane adding the products of the internal forces by the area on which they act $\Sigma n_i \cdot a_i$. Figure 26: The ties come loose when the membrane is pre-stressed.

In summary: the membrane restrains and stiffens de structure. it also replaces the ties, that come loose (fig.26). the panels interact, redistribute and balance the internal forces.

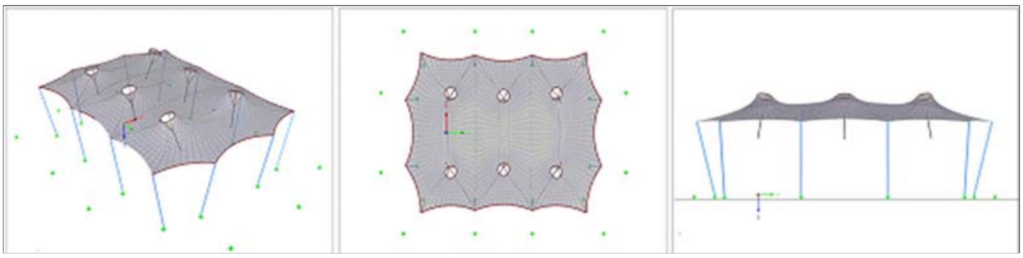


Figure 27: Replacing arches with flying masts.

All the above considerations regarding the refinement of the analysis of the initial model are OK but in fact, what greatly would improve the result would be changing the conceptual structural design. In this case avoiding bending, that is using only axial forces replacing arches with flying masts. Arches and interior masts disappear in exchange of flying masts and exterior ties (fig.27).

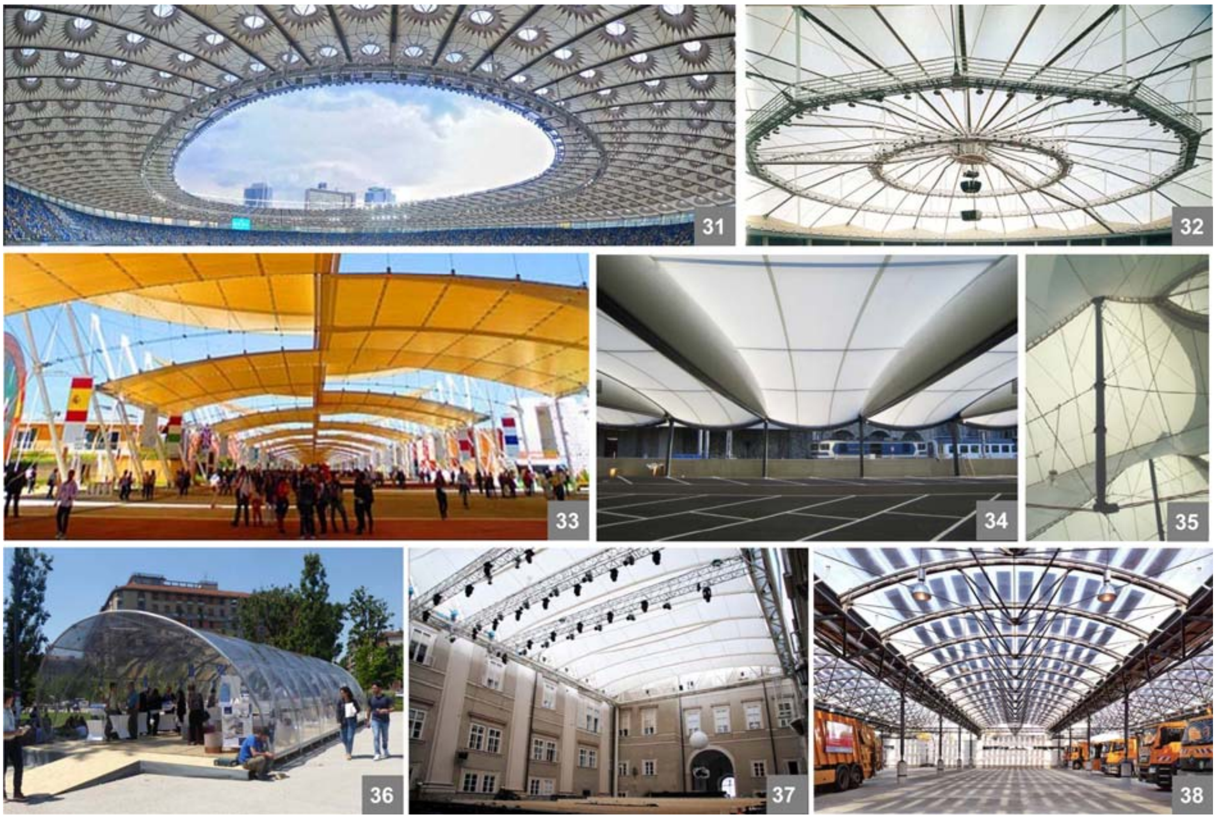
With respect to cutting pattern generation, it has to be mentioned that the analysis does not usually consider the influences of the width of the strips, the stiffness of the seams, the orientation of the fabric, its shear deformation and the pre-tensioning process. The flattening of the double curved surface is a geometrical process with no consideration of the stress distribution and the material behaviour. That's why in recent developments of cutting pattern methods the stress distribution is taken into account. As a result, the separation of the structural behaviour and the cutting pattern leads to highly inhomogeneous stress distributions which can be seen in wrinkles and measured in stresses which are higher than required (R.Wagner, 2005).

