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FROM FREEZING-INDUCED TO INJECTION-INDUCED NON-ISOTHERMAL SATURATED POROUS MEDIA FRACTURE

Yousef Heider

Institute of General Mechanics (IAM), RWTH Aachen University, Germany
RWTH Aachen University
Eilfschornsteinstr. 18, 52062 Aachen, Germany
e-mail: heider@iam.rwth-aachen.de, www.iam.rwth-aachen.de

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Abstract. The focus of this contribution is laid on different aspects and instances related to porous media fracture under non-isothermal conditions. This includes the extreme case of fracturing due to pore-fluid freezing, where the micro-cryo-suction plays an important role in generating the required stresses for crack onset. This also includes studying the instances related to hydraulic fracturing and heat transfer under non-isothermal conditions. In all cases, the continuum mechanical modeling of the induced fractures is based on macroscopic porous media mechanics together with the phase-field method (PFM) for fracture modeling. For the micro-cryo-suction in saturated porous media, the water freezing is treated as a phase-change process. This is modeled using a different phase-field approach, in which the thermal energy derives the phase change and, thus, leads to the occurrence of micro-cryo-suction. Two numerical examples are presented to show the effectiveness of the proposed modeling frameworks.

1 INTRODUCTION

Under non-isothermal conditions, several interesting processes happen in multiphase porous materials. In the underlying contribution, two of them are briefly addressed within a numerical framework; the pore-fluid freezing with the possible onset of cryosuction-induced fracturing and the topic of non-isothermal hydraulic fracturing and heat exchange in the convection-dominated regime.

1.1 Porous media freezing

The freezing of fully-saturated and partially-saturated porous materials, such as ground freezing, is considered a challenging problem for structures in cold regions. However, this can also be applied as a supporting technology in the construction of tunnels using artificial ground freezing (AGF), see, e.g., [36, 37, 34] for references. Modeling the mechanical responses in porous media freezing requires understanding and investigations at different scales. In this, laboratory experiments with special boundary conditions can be applied to validate the numerical results as can be found in, e.g., [37]. During pore-fluid freezing and fluid-solid interaction, one can distinguish between two states, i.e. the thermal transient state with ice penetration in the domain and the steady-state, where the cryosuction plays a major role in the fluid flow towards the ice-water interface and the evolution of ice lenses.

In the modeling of porous media freezing with the accompanying thermo-hydro-mechanical (THM) processes in this work, a continuum mechanical framework is applied, which combines the following aspects: (1) A macroscopic porous media model, which is capable to describe the multi-physical THM processes [5, 8, 9, 22, 23, 12, 13, 16, 27, 19, 6, 3, 7]. (2) A phase-field model for capturing the freezing as a phase-change process [20, 38, 4, 32, 2]. (3) A phase-field model for capturing the possible onset of fracturing and ice lenses formation [1, 29, 11, 18, 31, 19, 30, 26, 14, 17, 15]. (4) A constitutive model that captures the cryo-suction effects [39, 37].

1.2 Non-isothermal porous media fracture

Hydraulic fracturing under non-isothermal conditions and the fluid flow in the fracture network together with the heat exchange with the surrounding ambient are very important events in many engineering fields, such as in geothermal energy systems. If the targeted rock layers of a geothermal system have naturally low permeability, then the application of hydraulic fracturing to enhance this is an unavoidable solution, see, e.g., [28, 15] for review. Within this topic, the aim of the underlying contribution is to briefly highlight the modeling aspects of the thermal energy exchange between a low-temperature injection fluid and a high-temperature fractured and intact porous surrounding. While the fracture network can be modeled using the phase-field method (PFM), see, e.g, [19, 15, 26, 33], the heat exchange can be modeling following the fundamentals of the theory of porous media (TPM) with distinct temperatures for the solid and fluid phases, see, e.g., [21, 10, 35, 24, 36].

1.3 Content overview

To give an overview, the theoretical fundamentals related to the topic of freezing in saturated porous media and non-isothermal processes in hydraulic fracturing are briefly introduced in section 2. This is followed by a short description of the applied numerical schemes and challenges in section 3. Two numerical examples will then be presented in section 4, which is followed by the conclusions in section 5.

