THE SAFETY OF MASONRY ARCHES SUBJECT TO VERTICAL AND HORIZONTAL FORCES. A NUMERICAL METHOD BASED ON THE THRUST LINE CLOSEST TO THE GEOMETRICAL AXIS

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Abstract. *In this paper the topic of the safety assessment of masonry arches based upon their geometry is investigated. The theoretical background is the Heymanian master safe theorem along with the no-tension assumption of masonry. The continuous arch is analyzed considering a discrete pattern of vertical loads, such as those of the self-weight and superimposed loads. Among all the lines of thrust contained within the profile of the arch, the one closest to the geometrical axis can be considered to be the best one thanks to the minimum bending moment and shear force present in each cross section. A numerical procedure for computing the line of thrust closest to the geometrical axis of an arch subject to its self-weight has been recently formulated by the authors. This procedure accounts for this line of thrust by minimizing the distances between the geometrical axis of the arch and the thrust line. In order to consider the action of both vertical loads and horizontal forces proportional to the vertical ones, such as those provoked by an earthquake, an extension of this procedure is herein presented. The safety of the arch is finally assessed by computing a domain of equilibrium thrust lines within the profile of the arch which provides, in analogy with the Heymanian geometrical factor of safety, the full range factor of safety. The procedure is described in the paper and illustrated with regards to the analysis of arches subject only to vertical loads and arches subject to also horizontal forces.*

1 INTRODUCTION

In the literature, the mechanical behaviour of masonry arches is investigated using different approaches. With the purpose of computing the actual stress state in an arch, some authors have proposed methodologies that are strictly connected to the need of assessing the properties of masonry and estimating its actual "deformation capacity" [1]. Conversely, other authors, to assess the safety level of an arch or, vice-versa, the vulnerability level, have formulated methodologies based on the limit analysis principles.

In the context of limit analysis, two approaches can be used: the thrust line method [2-6], based on the static theorem, and the mechanism method [7-12], based on the kinematic theorem. Leaving aside the mechanism method, which requires all the kinematically compatible mechanisms to be defined and which is, therefore, a very computationally expensive method, the thrust line method assesses the safety searching for the existence, within the profile of the arch, of any line of thrust in equilibrium with the loads acting on it, according to the famous assumptions formulated by Heyman [13]: no tensile strength, infinite compressive strength, sliding failure cannot occur due to the presence of friction. Under these assumptions, the material can be considered non-deformable and the arch can be modelled as an assemblage of rigid blocks.

Recently the authors of this paper have formulated a method for the analysis of masonry arches based on the detection of the line of thrust closest to the geometrical axis [14,15], checking at a later stage that it is also within the profile. Among the ∞^3 likely lines of thrust which depict the equilibrium condition between loads and abutment reactions, this is the best one because it corresponds to an internal stress state of almost only compression. Unlike other methods present in the literature, such as that proposed by Heyman in [13], which computes the line of thrust closest to the geometrical axis of an arch exploiting an iterative method that proceeds by trial and errors, and those capable of computing the thrust network or the thrust surface closest to the mid-surface of a 3D-vault [16-20], the method presented herein is a one-step procedure that provides the exact solution in “closed-form”, thus reducing the computational time.

In this paper the above mentioned procedure is reformulated and extended in order to account also for a set of horizontal forces, proportional to the vertical ones, capable of simulating the action of an earthquake. Such horizontal forces are computed, in an innovative way, with a reverse method, exploiting the analogy between an inclined plane, on which the structure is “virtually” placed, and the inclination of the resultant actions of the vertical and horizontal loads, thus avoiding the need of inputting them directly in the mechanical model of the structure. The safety of the arch is assessed extending the Full-Range Factor Method (FRS Method) formulated by the authors in [14,15] for the vertical load case to the combination of this load case with the set of horizontal forces, and it is exploited to also compute the seismic factor.

2 LINE OF THRUST CLOSEST TO THE GEOMETRICAL AXIS

Let us consider a continuous arch, subject to a discrete pattern of ‘n’ vertical loads F_1, F_2,…, F_n. Let us subdivide the arch into ‘n’ elements (Fig. 1a), in such a way as to apply each load F_i ($1 \leq i \leq n$) to the centroid G_i of each element. In so doing, in case of a rigid block structure, the elements correspond to the rigid blocks and the lines of separation between the elements correspond to the actual joints.

For the assigned load condition, a family of ∞^3 funicular polygons can be drawn, each one describing a specific condition of equilibrium between loads and abutment reactions. Furthermore, since these funicular polygons must describe the flow of internal forces between the elements of the arch, they are, de facto, the polygons of successive resultants and, therefore, are referred to as lines of thrust as well.

In order to draw a specific line of thrust, three parameters or conditions must be fixed. In the present approach, the parameters that have been fixed are the ordinate of points A and B, through which the first and the last side of the line of thrust must pass respectively, and the value H of the thrust, which is everywhere constant throughout the arch due to the solely presence of vertical loads.