Figure 28: Schlaich & Bergermann, 1998: National Swimming Pool, Kuala Lumpur. The parallel membrane strips running towards the edge reflect the distributed stress caused by wind suction. The radial pattern of the high points reflects the concentration of loads due to snow. Figure 29: The layout of the patterns minimizes the waste but the result is messy and confusing. Figure 30: Matti Orpana, Tensotech: 14.300 m^2 inflatable football field in only one panel to avoid joining on site.

Other concerns are the translucency of the membrane and the size of the panels. Since the membranes are translucent, even transparent, the shape can be understood from the cutting pattern due to the seams that are visible against the light. Therefore, care must be given so that the perception of the overall surface is consistent with the spatial configuration (figs.28 and 29). Regarding the size and number of panels, there is a tendency to manufacture only one (or just a few) panel(s) to reduce the membrane connections on site, saving labour and shortening the delivery time in exchange for complicating the handling (fig. 30). In addition, fewer panels entails the continuity of the membrane, which is favourable because it balances tensions that are not transmitted to the supporting structure, if any.

4 IMPROVEMENTS

Many designs assume an appropriate design based on lightness, avoiding bending by following the load paths with the right combination of curvature, depth and pre-sress. They improve the behaviour of the structure reaching some successful achievements represented in figures 31 to 38. The masts in particular accept different strategies to cope with over-dimensioning imposed by buckling. They include the use of circular hollow steel sections and bolted connections not protruding from the profile of the section together with tapered, trussed, tied, coupled or branched masts (fig.39).



Improvements. Figure 31: Spoked-wheels. Figure 32: Cable-domes. Figure 33: Cable-beams. Figure 34: The Tensairity system and large inflatable structures. Figure 35: Flying masts. Figure 36: Active bending. Figure 37: Retractable roofs. Figure 38: Environmentally performing and energy harvesting.

Figure 39: Improvement of masts: circular hollow steel sections, bolted connections not protruding from the profile of the section, tapered, trussed, tied, coupled or branched (J.Llorens, 2019).

Related with improvements of the structural efficiency, it is pertinent to recall the evaluating method developed by the Institute for Lightweight Structures, 1979. The structural behaviour can be evaluated using the values “Tra” and “Bic” in order to minimize material/energy input, as natural structures do (fig.40).