2 THEORETICAL FUNDAMENTALS

2.1 Development of the multiphase continuum porous media frost action model

A mathematical model that describes the major physical processes in saturated-soil freezing is applied in this work. In this, we proceed from a saturated porous material φ, consisting of an immiscible solid phase φ^S and a fluid phase φ^F. The pore-fluid therein can be found in a liquid state ($\overline{\varphi}^L$), in an ice state ($\overline{\varphi}^I$), or in a liquid-ice mixed state; $\varphi^F = \bigcup_\alpha \overline{\varphi}^\beta = \overline{\varphi}^L \cup \overline{\varphi}^I$. Within this continuum mechanical approach, one defines the volume fractions n^α, the partial densities ρ^α, and the intrinsic densities $\rho^{\alpha R}$ for each constituent, where $\rho^\alpha := n^\alpha \rho^{\alpha R}$ and $\alpha \in \{S, F\}$. The variation of the solid volume fraction can be expressed in terms of the initial solidity n_0^S and the solid displacement $\mathbf{u}_S$ as $n^S \approx n_0^S\,(1 - \operatorname{div} \mathbf{u}_S)$. Moreover, the density of the assumed barotropic fluid constituent ρ^{FR} is formulated as a function of the fluid compressibility κ^F and the effective pore pressure p. For the kinematics, a Lagrangian description is applied for the solid phase deformation using the solid displacement $\mathbf{u}_S$ and velocity $\mathbf{v}_S$, whereas an Eulerian description is considered for the fluid phase motion via the seepage velocity $\mathbf{w}_F = \mathbf{v}_F - \mathbf{v}_S$. The reversible water/ice phase exchange is captured using a diffusive phase-field method (PFM) for phase-change materials. In this, a scalar-valued phase-field variable $\phi^F(\mathbf{x}, t)$ is used to indicate the states, i.e., $\phi^F = 1$ for the liquid and $\phi^F = 0$ for the ice states. Moreover, employing the PFM allows for a unified kinematic treatment of the ice and water phases. The formation of the ice phase is associated with the onset of cracks, which can be modeled using a phase-field fracture approach. For this, a scalar-valued phase-field variable $d^S(\mathbf{x}, t)$ is defined to indicate the states, i.e., $d^S = 1$ for the cracked state, $d^S = 0$ for the intact material state, and $0 < d^S < 1$ for the diffusive interface.

In the freezing process, the micro-cryo-suction is responsible for driving the pore liquid towards the frozen zone. The realization of this effect within continuum mechanics is done through a phenomenological retention-curve-like formulation. In this, the cryogenic suction due to the ice/water interface tension, i.e., $\mathcal{S}_{cryo} := p^{IR} - p^{LR}$, can be expressed by the following relationship

$$\mathcal{S}_{cryo} = p^{IR} - p^{LR} = \mathcal{N}\left[(\chi^L)^{-\frac{1}{m}} - 1\right]^{1-m} \qquad \text{with} \qquad p^{FR} = p^{LR} + \left(1 - \chi^L\right)\mathcal{S}_{cryo}\,, \tag{1}$$

where p^{LR} and p^{IR} are the pressures of the liquid water and the ice crystals, respectively, and p^{FR} is the net pore pressure. $\mathcal{N}$ and m are model parameters and the liquid water saturation χ^L is represented by ϕ^F within this unified water/ice kinematics treatment. More details can be found in [36, 37]. Assuming quasi-static, non-isothermal process and negligible body forces, the governing balance relations are (1) **Overall momentum balance**, (2) **Fluid mass balance**, (3) **Fluid momentum balance**, (4) **Mixture energy balance**, (5) **Phase-field equation for the phase-change process**. (6) **Phase-field equation for the fracture process**. In the soil freezing example, an equilibrium thermal model is employed, where Kelvin's temperatures of all constituents are equal ($\theta^S = \theta^F = \theta$). Moreover, simplifications are made for the energy balance and phase-field evolution by neglecting some non-significant terms.