Referring to the generic node 'i' of the line of thrust (Fig. 1b), the equilibrium condition of the vertical forces is provided by Eq. 1:

$$(\tan\alpha - \tan\beta) = \frac{F_i}{H} \tag{1}$$

where α and β are the angles of inclination of the two sides of the line of thrust connected to the node 'i'.

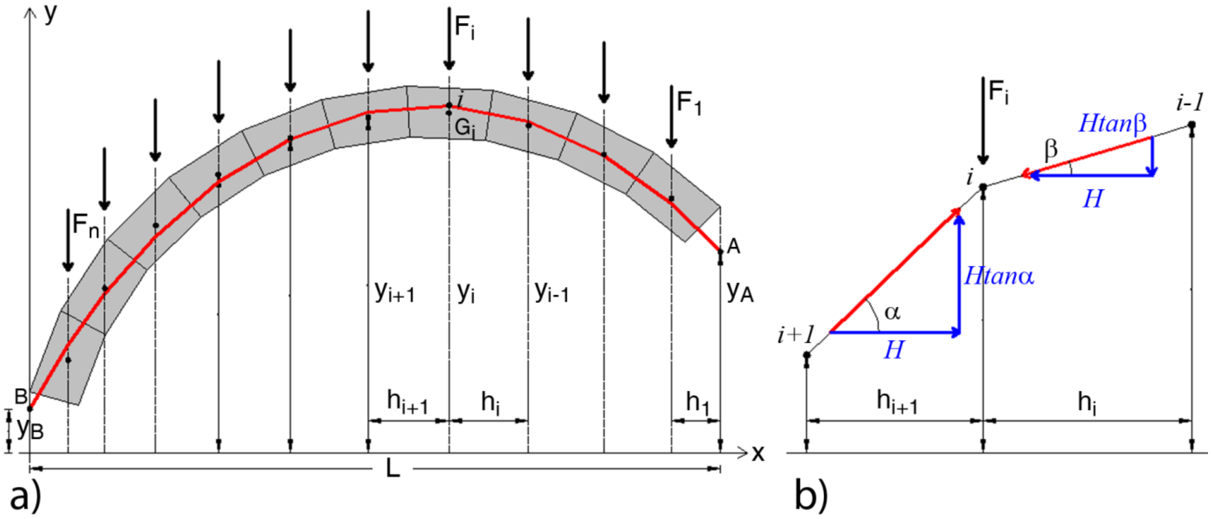

Figure 1: a) Reference discrete arch model; b) detail of the generic node 'i' of the line of thrust

It is worth noting that the equilibrium condition of the horizontal forces is meaningless because, as aforementioned, the thrust H is constant.

The tangents of angles α and β can be expressed, in geometrical form, by Eq. 2:

$$\begin{aligned} \tan\alpha &= \frac{Y_i - Y_{i-1}}{h_i} \\ \tan\beta &= \frac{Y_{i+1} - Y_i}{h_{i+1}} \end{aligned} \tag{2}$$

As a consequence, putting $h_i = x_i - x_{i-1}$ the variable horizontal distance between the centroids of two generic subsequent elements 'i-1' and 'i', that is between the directions of the load vectors F_{i-1} and F_i acting on such elements, Eq. 1 can be replaced by Eq. 3:

$$\left(\frac{-1}{h_i}\right) \cdot Y_{i-1} + \left(\frac{h_i + h_{i+1}}{h_i \cdot h_{i+1}}\right) \cdot Y_i + \left(\frac{-1}{h_{i+1}}\right) \cdot Y_{i+1} = \frac{F_i}{H} \tag{3}$$

Extending Eq. 3 to all nodes of the thrust line, the following system of linear equations is obtained:

$$\begin{cases}\left(\frac{-1}{h_1}\right)\cdot Y_0+\left(\frac{h_1+h_2}{h_1\cdot h_2}\right)\cdot Y_1+\left(\frac{-1}{h_2}\right)\cdot Y_2=\frac{F_1}{H}\\ \left(\frac{-1}{h_2}\right)\cdot Y_1+\left(\frac{h_2+h_3}{h_2\cdot h_3}\right)\cdot Y_2+\left(\frac{-1}{h_3}\right)\cdot Y_3=\frac{F_2}{H}\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ \left(\frac{-1}{h_i}\right)\cdot Y_{i-1}+\left(\frac{h_i+h_{i+1}}{h_i\cdot h_{i+1}}\right)\cdot Y_i+\left(\frac{-1}{h_{i+1}}\right)\cdot Y_{i+1}=\frac{F_i}{H}\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ \left(\frac{-1}{h_n}\right)\cdot Y_{n-1}+\left(\frac{h_n+h_{n+1}}{h_n\cdot h_{n+1}}\right)\cdot Y_n+\left(\frac{-1}{h_{n+1}}\right)\cdot Y_{n+1}=\frac{F_n}{H}\end{cases} \tag{4}$$

where the ordinates Y_i of the nodes 'i' of the line of thrust and the thrust H are the unknowns to be computed.