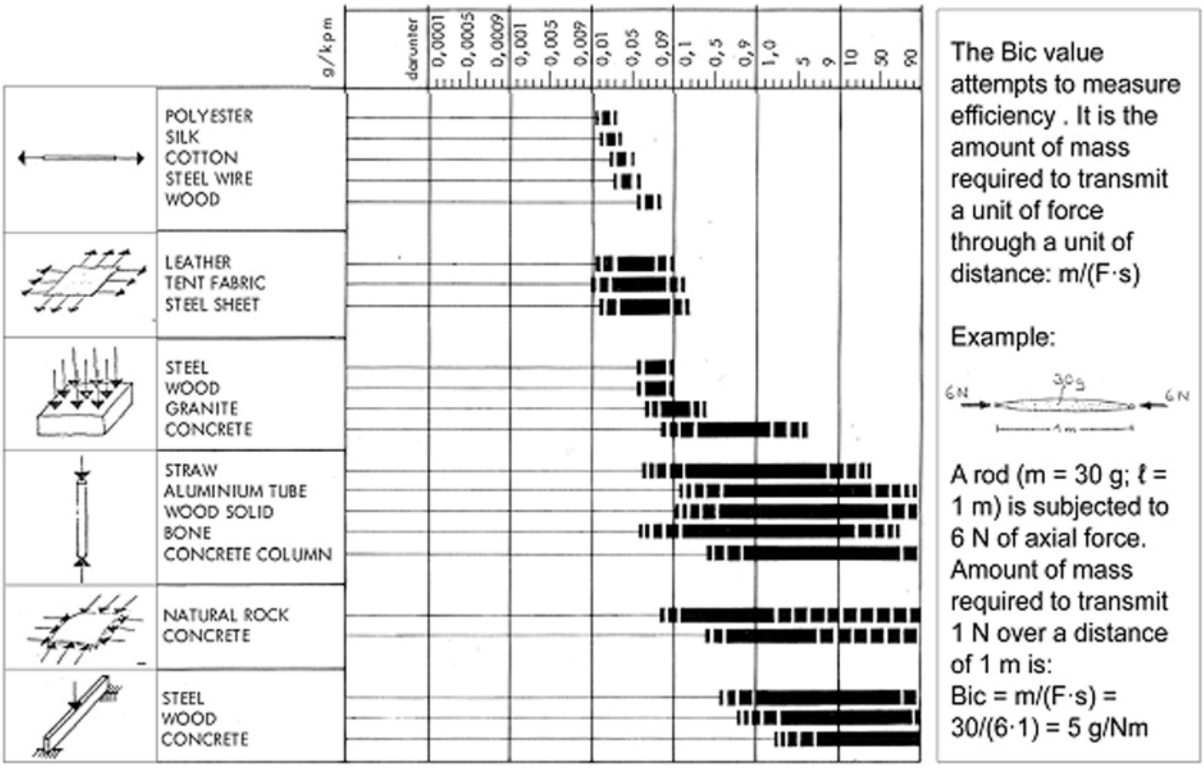


Figure 40: Structures subject to tension hold a low Bic value, while structures subjected to compression and bending result in a high Bic value. Low Bic value means more efficiency (IL, 1979).

Tra = F · s (Tra: ability of transmitting forces; F: force; s: transmission distance or force path). Given a load to be transferred, a low Tra value results in a more lightweight structure. Bic = m / Tra = m /(F ·s). Bic is the ratio of the mass of the structure to its capability to transmit forces, so it takes into account the mass m of the structure and illustrates the mass used in order to transfer a force F along a distance s. It is a way to measure the amount of mass (expenditure) required under a given load condition to transmit a unit load over a unit distance.

5 CASE STUDIES, BEST PRACTICES AND BORDERLINE CASES

Applying the aforementioned principles, the designs may be improved as those presented in figures 41 and 42 (F.Alvarado, 2014).