In constitutive modeling, the principle of effective stresses is applied, where the total stresses are split into an effective term and a pressure-dependent term. The solid effective stress $\boldsymbol{\sigma}^S_E$ of the initially linear elastic brittle material is formulated as a function of the phase-field variable d^S to capture the stiffness degradation due to crack onset and propagation. The stiffness degradation together with the realization of the cryo-suction effects allows for capturing the formation of ice lenses, as will also be shown in the numerical examples. The fluid effective stress is formulated in a way that the parameters, like viscosity, are functions of the phase-field variable ϕ^F. More details about the constitutive models can be found in [36, 37].

2.2 Modeling of hydraulic fracturing and heat transfer under non-isothermal conditions

The continuum mechanical framework herein proceeds from the assumption of biphasic material with materially compressible pore fluid and incompressible solid phase. Unlike the model of porous media freezing, the underlying non-isothermal hydraulic fracture model assumes distinct temperatures of the solid and the fluid phases, i.e. the two-phase porous material is under local thermal non-equilibrium conditions. Additionally, no mass exchange takes place between the phases and the assumption of quasi-static processes is adopted. The governing equations to describe such processes are the following: (1) **Overall momentum balance**, (2) **Fluid mass balance**, (3) **Fluid momentum balance**, (4) **Fluid energy balance**, (5) **solid energy balance**, (6) **Phase-field equation for the fracture process**. More details about the governing equations and the related constitutive models can be found in, e.g., [25]. Of very important topics in this is the definition of intrinsic permeability, which becomes strongly anisotropic in the presence of cracks, the definition of Fourier's law of heat conduction for both the solid and fluid phases, and the definition of the energy production term that governs the heat exchange between the constituents. The energy production is a function of the specific heat transfer surface area parameter, which is formulated in terms of the phase-field damage parameter to capture the change in the contact areas between the fluid and solid in the fractured zones. For simplicity, the latent energy is neglected in this treatment.

3 NUMERICAL TREATMENT AND STABILITY CHALLENGES

The finite element method (FEM) is used in the numerical solution of initial-boundary value problems (IBVPs) of both freezing-induced fracture in porous media and heat flow within the non-isothermal hydraulic fracture. For the freezing-induced fracture problem, the primary variables to be figured out are $\boldsymbol{\xi}_{\text{Freez}}(\mathbf{x},t) := [\mathbf{u}_S\ p\ \mathbf{u}_F\ \mathbf{v}_F\ \theta\ \phi^F\ d^S]^T$. For the heat flow within non-isothermal hydraulic fracture, the primary variables are $\boldsymbol{\xi}_{\text{HydF}}(\mathbf{x},t) := [\mathbf{u}_S\ p\ \theta^F\ \theta^S\ d^S]^T$, where the fluid velocity $\mathbf{v}_F$ is computed in a strong form via the Darcy's law and using the computed pressure. In the derivation of the weak formulation, boundary conditions are considered for each of the variables. Thus, considering $\mathcal{B}$ as the spatial domain with $\partial\mathcal{B}$ as its boundary, this is split for each variable into Dirichlet ($\partial\mathcal{B}_D$) and Neumann ($\partial\mathcal{B}_N$) boundaries, whereas $\partial\mathcal{B}_D \cup \partial\mathcal{B}_N = \partial\mathcal{B}$ and $\partial\mathcal{B}_D \cap \partial\mathcal{B}_N = \emptyset$. For the time discretization of the coupled equations, the second-order implicit Backward Difference Formula (BDF2) is used, which is available in the considered FE package FlexPDE 7.16. For the spatial discretization, triangular, equal-order, quadratic shape functions are applied for all variables.

