14th World Congress in Computational Mechanics (WCCM)
ECCOMAS Congress 2020
19 – 24 July 2020, Paris, France
F. Chinesta, R. Abgrall, O. Allix and M. Kaliske (Eds)

GEOMISO TNL: A SOFTWARE FOR NON-LINEAR STATIC T-SPLINE-BASED ISOGEOMETRIC ANALYSIS OF COMPLEX MULTI-PATCH STRUCTURES

PANAGIOTIS L. KARAKITSIOS[1], GEORGIOS I. KARAISKOS[2], ATHANASIOS I. LEONTARIS[3] AND PANAGIOTIS F. KOLIOS[4]

[1] Geomiso Company
Themistokleous 19, Farsala
pkarakitsios@geomiso.com, https://www.geomiso.com/

[2] Geomiso Company
Themistokleous 19, Farsala
gkaraiskos@geomiso.com, https://www.geomiso.com/

[3] Geomiso Company
Themistokleous 19, Farsala
aleontaris@geomiso.com, https://www.geomiso.com/

[4] Geomiso Company
Themistokleous 19, Farsala
pkolios@geomiso.com, https://www.geomiso.com/

Keywords: Isogeometric analysis, T-splines, NURBS, Non-linearity, Patch, Software

Summary. A new software, Geomiso TNL, is proposed to facilitate the use of isogeometric analysis and 3D design with NURBS and T-splines. Its dual nature eliminates geometric errors by merging geometric design with mesh generation into a single procedure. It is based on the isogeometric method, the powerful generalization of the traditional finite element method. This paper presents four sample applications in non-linear solid and structural mechanics. This software is seen to handle these situations remarkably well, as the numerical examples exhibit significantly improved accuracy of the results, such as displacement, strain and stress fields, and reduced computational cost when compared with finite element analysis. It is argued that Geomiso TNL is a new, more efficient, alternative to finite element software packages and possesses several advantages.

European Union
European Regional Development Fund
HELLENIC REPUBLIC
MINISTRY OF ECONOMY & DEVELOPMENT
SPECIAL SECRETARY FOR ERDF & CF
MANAGING AUTHORITY OF EPAnEK
EPAnEK 2014-2020
OPERATIONAL PROGRAMME
COMPETITIVENESS
ENTREPRENEURSHIP
INNOVATION
ΕΣΠΑ 2014-2020
ανάπτυξη - εργασία - αλληλεγγύη
Partnership Agreement 2014 - 2020
Co-financed by Greece and the European Union

1 INTRODUCTION

In this paper we introduce a new software for applications on isogeometric analysis and 3D design with NURBS and T-splines, which is a viable alternative to finite element software packages with several advantages, as it facilitates the geometry modeling within analysis and offers superior quality numerical results in various applications. Geomiso TNL, which provides non-linear static T-spline-based isogeometric analysis with the ability of analyzing multipatch complex structures, satisfies the rising industrial need for technical software of fully integrated CAD/CAE nature. It provides an innovative way to merge geometric design with mesh generation into a single procedure by creating, with its hybrid graphical user interface, 3D models as tensor product grids. This hybrid software, used for both design and analysis, has many features in common with both FEA software and design programs. The isogeometric method, in combination with material non-linearity, has attracted increasing attention, as a result of the industrial need for high product quality, coupled with increasingly stringent safety.

Applications to plane strain/stress and 3D problems are demonstrated with a comparison between Geomiso TNL and FEA programs. We compare the accuracy of the numerical results for typical situations of academic and industrial interest arising in structural mechanics, while we conduct parametric investigations on the effects of control point number, element number and polynomial order of shape functions. This new software represents improvements over FEA software, as higher accuracy, robustness and stability level is accomplished with considerably reduced computational cost.

This paper is structured as follows. In Section 2 we exhibit an overview on IGA, multipatch NURBS models, T-splines and material non-linearity. Section 3 refers to the dual CAD/CAE nature of Geomiso TNL. Four applications, and more specifically, a multipatch NURBS model, a T-spline model of a stadium roof, a steel plate with rectangular cross-section under tensile load and a steel beam under pure bending are presented followed by comparison between Geomiso TNL, FEA and exact solution in Section 4, while conclusions are drawn in Section 5.

2 A BRIEF OVERVIEW ON ISOGEOMETRIC ANALYSIS

Isogeometric analysis is a methodology that has come to bridge the gap between computer-aided design and finite element analysis. It was introduced by Hughes et al. [1] and since then it has attracted a lot of attention for solving boundary value problems as a result of using the same shape functions, means splines, functions commonly used in CAD, for both describing the domain geometry and building the numerical approximation of the solution. The utilization of the exact geometry mesh for analysis eliminates the geometric errors, while there is no need of repeating the geometry design for refinement purposes [2]. In contrast, remeshing with more and smaller elements is the standard technique in FEA, as it cannot fully utilize the available data of the exact geometry mesh. Popular NURBS and sophisticated T-splines are proved the most suitable shape functions and a mighty tool for isogeometric analysis [3].

