15th World Congress on Computational Mechanics (WCCM-XV)
8th Asian Pacific Congress on Computational Mechanics (APCOM-VIII)
Virtual Congress: 31 July – 5 August 2022
S. Koshizuka (Ed.)

A TIME CONSISTENT METHOD BY PRECONDITIONING OF THE DIFFUSION TERM FOR UNSTEADY GAS-LIQUID TWO-PHASE FLOWS

TIANMU ZHAO[1] AND BYEONG ROG SHIN[2]

[1] Interdisciplinary Graduate School of Agriculture and Engineering, University of Miyazaki
Miyazaki 889-2192, Japan
z370701@student.miyazaki-u.ac.jp

[2] Mechanical Engineering Program, University of Miyazaki
Miyazaki 889-2192, Japan
brshin@cc.miyazaki-u.ac.jp

Key words: Gas-Liquid Two-Phase Flow, Time Consistent Preconditioning, Modified Roe Scheme, FDM, Shock Tube Problem

Abstract. *A time accurate and high resolution numerical method for gas-liquid two-phase flows is proposed. The artificial viscous terms in the flux splitting of upwinding are derived by using the preconditioner to enhance the stability of computation for compressible and incompressible combined flow with arbitrary void fractions. A homogeneous equilibrium gas-liquid two-phase model taken account of the compressibility of mixed media is used. A finite-difference 4th-order Runge-Kutta method and a Roe-type flux splitting method with the MUSCL TVD scheme are employed. By this method, a one-dimensional two-phase shock tube problem was computed and confirmed the applicability to the unsteady and arbitrary Mach number flow problems. Detailed observations of shock and expansion wave propagations through the gas-liquid two-phase media and comparisons of predicted results with exact solutions are made.*

1 INTRODUCTION

Gas-liquid multiphase flow is well encountered in engineering problems of boiling, aerosol, phase change and the flow through hydro machines and under water vehicles, moving with a high-speed in working fluid of liquid state. As cavitation bubble occurs and collapses near the surface of the body as an example, it causes noise and vibration and damages to the hydraulic machine system. In the sense of reducing these unfavourable impacts from the gas-liquid multiphase flow, therefore, accurate prediction and estimation of such flow is very important.

To understand the behavior of collapsing of cavitation bubbles, some efforts to propose cavity flow model for numerical simulations [1-3] and, analytical and experimental method for shock-bubble interaction problems [4,5] have been made. However, due to the strong and complicated unsteady flow phenomenon such as phase changes, the co-existence of compressible and incompressible flow, vortex shedding and turbulence of cavitating flow, the mathematical expression of the flow as well as a development of numerical method is not established yet. Recently, Shin et al. [6,7] has proposed a mathematical cavity flow model based on a homogeneous equilibrium model taking account of the compressibility of the gas-liquid two-phase media. With this model, the mechanism of developing cavitation has been investigated through the application to a couple of cavitating flows problems [8,9]. These schemes were extended to preconditioned dual time-stepping methods to treat both compressible and incompressible flow effects associated with very large range of sound speeds, which can arise in cavitating flows with multi-rates of void fraction [10].

The purpose of this paper is to extend previous high resolution scheme [10] with the 3rd-orde MUSCL TVD scheme to a time consistent method for solving high speed gas-liquid two-phase flows with arbitrary Mach numbers. In order to obtain a stable and accurate treatment of gas-liquid interfaces considered by contact discontinuity, artificial viscous terms in the flux splitting on the upwinding are modified by using the preconditioner. As numerical examples, gas-liquid two-phase shock tube problems with arbitrary void fractions were computed and checked the applicability to the unsteady problem. Unsteady shock wave phenomena in the gas-liquid two-phase media are investigated.