System of Eq. 4 shows the indeterminacy of the problem of equilibrium of the arch, which is indeed a statically indetermined structure to third degree: there are three more unknowns than the number of equations.

System of Eq. 4 is then rewritten, in matrix form, in such a way as to isolate the three redundant unknowns from the general system, i.e. the thrust H and the ordinates Y_0 and Y_{i+1} of points A and B respectively, and to define three vectors:

- vector T_1, whose entries are the values of loads F_i, which is then multiplied by the constant $K=H^{-1}$;
- vector T_2, whose only non-null entry is the coefficient $1/h_1$ of the unknown Y_0;
- vector T_3, whose only non-null entry is the coefficient $1/h_{n+1}$ of the unknown Y_{n+1}.

Finally, defining D as the matrix of the coefficients of the unknowns, Eq. 4 becomes:

$$[D]\{Y\}=\{T_1\}\cdot K+\{T_2\}\cdot Y_0+\{T_3\}\cdot Y_{n+1} \tag{5}$$

where:

$$D=\begin{bmatrix}\left(\frac{h_1+h_2}{h_1\cdot h_2}\right) & \left(\frac{-1}{h_2}\right) & 0 & 0\\ \left(\frac{-1}{h_2}\right) & \left(\frac{h_2+h_3}{h_2\cdot h_3}\right) & \left(\frac{-1}{h_3}\right) & 0\\ \dots & \dots & \dots & \dots\\ 0 & \left(\frac{-1}{h_i}\right) & \left(\frac{h_i+h_{i+1}}{h_i\cdot h_{i+1}}\right) & \left(\frac{-1}{h_{i+1}}\right)\\ \dots & \dots & \dots & \dots\\ 0 & 0 & \left(\frac{-1}{h_n}\right) & \left(\frac{h_n+h_{n+1}}{h_n\cdot h_{n+1}}\right)\end{bmatrix},\qquad Y=\begin{Bmatrix}Y_1\\Y_2\\\dots\\Y_i\\\dots\\Y_n\end{Bmatrix},$$

$$T_1=\begin{Bmatrix}F_1\\F_2\\\dots\\F_i\\\dots\\F_n\end{Bmatrix},\qquad T_2=\begin{Bmatrix}1/h_1\\0\\\dots\\0\\\dots\\0\end{Bmatrix},\qquad T_3=\begin{Bmatrix}0\\0\\\dots\\0\\\dots\\1/h_{n+1}\end{Bmatrix} \tag{6}$$

Putting $\{R_1\}=[D]^{-1}\{T_1\}$, $\{R_2\}=[D]^{-1}\{T_2\}$, $\{R_3\}=[D]^{-1}\{T_3\}$, the solution of Eq. 5 provides the ordinates of the line of thrust, collected in the entries of vector Y:

$$\{Y\}=\{R_1\}\cdot K+\{R_2\}\cdot Y_0+\{R_3\}\cdot Y_{n+1} \tag{7}$$

Eq. 7 provides the ordinates of the nodes of all ∞^3 funicular polygons associated to the load condition expressed by vector T_1, depending on the three redundant unknowns mentioned above. Therefore, in order to compute the ordinates of the nodes of the line of thrust closest to the geometrical axis, the three redundant unknowns shall be firstly calculated in such a way as to respect the above requirement of “closeness”. To calculate them, a procedure that minimizes the distances between the nodes of the researched thrust line and the centroids of the elements, is formulated.

In order to make the negative signs of this difference irrelevant in the minimization procedure, the proposed method computes the function S that expresses the square of the sum of the distances ΔY_i between the ordinates Y_i of the nodes ‘i’ and the ordinates Y_{Gi} of the centroid G_i of the elements, as follows:

$$S = \sum_{i=1}^{n}(\Delta Y_i)^2 = \sum_{i=1}^{n}(Y_i - Y_{Gi})^2 = (\{R_1\} \cdot K + \{R_2\} \cdot Y_0 + \{R_3\} \cdot Y_{n+1} - \{Y_G\})^2 \tag{8}$$

Function S is minimized by expressing the partial derivatives of it, with respect to the three unknowns, equal to zero:

$$\begin{aligned} &\frac{\partial S}{\partial K}(Y_0, Y_{n+1}, K) = 0 \\ &\frac{\partial S}{\partial Y_0}(Y_0, Y_{n+1}, K) = 0 \\ &\frac{\partial S}{\partial Y_{n+1}}(Y_0, Y_{n+1}, K) = 0 \end{aligned} \tag{9}$$

