A posteriori error estimates of elliptic and parabolic equations for the weak Galerkin finite element methods

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ABSTRACT

The practice of engineering calculation shows that the adaptive finite element method is quite an efficient numerical method for solving partial differential equations, where the reliable and effective a posteriori error estimator plays a crucial role in the implementation. In this talk, we first design a weak gradient recovery type a posteriori error estimator of the weak Galerkin finite element method for the elliptic equation. Then, for the Crank-Nicolson weak Galerkin finite element numerical discretization of the linear parabolic equation, we establish its time-space a posteriori error estimation by the elliptic reconstruction technique, where the numerical error is split into the elliptic error and the parabolic error by introducing the corresponding elliptic equation. The theory shows that any a posteriori error estimator of the elliptic equation can be taken to control the elliptic error, and the error of numerical solutions between two adjacent time layers can be used to control the parabolic error. The rationality of the error estimators is verified numerically according to the time-space adaptive algorithm. Finally, the conclusion of the linear parabolic equation is extended to the SAV weak Galerkin finite element discretization of the Allen-Cahn equation. Exploiting the time-space adaptive algorithm, the dynamic interface evolution of the Allen-Cahn equation is simulated by some benchmark numerical examples, which validates the effectiveness of the time-space error estimators.

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