2 NUMERICAL METHOD

2.1 Gas-liquid two-phase model

Gas-liquid two-phase flow is modeled to a pseudo single-phase flow by using the concept of the homogeneous equilibrium model [6] and the reconstruction of equation of state. In this study, Tammann's equation of state [11], $p + p_c = \rho_l K(T + T_c)$ is used for the liquid phase. Here p, ρ and T are mixture pressure, density and temperature, respectively. The subscript l represents the liquid phase. K, p_c and T_c are the liquid, pressure and temperature constants of liquid. On the order hand, the gas phase is assumed as an ideal gas with the equation of state of $p = \rho_g RT$, where R is the gas constant and the subscript g represents the gas phase. The density ρ of the two-phase medium is expressed by combining linearly the gas phase density ρ_g and the liquid phase density ρ_l with the local void fraction α (gas volume fraction) and the quality Y. By assuming the local equilibrium conditions, the equation of state becomes,

$$\rho = \frac{p(p + p_c)}{K(1 - Y)p(T + T_c) + RY(p + p_c)T} \tag{1}$$

In this model, the apparent compressibility is considered, and the speed of sound c is exactly derived by using thermodynamic relations, $c^2 = \rho C_p / (\rho_T + \rho C_p \rho_p)$. Here, ρ_T and ρ_p represent $\partial\rho/\partial T$ and $\partial\rho/\partial p$, respectively. $C_p$is the specific heat capacity at constant pressure of $C_p = YC_{pg} + (1 - Y)C_{pl}$. The relation between the α and the quality Y is given by $\rho(1 - Y) = (1 - \alpha)\rho_l$ and $\rho Y = \alpha\rho_g$, where

$$\alpha = \frac{RY(p + p_c)T}{K(1 - Y)p(T + T_c) + RY(p + p_c)T} \tag{2}$$

2.2 Governing equations and preconditioned stability term

Based on the above model concept, the one dimensional Euler equations for the mixture mass, momentum, energy and the gas-phase mass conservation can be written as follows,

$$\frac{\partial \boldsymbol{Q}}{\partial t}+\frac{\partial \boldsymbol{E}}{\partial x}=0 \quad with \quad \boldsymbol{Q}=\begin{bmatrix}\rho\\ \rho u\\ e\\ \rho Y\end{bmatrix} \quad and \quad \boldsymbol{E}=\begin{bmatrix}\rho u\\ \rho u^2+p\\ (e+p)u\\ \rho uY\end{bmatrix} \tag{3}$$

where, u and e in the unknown variable vectors $\boldsymbol{Q}$ and flux vectors $\boldsymbol{E}$ are velocity and total energy, respectively.

The hydraulic flow with hydraulic transients and hydroacoustics such as cavitating flow has both compressible and incompressible flow characteristic. For this kind of flow, a compressible flow model which can handle the incompressible flow is advantageous. For this purpose, artificial compressible method and preconditioning method [12,13] have been developed and used in the steady state computation. In general, these methods are not consistent in time because the time derivative is multiplied by the preconditioning matrix. For unsteady flow computation, therefore, the preconditioning method for steady problems should be improved to that of consistent in time. In this paper, according to the basic concept of high-order upwind scheme, a modification of artificial viscous terms in Roe's flux difference splitting was examined: the artificial viscous terms in the flux splitting is modified by the preconditioning matrix to enhance the numerical stability at the treatment of shock interfaces in the two-phase media. To obtain the preconditioning matrix of the non-conservative system controllable the propagation of acoustic waves, the conserved variables $\boldsymbol{Q}$ in equation (3) is transformed to primitive variables $\boldsymbol{W}$ as follows:

$$\Gamma^{-1}\frac{\partial \boldsymbol{W}}{\partial t}+\frac{\partial \boldsymbol{E}}{\partial x}=0 \quad with \quad \boldsymbol{W}=[p,u,T,Y]^T \quad and \quad \Gamma^{-1}=\frac{\partial \boldsymbol{Q}}{\partial \boldsymbol{W}} \tag{4}$$