Inputting Eq. 8 into Eq. 9 and solving the partial derivatives, leads to the system of three linear equations that follows:

$$\begin{cases} \{R_1\}^2 \cdot K + \{R_1\}\{R_2\} \cdot Y_0 + \{R_1\}\{R_3\} \cdot Y_{n+1} - \{R_1\}\{Y_G\} = \{0\} \\ \{R_2\}^2 \cdot Y_0 + \{R_1\}\{R_2\} \cdot K + \{R_2\}\{R_3\} \cdot Y_{n+1} - \{R_2\}\{Y_G\} = \{0\} \\ \{R_3\}^2 \cdot Y_{n+1} + \{R_1\}\{R_3\} \cdot K + \{R_2\}\{R_3\} \cdot Y_0 - \{R_3\}\{Y_G\} = \{0\} \end{cases} \tag{10}$$

which can be rewritten in matrix form as:

$$\begin{bmatrix} \{R_1\}^2 & \{R_1\}\{R_2\} & \{R_1\}\{R_3\} \\ \{R_1\}\{R_2\} & \{R_2\}^2 & \{R_2\}\{R_3\} \\ \{R_1\}\{R_3\} & \{R_2\}\{R_3\} & \{R_3\}^2 \end{bmatrix} \cdot \begin{Bmatrix} K \\ Y_0 \\ Y_{n+1} \end{Bmatrix} = \begin{Bmatrix} \{R_1\}\{Y_G\} \\ \{R_2\}\{Y_G\} \\ \{R_3\}\{Y_G\} \end{Bmatrix} \tag{11}$$

or, more compactly as:

$$[N]\{P\} = \{W\} \tag{12}$$

The solution of Eq. 12 provides the value of the three redundant unknowns, stored in vector P:

$$\{P\} = [N]^{-1}\{W\} \tag{13}$$

At this stage, entries of vector Y are reversely computed inputting the values of the three unknowns provided by Eq. 13 into Eq. 7.

Finally, to plot the line of thrust closest to the geometrical axis the abscissa of point A, the abscissas of the centroids G_i of the elements and the abscissa of point B are collected, in sequence, in vector X and the size of vector Y is expanded by concatenating the ordinate Y_0 of point A, inputted in the first entry, the entries stored in the original vector, and, finally, the ordinate Y_{n+1} of point B, inputted in the last entry (Eq. 14):

$$X = \begin{Bmatrix} X_0 \\ X_1 \\ \dots \\ X_i \\ \dots \\ X_{n+1} \end{Bmatrix}, \qquad Y = \begin{Bmatrix} Y_0 \\ Y_1 \\ \dots \\ Y_i \\ \dots \\ Y_{n+1} \end{Bmatrix}, \tag{14}$$

The line of thrust is graphically obtained plotting the poly-line of vertices (X_i,Y_i).

3 EXTENSION TO THE HORIZONTAL LOAD CASE

In the approach of both linear and nonlinear static analysis, the effect of an earthquake can be represented through a pattern of horizontal forces proportional to the vertical ones. The procedure that computes the line of thrust closest to the geometrical axis of an arch under vertical loads is herein extended in order to also consider such horizontal forces.

The procedure takes into account the effect of the horizontal forces by exploiting the analogy between an inclined plane, on which the structure is "virtually" placed, and the inclination of the resultant actions of the vertical and horizontal loads, thus avoiding the horizontal forces to be inputted directly into the mechanical model. According to this analogy, the effect of an horizontal load acting together with a vertical one can be accounted for by rotating the structure at the same angle as that which corresponds to the action line of the resultant.

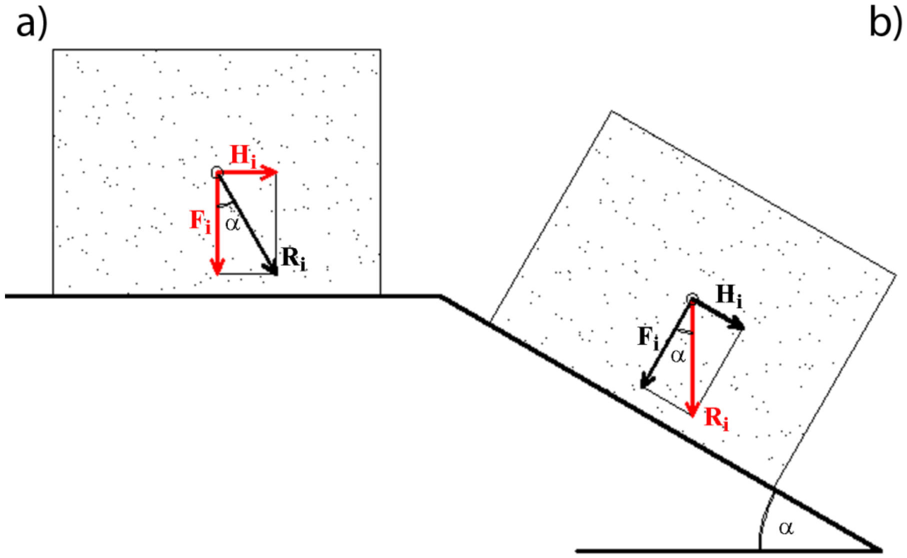

Figure 2: Analogy between inclined plane and seismic effect

Put simply, let us refer to Fig. 2, which shows the generic element 'i' of the mechanical model. Fig. 2a shows the actual position of the element, while Fig. 2b shows the same element placed on an inclined plane rotated at angle α.