Best practices also illustrate different solutions to adequately consider the characteristics of structural membranes. To mention only a few: the Dance Pavilion at the Federal Garden Exhibition, Cologne 1957, the Aviary of the Munich Zoo 1980, The Pier 6 Concert Pavilion, Baltimore 1990, the Truck Depot of the Office for Waste management, Munich (1999 former version), the Roller Skate Park, Paris 2006, the Penny Market, Karlsruhe 2012, the Velden am Wörther See Gemonaplatz 2015 and the Jeddah Hangar 2021.

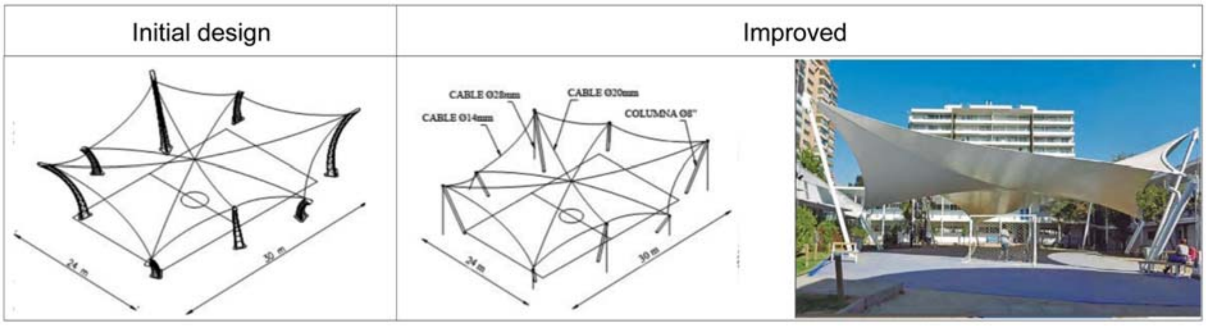


Figure 41: Leonardo da Vinci School, Santiago de Chile (by Espacio Cubierto). A 720 m^2 roof for the courtyard was planned to be supported by 8 trussed masts of two heights in order to have 4 high points and 4 low points to provide double curvature (9.510 kp = 15,9 kp/m^2). The improvement consisted in tying the masts so that bending was replaced by axial forces of tension and compression (1.639 kp = 3,08 kp/m^2).

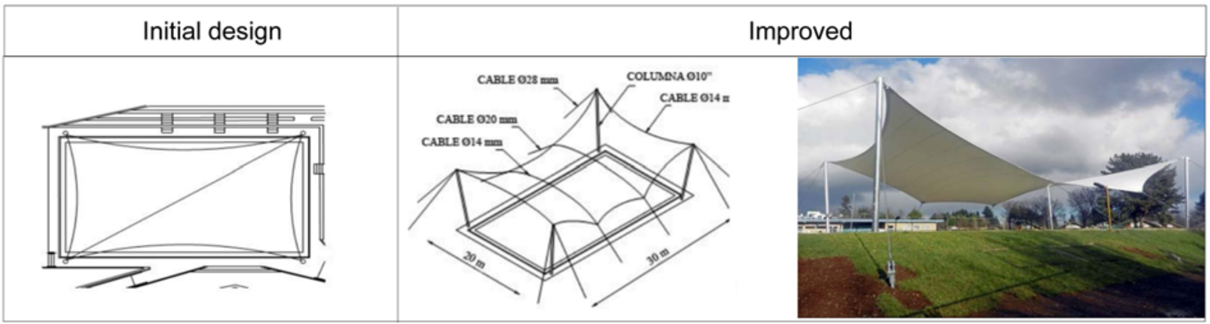


Figure 42: Frontera University Amphitheatre, Temuco (by Espacio Cubierto). A 800 m^2 hypar enclosed in a frame (6.297 kp = 15,47 kp/m^2) was improved by reducing the height and replacing the frame with two valley cables tied to the ground so that bending was replaced by axial forces of tension and compression (1.544 kp = 2,95 kp/m^2).

It should be mentioned however that in some cases the recommended values are not respected. It is the case of (almost) flat surfaces that are increasingly used for aesthetic or practical reasons. (Almost) flat surfaces are feasible because the strain of the material provides curvature. Furthermore they need a sufficient amount of pre-stress.