To improve the stability for the multiphase problems with incompressible and compressible flow nature, a modified Roe's approximation [14] was used. The derivative of the flux vector $\boldsymbol{E}$ can be written with the numerical flux as $(\partial \boldsymbol{E}/\partial x)_i=(\boldsymbol{E}_{i+1/2}-\boldsymbol{E}_{i-1/2})/\Delta x$, and the flux $\boldsymbol{E}_{i+1/2}$ modified by using the Γ^{-1} and Γ_p as

$$\boldsymbol{E}_{i+1/2}=(1/2)\left\{\boldsymbol{E}(\boldsymbol{Q}^L_{i+1/2})+\boldsymbol{E}(\boldsymbol{Q}^R_{i+1/2})-\Gamma^{-1}_{i+1/2}(\boldsymbol{L_p}|\Lambda|\boldsymbol{L_p^{-1}})_{i+1/2}(\boldsymbol{W}^R_{i+1/2}-\boldsymbol{W}^L_{i+1/2})\right\} \tag{5}$$

where, Λ, $\boldsymbol{L_p}$ and $\boldsymbol{L_p^{-1}}$ are matrices of eigenvalues and the left eigenvectors of $\Gamma_p(\partial E/\partial Q)\Gamma^{-1}$. The stability terms of third terms in Eq.(5) can also be consisted of $\Gamma^{-1}_{pi+1/2}$ instead of $\Gamma^{-1}_{i+1/2}$. Γ_p^{-1} is preconditioner of the non-conservative system which is formed by the addition of the vector $\theta[1,u,H,Y]^T$ to the first column of the Γ^{-1} as,

$$\Gamma_p^{-1}=\begin{bmatrix}\theta+\frac{\partial\rho}{\partial p} & 0 & \frac{\partial\rho}{\partial T} & \frac{\partial\rho}{\partial Y}\\ u\left(\theta+\frac{\partial\rho}{\partial p}\right) & \rho & u\frac{\partial\rho}{\partial T} & u\frac{\partial\rho}{\partial Y}\\ H\left(\theta+\frac{\partial\rho}{\partial p}-1\right) & \rho u & \rho C_p+H\frac{\partial\rho}{\partial T} & H\frac{\partial\rho}{\partial Y}\\ Y\left(\theta+\frac{\partial\rho}{\partial p}\right) & 0 & Y\frac{\partial\rho}{\partial T} & \rho+Y\frac{\partial\rho}{\partial Y}\end{bmatrix}$$

where, parameter θ is chosen by Weiss & Smith [15]. $\boldsymbol{W}^{L,R}_{i+1/2}$ is obtained by applying the third-order MUSCL TVD scheme [16] as followings.

$$\begin{aligned} \boldsymbol{W}_{i+1/2}^{L} &= \boldsymbol{W}_i + (1/4)\{(1-\kappa)D^{+}\boldsymbol{W}_{i-1/2} + (1+\kappa)D^{-}\boldsymbol{W}_{i+1/2}\} \\ \boldsymbol{W}_{i+1/2}^{R} &= \boldsymbol{W}_{i+1} - (1/4)\{(1-\kappa)D^{-}\boldsymbol{W}_{i+3/2} + (1+\kappa)D^{+}\boldsymbol{W}_{i+1/2}\} \end{aligned} \tag{6}$$

Here, the flux-limited values of $D\boldsymbol{W}$ and the minmod function are determined by

$$\begin{aligned} D^{+}\boldsymbol{W}_{i-1/2} &= \mathrm{minmod}(\delta\boldsymbol{W}_{i-1/2}, b\delta\boldsymbol{W}_{i+1/2}), \\ D^{-}\boldsymbol{W}_{i+1/2} &= \mathrm{minmod}(\delta\boldsymbol{W}_{i+1/2}, b\delta\boldsymbol{W}_{i-1/2}), \\ \delta\boldsymbol{W}_{i+1/2} &= \boldsymbol{W}_{i+1} - \boldsymbol{W}_i \\ \mathrm{minmod}(x,y) &= \mathrm{sign}(x)\max[0, \min\{|x|, y\,\mathrm{sign}(x)\}] \end{aligned}$$