In the actual position (Fig. 2a), the element is subject to the joined action of the vertical force F_i and the horizontal action H_i provoked by the earthquake, the horizontal force proportional to the vertical one being computed as:

$$H_i = \lambda F_i \tag{15}$$

where λ is the seismic factor, depending on the seismic zone or the seismicity of the territory.

According to the aforementioned analogy, the joined action of the vertical and horizontal forces can be taken into account by replacing them with a “virtual” vertical force R_i whose length corresponds to the resultant of the forces F_i and H_i (Fig. 2b):

$$R_i = \frac{F_i}{\cos(\alpha)} \tag{16}$$

With this approach, the seismic factor λ corresponds to the tangent of the angle at which the element is rotated:

$$\lambda = \frac{H_i}{F_i} = \tan(\alpha) \tag{17}$$

Therefore, the seismic response of the structure for an assigned seismic factor λ is assessed by rotating the structure at the angle provided by reversing Eq. (17), that is:

$$\alpha = \arctan(\lambda) \tag{18}$$

For clarity, the main steps of the procedure for assessing the seismic response of the structure using the FRS Method (Section 2) are listed below:

1) Compute the angle α (Eq. 18) for the assigned value of the seismic factor λ;
2) Compute the vertical forces R_i (Eq. 16) acting on each element of the structure;
3) Rotate the structure at angle α;
4) Compute the line of thrust closest to the geometrical axis, using the procedure described in Section 2.

4 SAFETY ASSESSMENT

The procedure described in Section 2 and 3 allows one to compute, for any assigned load condition of vertical forces and horizontal forces proportional to the vertical ones, the line of thrust closest to the geometrical axis.

In order to assess if the arch is safe, a comparison between the shape of the line of thrust and the profile of the arch must be carried out. However, the line of thrust closest to the geometrical axis, provided by the computation, could also be somewhere out of the profile of the arch, without mandatorily meaning that the arch is unsafe.

In fact, the line of thrust closest to the geometrical axis is only one of all ∞^3 funicular polygons associated to the load condition. According to the static theorem of limit analysis, it can be vertically shifted in order to check if it stands within the profile of the arch. By doing so, we are exploring all ∞^1 funicular polygons parallel to that provided by the computation.

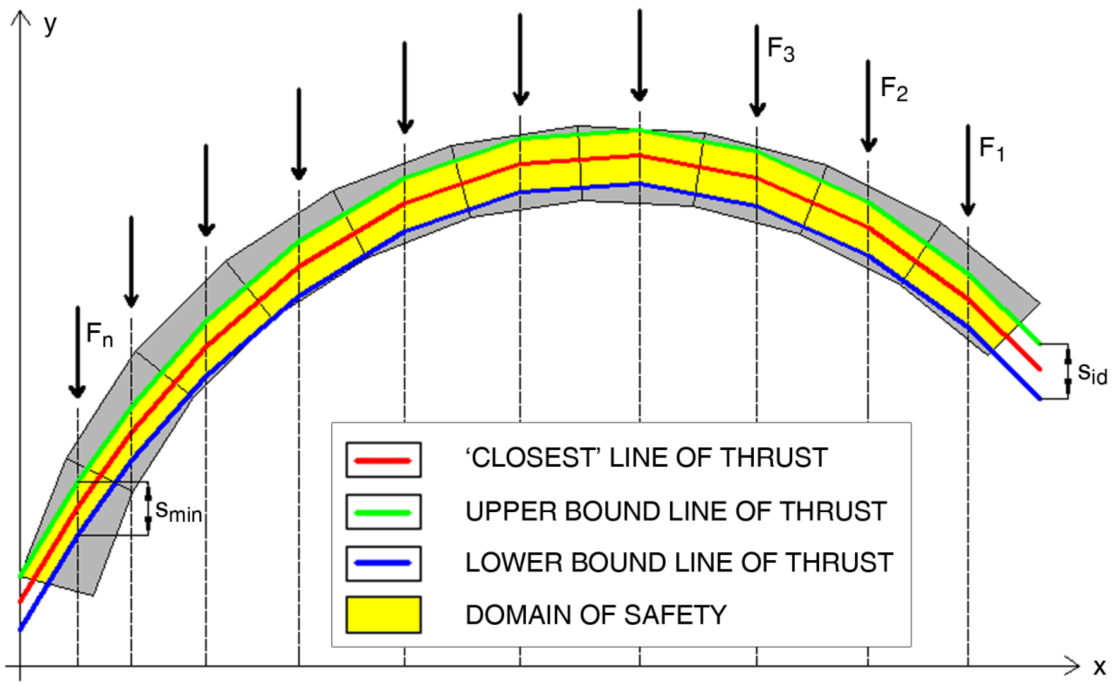

Figure 3: Safety assessment procedure

With this in mind, the authors have developed a procedure to assess the degree of safety of an arch based on the "capacity" of the line of thrust closest to the geometrical axis to be vertically shifted within the profile of the arch (Fig. 3). Two limit positions can be obtained: the lines of thrust, parallel to the original one, tangent to the extrados and intrados of the arch, respectively, in at least one point while still remaining contained within its profile. In authors' formulation, these two polygons identify a domain of admissible lines of thrust (i.e. equilibrium states) and are the upper and lower bound of this region respectively. In other words, this region identifies all the ∞^1 lines of thrust that lie within the profile of the arch and that, as a consequence, describe different safety conditions.