A number of numerical stability challenges can be identified in solving such problems, which are connected to the limits of very low permeability, very low compressibility of the fluid, high fluid viscosity, near crack-domain interfaces, next to drained boundaries, and for high fluid flow regimes. In this, it is crucial to fulfilling the inf–sup condition to get a stable and unique solution, see, e.g., [36]. To overcome the challenges connected with low permeability, high fluid viscosity, and incompressible pore fluid, the approach of quasi-compressibility is applied. In this, the stability is enforced by adding a stabilization term of the form $\operatorname{div}(\alpha_{\text{st}}\operatorname{grad}\dot{p})$ to the fluid mass balances with $\alpha_{\text{st}} > 0$ as a small scalar-valued stabilization parameter. For the other source of instability, i.e., with high fluid flow in the cracks and convection-dominated heat transport, a different numerical treatment needs to be implemented. To better understand the problem, the fluid energy balance can be written as a simple convection-diffusion equation as

$$\underbrace{\mathcal{A}\,\mathbf{w}_F \cdot \operatorname{grad}\theta^F}_{\text{Convective transport}} - \underbrace{\mathcal{D}\operatorname{div}\operatorname{grad}\theta^F}_{\text{Diffusive transport}} + \mathcal{B} = 0 \tag{2}$$

with $\mathcal{A}$, $\mathcal{B}$, $\mathcal{D}$ being scalar-valued terms that can be determined based on the comparison with the energy balance equation. In the convection-dominated state, a stabilization technique based on adding an artificial thermal diffusivity term $\overline{\mathcal{D}}$ can be applied. In particular, $\overline{\mathcal{D}}$ is added to the original thermal diffusivity term $\mathcal{D}$ and chosen as small as possible to enhance the stability but not significantly deteriorate the accuracy.

4 NUMERICAL EXAMPLES

Two related numerical examples are presented in the following discussion and solved in the FE package FlexPDE 7.16 .

4.1 Freezing and ice-lenses formation in saturated porous media

The objective of this example is to show the capabilities of the proposed model in capturing important THM responses during the phase transition of the pore fluid from liquid water to ice and the ability to model the onset of fracture and the formation of ice lenses. The geometry and boundary conditions are illustrated in Figure 1, left. In this, the bottom temperature is kept above freezing temperature and constant ($\overline{\theta}_b$=278[K]), whereas the upper temperature is reduced gradually from $\overline{\theta}_t$=273[K] to $\overline{\theta}_t$=253[K] within 25[h]. The initial temperature is set to θ_0=278[K]. The material parameters in this example are mostly taken from [37]. In this, we consider a heterogeneous distribution of the initial solid volume fraction n^S_{0S} in the range $[0.56, 0.68]$ and for the elasticity modulus E^S in the range $[656600, 683400]\,\mathrm{N/m^2}$ as illustrated in Figure 1, middle. The frost heave evolution during the freezing process is illustrated in Figure 1, right.

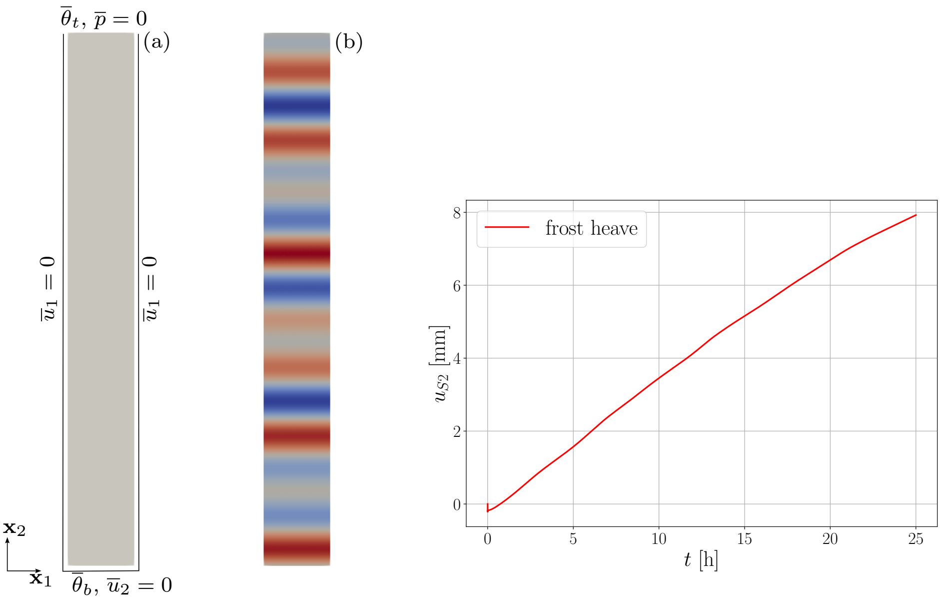

Figure 1: The considered geometry and boundary conditions in the numerical modeling of top freezing problem (left), the heterogeneous distribution of the elastic modulus and initial porosity (middle), and the frost heave evolution (right).