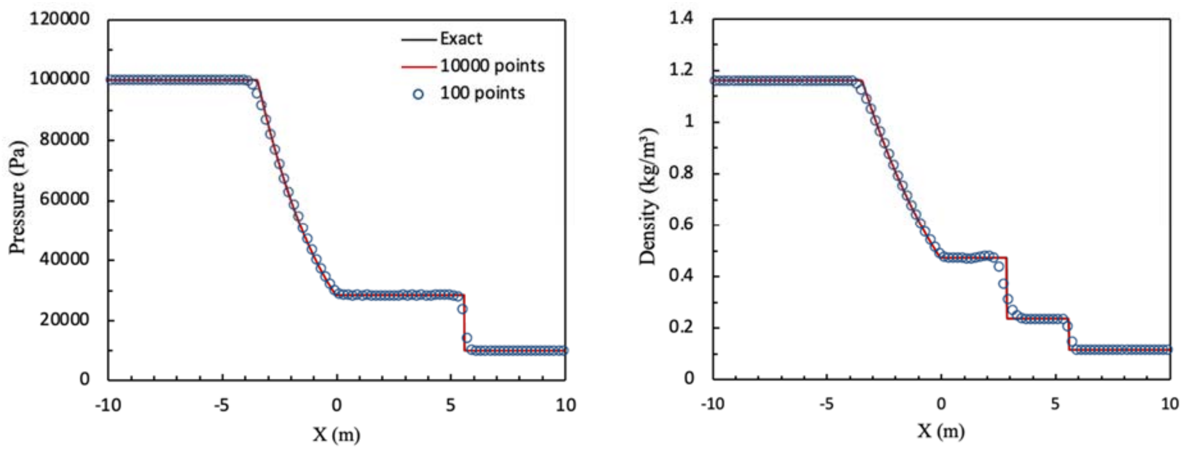
The linear combination parameter κ is determined by the rang of $-1 \le \kappa \le 1$ and has an effect on the accuracy. On the other hand, the slope of the flux in the minmod function is controlled by the limiter b. The rang of b, $1 \le b \le (3-\kappa)/(1-\kappa)$, is determined by the condition of TVD stability.

For time accurate solutions, the following 4th-order Runge-Kutta explicit method in finite difference discretization is used in equation (4) with $\boldsymbol{L}(\boldsymbol{Q}) = \partial\boldsymbol{E}/\partial x$ because it is capable of capturing linear as well as non-linear waves and can resolve contact discontinuities.

$$\begin{aligned} \boldsymbol{W}^{1} &= \boldsymbol{W}^{n} - \Delta t/4\Gamma\boldsymbol{L}(\boldsymbol{Q}^{n}) \qquad & \boldsymbol{W}^{2} &= \boldsymbol{W}^{n} - \Delta t/3\Gamma\boldsymbol{L}(\boldsymbol{Q}^{1}) \\ \boldsymbol{W}^{3} &= \boldsymbol{W}^{n} - \Delta t/2\Gamma\boldsymbol{L}(\boldsymbol{Q}^{2}) \qquad & \boldsymbol{W}^{n+1} &= \boldsymbol{W}^{n} - \Delta t\Gamma\boldsymbol{L}(\boldsymbol{Q}^{3}) \end{aligned} \tag{7}$$

3 NUMERICAL RESULTS

The present numerical method has been validated by using the Riemann problem suggested by Sod [17] as a standard test problem. The domain is x of [-10 m, 10m]. Initial conditions of left (L)- and right (R)-hand side at discontinuous surface (x = 0m) at T=300K for a given void fraction α_i are as followings: p_L= 0.1MPa, u_L= 0m/s, α_L= α_i and p_R= 0.1MPa, u_R= 0m/s, α_R= α_i.