In order to assess the degree of safety with this approach, the "full range factor of safety" has been defined: it is the ratio between the thickness of the domain and the thickness of the arch, in line with the "geometrical factor of safety" proposed by Heyman in [13]. However, in authors' opinion, the factor that best points at the degree of safety of the arch is the reciprocal of such coefficient, that we have denoted as "performance factor":

$$\mathrm{p.f.} = \frac{s_{id}}{s_{min}} \qquad 0 \leq \mathrm{p.f.} \leq 1 \tag{19}$$

It is worth noting that, in line with the formulation of the procedure above described, the thickness s_{min} of the arch to be inputted in Eq. 19 is the minimum vertical thickness among all the thicknesses measured in correspondence to the action lines of loads and that also the thickness of the domain, s_{id}, is vertically measured, and, for this reason, is a constant value.

According to Eq. 19, the safest arch is that in which the line of thrust can be vertically shifted upwards and downwards so that the distance between the upper and lower thrust lines is exactly equal to the thickness of the arch (s_{id} = s_{min}). It occurs in the solely case of an inverted catenary-shaped arch [14] and provides the performance factor equal to 1. Conversely, a performance factor equal to 0 points to an unsafe arch, because the upper bound and the lower bound thrust lines coincide and identify a domain of zero thickness. Therefore, summarizing, the performance factor ranges in the narrow interval [0,1], thus indicating intermediate safety levels.

In the case of an arch subject to a predetermined vertical load condition and horizontal forces computed based on an assigned seismic factor λ, Eq. 19 still holds and the safety level can still be assessed using the FRS approach.

However, in order to assess the seismic vulnerability level of an arch, that is the "seismic capacity" of the structure, the limit value of λ (hereafter referred to as λ_{lim}) is required to be computed.

In the approach of standard limit analysis, λ_{lim} is computed as the load multiplier that activates the collapse mechanism. Instead, with our approach, the seismic factor λ is maximum (i.e. $\lambda = \lambda_{lim}$) when the thickness of the domain of safety is zero:

$$\lambda = \lambda_{\mathrm{lim}} \;\; \rightarrow \;\; \alpha = \alpha_{\mathrm{max}} \; : \; s_{\mathrm{id}} = 0 \tag{20}$$

Therefore, in order to compute λ_{lim}, an iterative analysis that runs repeatedly the four-step procedure described at the end of Section 3, is proposed. Such a procedure could be executed increasing, step by step, the angle of the plane on which the arch stands.

However, it is worth noting that the horizontal forces being simulating the seismic action, the seismic factor λ ranges in the interval [0,1] and, in agreement with Eq. 18, the inclination angle of the structure ranges in the interval [0°,45°] accordingly. Therefore, in order to obtain a more accurate value of the limit seismic factor λ_{lim}, we have preferred to use the bisection method in such an interval. The iterative procedure converges when the thickness of the domain becomes zero.

5 NUMERICAL EXAMPLES

5.1 Arch subject to vertical loads

In this section the safety level of a non-symmetric segmental arch is investigated. The arch under study covers a span of 4.70 meters, has an angle of embrace of 135° (with the right abutment positioned at 25° and the left one at 165° from the horizontal reference line), has a constant thickness of 30 cm and is subdivided in 20 elements, each one subject only to its self-weight. Although the specific weight used for the computation is 18kN/m^3, it is also worth noting that this value is irrelevant, since all the elements have the same size.

Figure 4 shows the line of thrust closest to the geometrical axis (the thicker poly-line) which is everywhere within the profile of the arch. Furthermore, the upper and lower bound lines of thrust that border the domain of safety are also shown. The computations, performed with the computer program *ArchiVAULT*, in which the procedure has been implemented, have provided the vertical thickness of the domain of 18.52 cm, the full range factor of safety of 1.6177 and the performance factor of 0.6182, a value that points to a rather safe arch.

