A Convergence Proof for Adaptive Parametric PDEs with Unbounded Coefficients

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ABSTRACT

Numerical methods for random parametric PDEs can greatly benefit from adaptive refinement schemes, in particular when functional approximations are computed as in stochastic Galerkin methods with residual based error estimation. From the mathematical side, especially when the coefficients of the PDE are unbounded, solvability is difficult to prove and numerical approximations face numerous challenges. In this talk we generalize the adaptive refinement scheme for elliptic parametric PDEs introduced in [1, 2] to unbounded (lognormal) diffusion coefficients [3]. The algorithm is guided by a reliable error estimator which steers both the refinement of the spacial finite element mesh and the enlargement of the stochastic approximation space. As the algorithm relies solely on (a sufficiently good approximation of) the Galerkin projection of the PDE solution and the PDE coefficient, it can be used in a non-intrusively manner, allowing for applications in many different settings. We prove that the proposed algorithm converges and even show evidence that similar convergence rates as for intrusive approaches can be observed.

REFERENCES

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