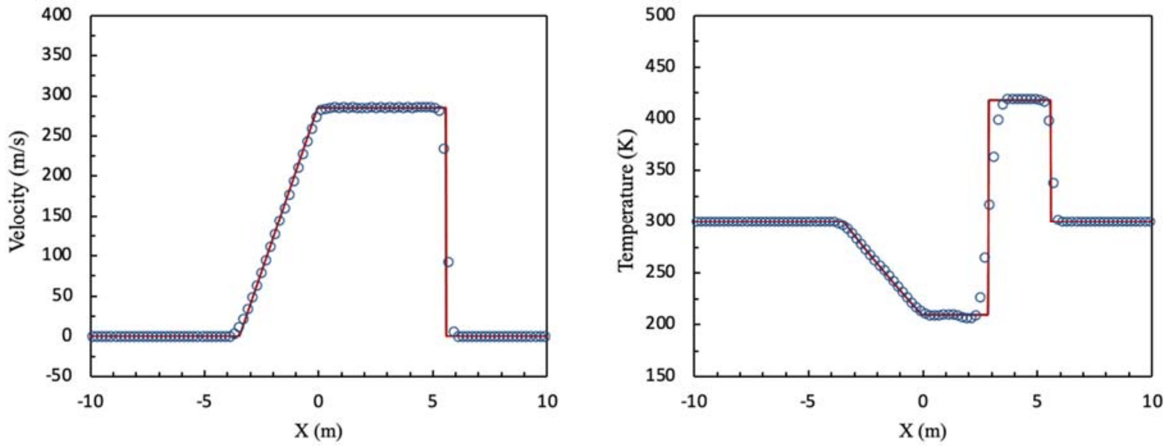
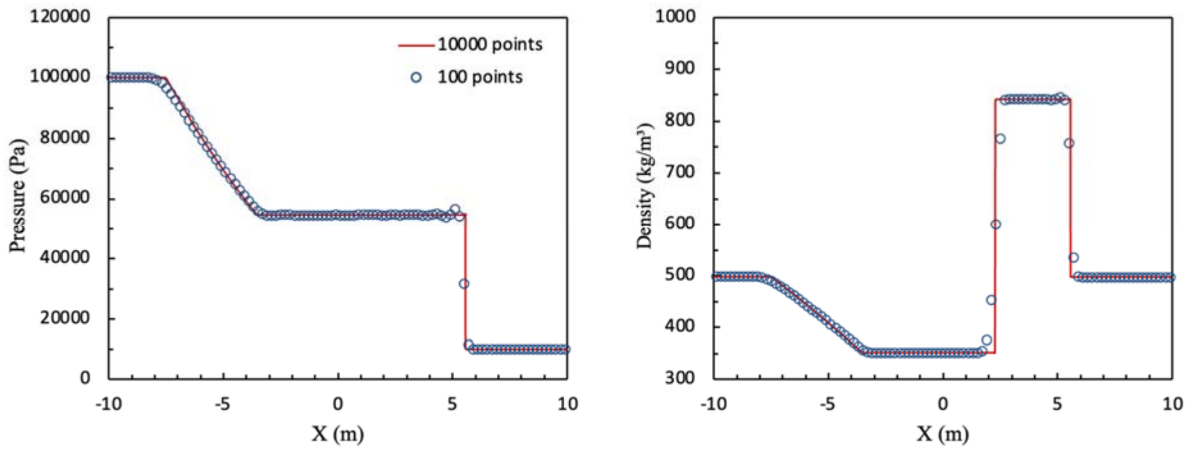


FIGURE 1: Computational results of pressure, density, velocity and temperature distributions for ideal gas at α_i = 100%

Figure 1 shows comparisons with exact solution for the shock tube problem of ideal gas (α_i =100%) with the ratio of specific heats γ =1.4 at t =0.01s. The results obtained by present time consistent method with grid points of 10,000 (red line) matched with exact solutions. The coarse grid of 100 (symbols) is also fairly well predicted for the comparison with exact ones except small dissipation at discontinuity.