6 CONCLUSIONS

Although design tools have progressed, some membrane structures are still designed without taking advantage of their structural characteristics. The result is usually a disproportionate steel structure that is cladded to generate the (arbitrary) projected shape. In order to achieve a good result, it is (highly) recommended to respect the principles of only tension, funicularity, curvature and pre-stressing, as well as to take advantage of the available design methods to properly determine the form (in equilibrium), the loads and the hybrid behaviour of the structure. The most significant variables can also be parameterized, just as bending avoided in the supporting structure and compression optimized. Several improvements have been introduced that can be investigated further. Best practices could be looked up.

REFERENCES

F.Alvarado, 2014: “3 Casos de estructura tensadas como estrategia de optimización estructural”. IASS-SLTE Symposium, Brasilia.
H.Berger, 1996: “Light structures. Structures of light”. Birkhäuser, Basel
Ph.Block, 2019: "Strength through geometry": *https://www.scipedia.com/public/Block_2019a* (accessed 30/06/2021).
B.Bridgens & M.Birchall, 2012. “Form and function: the significance of material properties in the design of tensile fabric structures”. *Engineering Structures,* 44, p.1-12.
Dirección General de Agenda Urbana y Arquitectura, 2009: CTE DB SE-AE. Ministerio de Transportes, Movilidad y Agenda Urbana, Madrid
H.Engel, 2006: “Structure systems”. Hatje Cantz Verlag, Ostfildern
U.Gründig et al. 2017: “Computational modelling of lightweight structures”. Textile Roofs, Berlin.
C.G.Huntington, 2008: “Structures using uncurved or minimally curved tensioned fabric membranes”. *Structures Congress*, Vancouver, ASCE, Reston.
IL 21, 1979: “Form. Force. Mass”. Institute for Lightweight Structures, Stuttgart.
IL 18, 1987: “Forming bubbles”. Institute for Lightweight Structures, Stuttgart.
J.Knippers et al. 2011: “Construction manual for polymers + membranes”. Birkhäuser, Basel.
K.M.Koch, 2004: “Membrane structures”. Prestel, Munich
O.P.Larsen & A.Tyas, 2003: "Conceptual structural design". Thomas Telford, London.
J.Llorens, 2019: “Detailing masts”. 60th IASS Symposium – 9th International Conference on Structural Membranes and Inflatable Structures, Barcelona
E.Pujadas et al. 2021: “The Gem Tower: a hybrid renewable energy unit for festivals”. *Structural Engineering International,* Vol.31, Issue 2, p.249-254.
A.Ramírez Ponce, 2019: "¿Por qué proyectar y construir arcos, bóvedas y cúpulas?": *https://www.youtube.com/watch?v=NnJYXbgTMnU* (accessed 30/06/2021)
P.Rice, 1994: “An Engineer Imagines”. Artemis, London.
N.Stranghöner et al. 2016: “Prospect for European guidance for the structural design of tensile membrane structures”. Joint Research Centre, European Commission.
D.Ströbel, 2019: “Powerful tools for form finding, statics and patterning of textile structures” in Textile Roofs 2019, Archenhold Observatory, Berlin.
R.Wagner, 2005: “On the design process of tensile structures” in E.Oñate & B.Kröplin: “Textile composites and inflatable structures”. Springer, Dordrecht.
W.Zalewski & E.Allen, 1998: “Shaping structures: statics”. Wiley. NY.

WEB SITES

GeoGebra graphic static apps by J.M.Genesca: *https://www.geogebra.org/u/genes* (accessed 09/05/2021)
Membrane.online (formfinder): *https://membrane.online/project/19* (accessed 09/05/2021)
TensiNet projects database:
https://www.tensinet.com/index.php/projects-database/projects (accessed 09/05/2021)
Textile Construction and Tensile Structures. Joints, connections, fittings and anchors data bank: *https://wwwgat.webs.upc.edu/portada.html* (accessed 09/05/2021).