While the temperature decreases at the boundary and in the domain over time, the ice penetrates the domain as illustrated in Figure 2. In this, it is also interesting to see the formation of ice lenses at several intervals, which are captured via the phase-field variable d^S.

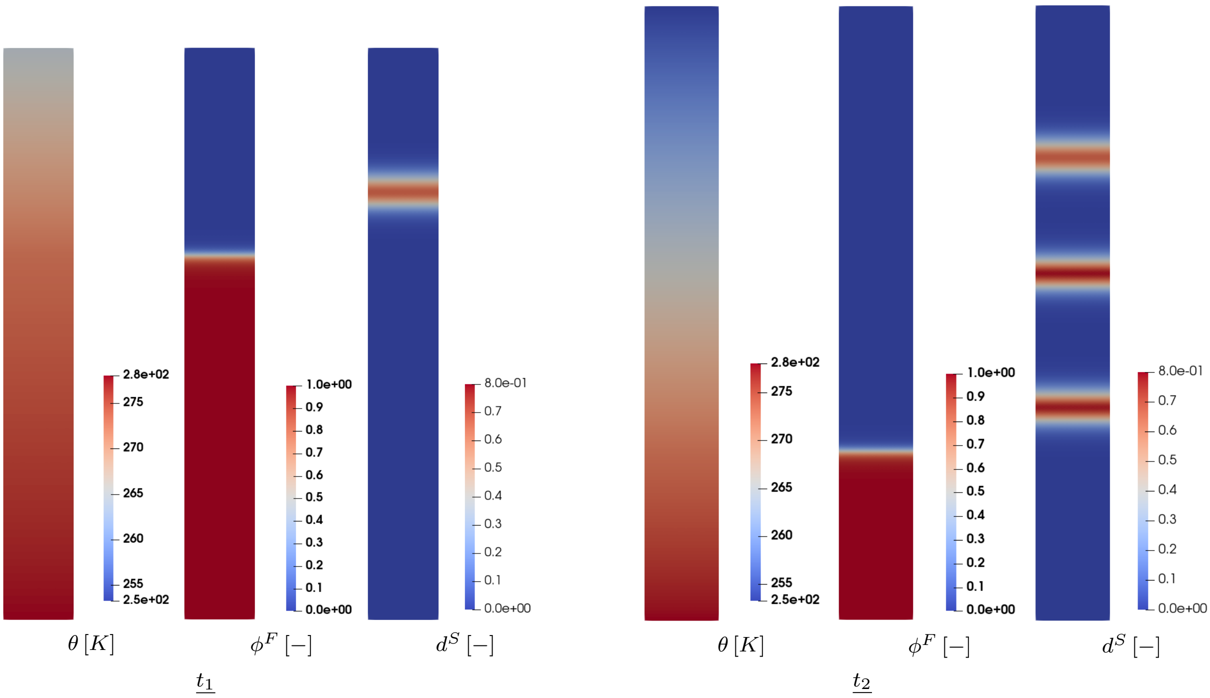

Figure 2: The temperature distribution θ, the phase change state ϕ^F, and the phase-field fracture variable d^S at two different times, i.e. $t_1 = 10.8\,[\mathrm{h}]$ and $t_2 = 25.0\,[\mathrm{h}]$.