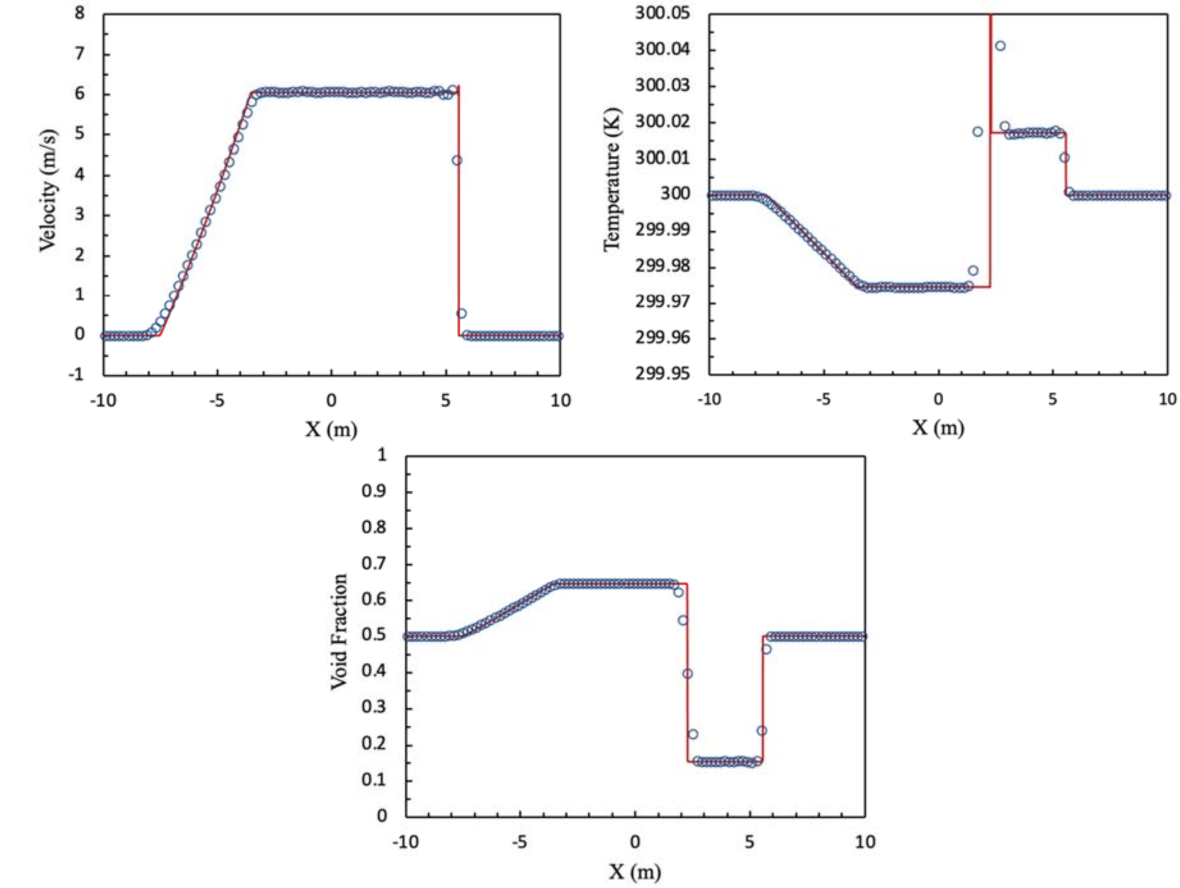
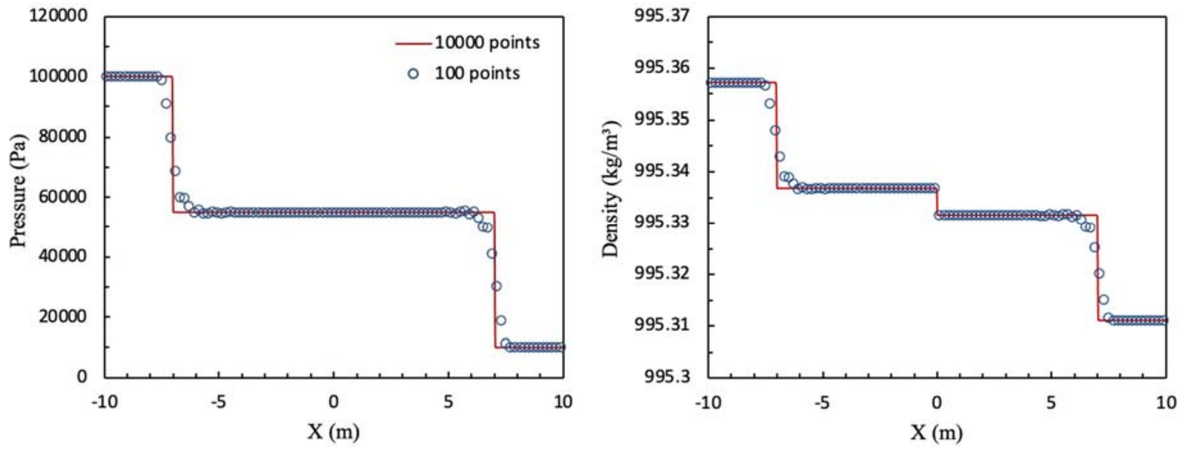