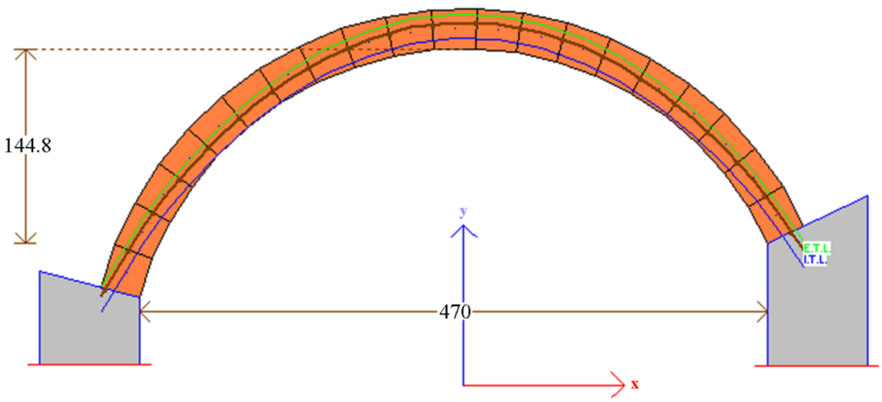

Figure 4: Safety assessment of an arch only subject to its self-weight

5.2 Arch subject to vertical and horizontal forces

According to the value of 0.6182 of the performance factor obtained for the load case of only self-weight in the previous section, the arch seems to be capable of supporting also a pattern of horizontal forces and, therefore, to resist a seismic event.

In this section the safety verification of the arch subject to its self-weight and a set of horizontal forces proportional to it is carried out and the limit value of the seismic factor is computed considering an earthquake acting both in +X and –X direction.

In order to simulate the action of horizontal forces in +X direction, the plane on which the structure stands has been tilted clockwise (Fig. 5a). The maximum angle in correspondence to which the thickness of the domain becomes zero is computed to be 19.88° (indeed, in Fig. 5a the upper and lower bound lines of thrust coincide). It is worth noting that, in correspondence to this value, the performance factor mandatorily becomes zero and the limit seismic factor is 0.3616.

Instead, in order to simulate the action of horizontal forces in -X direction, the plane on which the structure stands has been tilted counterclockwise (Fig. 5b). The maximum angle in correspondence to which the thickness of the domain becomes zero is computed to be 9.88°. Accordingly, the limit seismic factor is 0.1742, a much lower value than that obtained for the rightward forces analysis and, therefore, a much lower seismic capacity is shown.

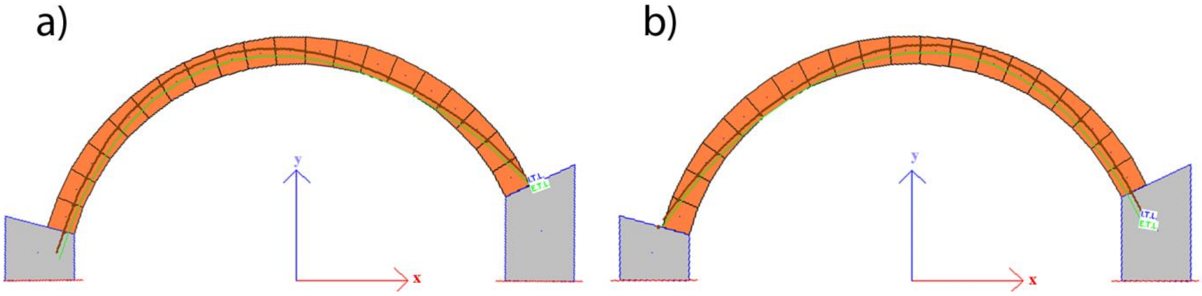

Figure 5: Safety assessment of an arch subject to its self-weight and a horizontal pattern of forces rightward (a) and leftward (b)

6 CONCLUSIONS

- In accordance with Heyman, collapse in masonry arches is not generally provoked by material failure because stresses are always rather small. Accordingly, to assess the safety a procedure based on the comparison between the shape and position of the thrust line and the profile of the arch has been formulated.
- The procedure, denoted as FRS Method, has been developed to compute the line of thrust closest to the geometrical axis of an arch subject to a set of vertical loads, the geometrical axis being represented through the poly-line that joins the element centroids.
- This procedure makes use of the finite difference method to minimize the distance between the ordinates of the element centroids and the ordinates of the nodes of the thrust line.
- Unlike other previous or recent methods in the literature, our method computes the “exact” solution avoiding both proceeding iteratively by trials and errors and the use of optimization techniques.
- Being mainly interested in assessing the behavior of arches in seismic regions, the FRS procedure has been extended in order to include the presence of horizontal forces proportional to the vertical ones.
- The value of the horizontal forces is computed, in an innovative way, exploiting the analogy between the inclined plane on which the structure stands and the inclination of the resultant of the vertical and horizontal forces. Thanks to this trick, the procedure, that minimizes the vertical distances between the ordinates of the element centroids and the ordinates of the nodes of the thrust line, still holds and can easily be exploited also for the seismic verification.
- The safety level of the arch is assessed defining the domain of equilibrium lines of thrust and computing the performance factor (PF), that is the reciprocal of the full range factor of safety (FRS).
- The domain of equilibrium lines of thrust and the FRS are exploited to estimate the seismic vulnerability of an arch by computing the limit seismic factor, which is achieved when the thickness of the domain of safety (i.e.: the PF) becomes zero.