4.2 Heat transport and exchange in porous media fracture network

The aim of this numerical example is to show the capability of the proposed non-isothermal TPM-PFM model (details in [25]) in capturing heat transfer and heat exchange in the fractured porous material. The crack topology is realized using the phase-field evolution equation by setting a higher value of the crack driving force in the cracked zone, while this driving force is set to zero in the intact zone. This is illustrated together with the overall geometry and the boundary conditions in Figure. 3. The material parameters are taken mainly from [25], whereas the intrinsic permeability of the intact zone $K^S_{0S} = 1 \cdot 10^{-14}\,\mathrm{m}^2$, for the horizontal cracks $K^S_{0S} = 1 \cdot 10^{-8}\,\mathrm{m}^2$, and for the inclined cracks $K^S_{0S} = 1 \cdot 10^{-9}\,\mathrm{m}^2$. For the boundary conditions, $\overline{q}_{\theta^S} = 0$ is applied at all boundaries and zero fluid heat flux $\overline{q}_{\theta^F} = 0$ together with $\overline{p} = 0$ are applied at all boundaries except for the left boundary. In the cooling process, a pressure of $\overline{p} = 1000\,\mathrm{N/m}^2$ is applied at the left-side boundary together with a fluid heat influx of $\overline{q}_{\theta^F} = 230.5\,(\theta_F - \theta_{F0})\,\mathrm{W/m}^2$ with $\theta_{F0} = \theta_{S0} = 328\,[\mathrm{K}]$ as the initial temperature of the fluid and solid phases, respectively.

Figure 4 shows the average fluid flow rate over time on the left (inlet) and right (outlet) boundaries of the domain together with the average temperature change over time on the left (inlet) and right (outlet) boundaries. This shows the gradual reduction of the outlet fluid temperature due to the heat exchange with the surrounding. Moreover, Figure 5 shows the contour plots of the fluid temperature θ^F in the domain (cracks and surrounding porous material)

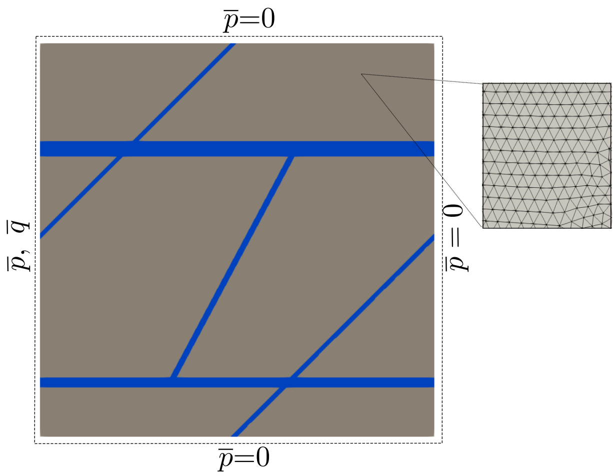

Figure 3: The geometry and boundary conditions considered in the numerical modeling of heat transmission in a fractured porous domain. The pre-existing cracked zone is colored blue, whereas the horizontal cracks have bigger width than the inclined cracks and, thus, higher permeability. For the boundary conditions, the upper and lower boundaries are fixed in the vertical direction ($u_{S2} = 0$), the lateral boundaries are fixed in the horizontal direction ($u_{S1} = 0$), zero solid heat flux $\overline{q}_{\theta^S} = 0$ at all boundaries, and zero fluid heat flux $\overline{q}_{\theta^F} = 0$ and undrained condition $\overline{p} = 0$ at all boundaries except for the left boundary.

at four sequential time points, which is in agreement with the heat exchange process.