FIGURE 2: Computational results of pressure, density, velocity, temperature and void fraction distribution for gas-liquid 2-phase media at $\alpha_i = 50\%$, time $t = 0.357$s



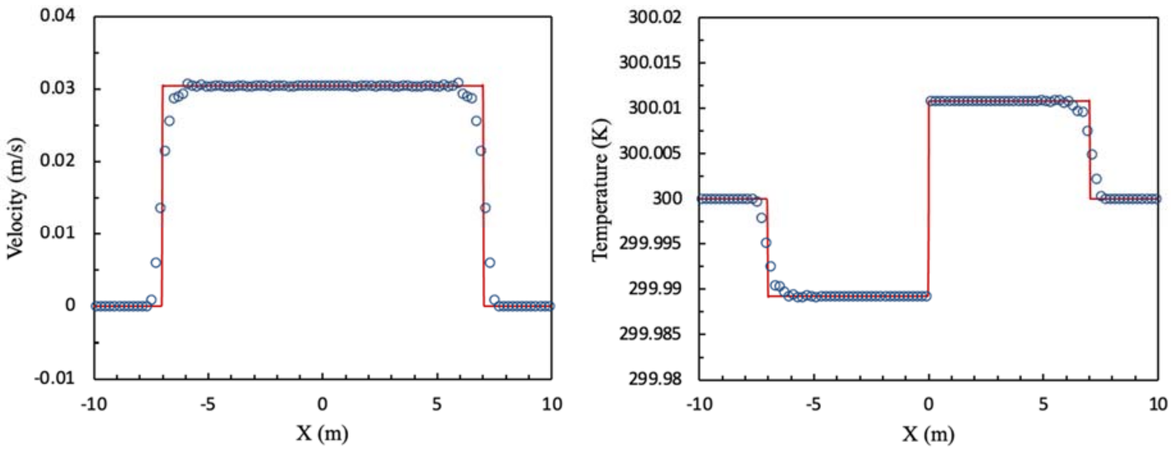


FIGURE 3: Computational results of pressure, density, velocity and temperature distribution for liquid phase at α_i = 0%, time $t = 0.00473$s

Based on the validity of the present method in Fig.1, the present method was applied to the two-phase shock tube flow in thermal process with arbitrary void fraction to check the applicability to the unsteady problem and investigate the characteristics of pressure waves propagating in the gas-liquid two-phase medium. Figure 2 shows calculated results at initial void fraction of 50%. The value of parameter θ for preconditioning was taken the order of Mach number. As seen in this figure, both results with 100 and 10,000 grid points were well predicted unsteady shock tube problems. However, the temperature profile at the place of density jump showed an overshoot even though the value is quite smell. Almost the same results were obtained by the stability term with $\Gamma_{pi+1/2}^{-1}$ instead of $\Gamma_{i+1/2}^{-1}$ in Eq.(5). Here, the result with 10,000 were regarded as an exact solution, which was coincide with solutions obtained without preconditioning [18]. In this flow case the compression wave is propagating with decreasing the void fraction because the compression wave compresses the two-phase medium. However, expansion wave shows the opposite behavior with increasing the void fraction, resulting the contact discontinuity exists and propagates toward right-hand side by the wave induced velocity. Pressure behind the shock wave is higher than single-phase of gas. Figure 3 shows another computational result for liquid phase at α_i = 0%. The expansion wave is propagating like a compression wave. It is different from the gas phase because there exists big difference of speed of sound and wave induced velocity between gas states and liquid states. Very small changes of density and velocity are observed as investigated in the previous work [18].