REFERENCES

[1] Alecci, V. Stipo, G. La Brusco, A., De Stefano, M. and Rovero, L. Estimating elastic modulus of tuff and brick masonry: a comparison between on-site and laboratory tests. *Constr. Build. Mater.* (2019) **204**:828-38.

[2] Block, P., Ciblac, T. and Ochsendorf, J. Real-time limit analysis of vaulted masonry buildings. *Comput. Struct.* (2006) **84**:1841-52.

[3] Huerta Fernández, S. Mechanics of masonry vaults: the equilibrium approach. In: Paulo Lourenço and Pere Roca (Eds.): *Proc. of the 3rd International Seminar, Guimarães*, Portugal (2001).

[4] Pugi, F. and Galassi, S. Seismic analysis of masonry voussoir arches according to the Italian building code. *Ing. Sismica-Ital.* (2013) **30,3**:33-55.

[5] Galassi, S., Ruggieri, N. and Tempesta, G. A novel numerical tool for seismic vulnerability analysis of ruins in archaeological sites. *Int. J. Archit. Herit.* (2018) **14(1)**: 1-

22, 10.1080/15583058.2018.1492647.
[6] Galassi, S. Ruggieri, N. and Tempesta, G. Ruins and archaeological artifacts: vulnerabilities analysis for their conservation through the original computer program BrickWORK. In: Aguilar, R., Torrealva, D., Moreira, S., Pando, M., Ramos, L.F. (Eds.), *Proc. of 11th International Conference on structural analysis of historical constructions (SAHC2018)*, Cusco, Perù (2018), pp. 1839-1848.
[7] Cavalagli, N. Gusella, V. and Severini, L. Lateral loads carrying capacity and minimum thickness of circular and pointed masonry arches. *Int. J. Mech. Sci.* (2016) **115**:645–56.
[8] Dimitri, R. and Tornabene, F. A parametric investigation of the seismic capacity for masonry arches and portals of different shapes. *Eng. Fail. Anal.* (2015) **52**:1–34.
[9] Zampieri, P., Simoncello, N. and Pellegrino, C. Structural behavior of masonry arch with no-horizontal springing settlement, *Frattura ed Integrità Strutturale* (2018) **43**:182-190, 10.3221/IGF-ESIS.43.14.
[10] Zampieri, P., Simoncello, N. and Pellegrino, C. Seismic capacity of masonry arches with irregular abutments and arch thickness. *Constr. Build. Mater.* (2019) **201**:786-806, 10.1016/j.conbuildmat.2018.12.063.
[11] Zampieri, P., Amoroso, M. and Pellegrino, C. The masonry buttressed arch on spreading support. *Structures* (2019) **20**:226-236, 10.1016/j.istruc.2019.03.008.
[12] Zampieri, P., Cavalagli, N., Gusella, V. and Pellegrino, C. Collapse displacements of masonry arch with geometrical uncertainties on spreading supports. *Comput. Struct.* (2018) **208**:118-129, 10.1016/j.compstruc.2018.07.001.
[13] Heyman, J. The safety of masonry arches. *Int. J. Mech. Sci.* (1969) **11,4**:363-85, 10.1016/0020-7403(69)90070-8.
[14] Tempesta, G. and Galassi, S. Safety evaluation of masonry arches. A numerical procedure based on the thrust line closest to the geometrical axis. *Int. J. Mech. Sci.* (2019) **155**:206-21, 10.1016/j.ijmecsci.2019.02.036.
[15] Galassi, S. and Tempesta, G. The Matlab code of the method based on the Full Range Factor for assessing the safety of masonry arches. *MethodsX* (2019) **6**:1521-42, 10.1016/j.mex.2019.05.033.
[16] O'Dwyer, D. Funicular analysis of masonry vaults, *Comput. Struct.* (1999) **73**:187-197, 10.1016/S0045-7949(98)00279-X.
[17] Block, P. and Ochsendorf, J. Thrust network analysis: a new methodology for three-dimensional equilibrium, *J. Int. Assoc. Shell. Spat. Struct.* (2007) **48(3).**
[18] Marmo, F. and Rosati, L. Reformulation and extension of the thrust network analysis, *Comput. Struct.* (2017) **182**:104-118, 10.1016/j.compstruc.2016.11.016.
[19] Block, P. and Lachauer, L. Three-dimensional funicular analysis of masonry vaults, *Mech. Res. Commun.* (2014) **56**:53-60, 10.1016/j.mechrescom.2013.11.010.
[20] Fraddosio, A., Lepore, N. and Piccioni, M.D. Thrust surface method: An innovative approach for the three-dimensional lower bound Limit Analysis of masonry vaults, *Eng. Struct.* (2020) **202**:109846, 10.1016/j.engstruct.2019.109846.