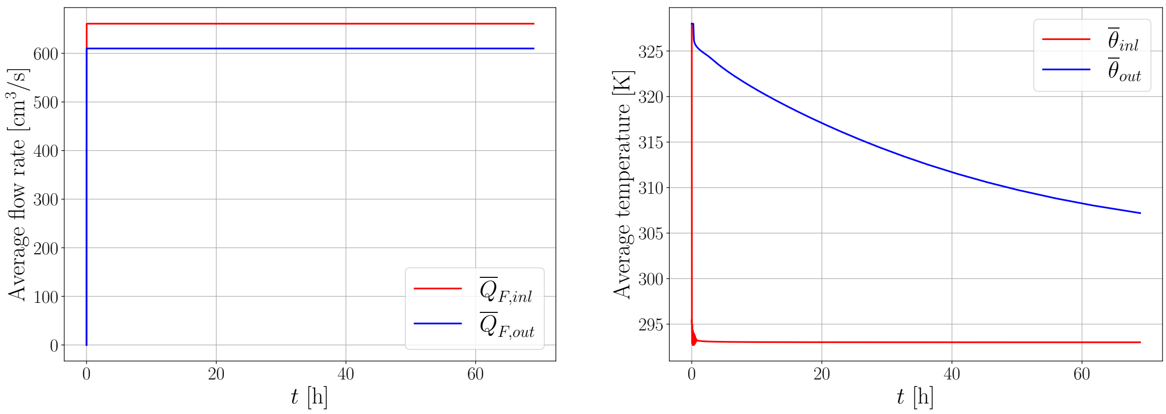

Figure 4: Average fluid flow rate over time on the left (inlet) and right (outlet) boundaries of the domain, where the upper and lower boundaries of the domain are open (left). Average fluid temperature change over time on the left (inlet) and right (outlet) boundaries of the domain (right).

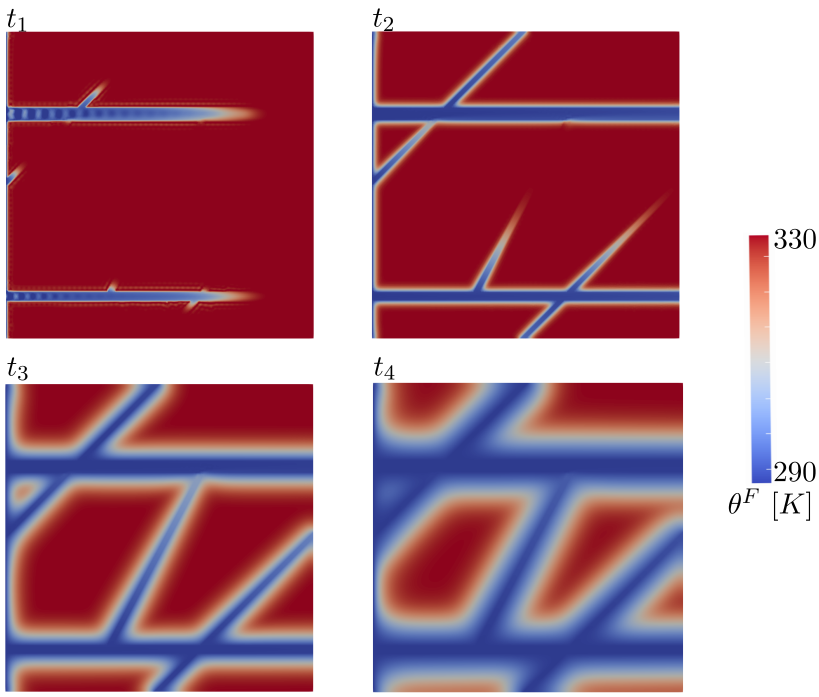

Figure 5: Contour plot of the fluid temperature θ^F in the domain at four time points, $t_1 = 0.2\,[h]$, $t_2 = 1.97\,[h]$, $t_3 = 16.4\,[h]$, $t_4 = 50\,[h]$. These plots are connected to the numerical solution with the non-isothermal TPM-PFM approach with a stabilization parameter $\overline{\mathcal{D}} = 10$.

5 CONCLUSIONS

Within the topics of non-isothermal processes in porous media, two events have been highlighted in this contribution. These are the freezing of the pore fluid of saturated porous media with the accompanying damages and micro-cryo-suction, and the heat transport and exchange within porous media hydraulic fracture. For the modeling, a continuum mechanical framework within the theory of porous media is presented. Two phase-field methods are introduced to model the phase-change process in freezing and to capture the onset of fractures and the topology of the predefined cracks. The numerical treatment is carried out using the finite element method, where a special focus is laid on the numerical challenges in extreme cases, such as the limits of very low permeability, very low compressibility of the fluid, high fluid viscosity, near crack-domain interfaces, next to drained boundaries, and for high fluid flow regimes with convection-dominated heat transport. Two numerical examples are also presented to show the capabilities of the proposed modeling framework and the stability of the solution algorithms.

Several topics remain open for future works, which include presenting more efficient solution algorithms, extending the models to accurately capture the different effects, and validating the models via comparison with experimental data.

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