4 CONCLUSIONS

A time consistent high resolution finite-difference method for gas-liquid two-phase flow was proposed and applied to the two-phase shock tube problem. In the proposed method, the artificial viscous terms in the flux splitting of upwinding are derived to improve the stability and used 4th-order Runge-Kutta method combined with MUSCL TVD scheme. A homogeneous equilibrium model of gas-liquid two-phase flows was applied.

Numerical results showed that the present high resolution method obtains a good prediction of pressure, density, velocity, temperature and void fraction distributions in comparison with exact solutions, and quite well simulated unsteady phenomena of the shock wave including the propagation of both compression and expansion waves. The reliability and applicability of the present method to unsteady flow problems with arbitrary void fraction and sound of speed were confirmed as consequence.

ACKNOWLEDGEMENTS

This work was partly supported by JSPS KAKENHI Grant Number JP20K04288.

REFERENCES

[1] Deshpande, M., Feng, J. and Merkle, C.L. Numerical modeling of the thermodynamic effects of cavitation. *ASME J. Fluids Engng* (1997) **119**:420-427.
[2] Singhal, A.K., Vaidya, N. and Leonard, A.D. Multi-dimensional simulation of cavitating flows using a PDF model for phase change. *ASME Paper* FEDSM97-3272 (1997).
[3] Merkle, C.L., Feng, J.Z. Buelow, P.E.O. Computational modelling of the dynamics of sheet cavitation. *Proc. 3rd Int. Sympo. on Cavitation* (1998) **2**:307-311.
[4] Bourne, N.K. and Field, J.E. Shock-induced collapse of single cavities in liquids. *J. Fluid Mech* (1992) **244**:225-240.
[5] Tomita, T. and Shima, A. Mechanisms of impulsive pressure generation and damage pit formation by bubble collapse. *J. Fluid Mech* (1986) **169**:535-564.
[6] Shin, B.R., Iwata, Y. and Ikohagi, T. A numerical study of unsteady cavitating flows using a homogenous equilibrium model. *Comput. Mech* (2003) **30**:388-395.
[7] Shin, B.R. Numerical analysis of unsteady cavitating flow by a homogeneous equilibrium model. *31st AIAA Fluid Dyn. Conf. AIAA Paper* 2001-2909 (2001) 1-10.
[8] Seo, J.H., Moon, Y.J. and Shin, B.R. Prediction of cavitation flow noise by direct numerical simulation. *J. Comput. Phys* (2008) **227**:6511-6531.
[9] Dittakavi, N., Chunekar, A. and Frankel, S. Large eddy simulation of turbulent-cavitation interactions in a venturi nozzle. *ASME J. Fluid Engng* (2010) **132**:121301-1-121301-11.
[10] Shin, B.R., Yamamoto, S. and Yuan, X. Application of preconditioning method to gas-liquid two-phase flow computations. *ASME J. Fluid Engng* (2004) **126**:605-612.
[11] Chen, H.T. and Collins, R. Shock wave propagation past on ocean surface. *J. Comput. Phys* (1971) **7**:89-101.
[12] Turkel, E. Preconditioned methods for solving the incompressible and low speed compressible equations. *J. Comput. Phys* (1987) **72**:277-298.
[13] Choi, Y.H. and Merkle, C.L. The application of preconditioning in viscous flows. *J. Comput. Phys* (1993) **105**:207-223.
[14] Roe, P.L. Approximate Riemann solvers, parameter vectors, and difference scheme. *J. Comput. Phys* (1981) **43**:357-372.
[15] Weiss, J.M. and Smith, W.A. Preconditioning applied to variable and constant density flows. *AIAA J.* (1995) **3**:2050-2057.
[16] van Leer, B. Towards the ultimate conservative difference scheme v. A second-order sequel to godunov's method. *J. Comput. Phys* (1995) **32**:101-136.
[17] Sod, G.A. A survey of several finite difference methods for systems of nonlinear hyperbolic conservation laws. *J. Comput. Phys* (1978) **27**: 1-31.
[18] Shin, B.R. A high resolution numerical scheme for a high speed gas-liquid two-phase-flow. *J. Mech. Sci. Tech* (2011) **25**:1373-1379.