European Union European Regional Development Fund
HELLENIC REPUBLIC MINISTRY OF ECONOMY & DEVELOPMENT SPECIAL SECRETARY FOR ERDF & CF MANAGING AUTHORITY OF EPAnEK
EPAnEK 2014-2020 OPERATIONAL PROGRAMME COMPETITIVENESS ENTREPRENEURSHIP INNOVATION
ΕΣΠΑ 2014-2020 ανάπτυξη - εργασία - αλληλεγγύη Partnership Agreement 2014 - 2020
Co-financed by Greece and the European Union

2.1 NURBS-based isogeometric analysis

NURBS has enjoyed wide popularity worldwide, as can exactly construct any conic section. NURBS-based isogeometric analysis is very well suited for computational analysis, leading to more accurate results in comparison with standard finite elements based on Lagrange polynomials [1]. Parameter space is important as all calculations take place in it, while index space plays an auxiliary role. NURBS geometries inherit properties, such as continuity and compact support for their basis functions. Due to their higher inter-element continuity, the overlapping is greater in comparison with FEA. Regularity –1 indicates discontinuity, while regularity 0 is the minimum continuity for interior knots [2].

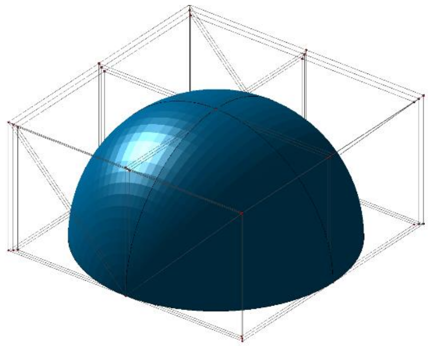


Figure 1: The control net and NURBS mesh for the Hagia Sofia Dome, designed in Geomiso TNL.

2.2 Multiple NURBS patches

In geometry modeling, parts of a product are modeled separately using a single patch. The discretized NURBS model is decomposed into patches, subdomains with the same material and geometry type. Each parent patch may contain subsidiary patches, consisted of knot spans, where C^{-1} or C^{0} continuity is attained across their interfaces. Complex, multiply connected domains, can frequently be handled quite simply by using multiple patches, while the tensor product structure of a single patch makes their representation poorly suited.

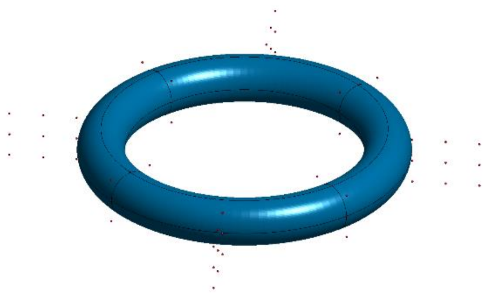


Figure 2: Torus designed in Geomiso TNL, using multiple NURBS patches.

Co-financed by Greece and the European Union

2.3 Building a T-spline model

NURBS were until lately the main shape functions used in isogeometric analysis [1]. In 2008, T-splines were introduced as a more efficient alternative that holds all the benefits of NURBS and exhibits more design capabilities, like watertightness, but also sophisticated implementation, that allow better handling more complex geometries and permits local refinement ensuring higher-order continuity and smoothness across patches [3]. Cubic T-splines can accurately design any geometry and make the problematic and often impossible merging of patches feasible. This generalization of NURBS defined by a grid of control points, where a row of control points can terminate without traversing the entire surface. Index space plays an important role, where junctions, anchors, local knot vectors, continuity reduction lines and elements are defined, while parameter space is auxiliary. Cartesian space represents the real geometry of the analyzed structure. T-splines allow us to build spaces that are complete up to a desired degree, as smooth as an equivalent NURBS basis, and capable of being locally refined, while keeping the original geometry and parameterization unchanged [3]. Each function has its own local knot vector, but these are inferred from a global structure, T-mesh, that encodes a topology and parameterization for the entire T-spline object. Index space of a T-mesh is a rectangular tiling of a region in $\mathbb{R}^2$, such that each edge of every rectangle has positive integer value and vertices connecting three edges, referred to as T-junctions. Each anchor will be used to infer local knot vectors, which will define a T-spline blending function. The continuity in physical space follows directly from that of its blending functions in parameter space. The local T-spline construction warrants closer consideration, particularly as it bears on numerical quadrature. The continuity of each blending function is determined from its local knot vector. A line of decreased continuity does not necessarily propagate throughout the domain and thus T-splines may have different smoothness within a T-mesh [3].

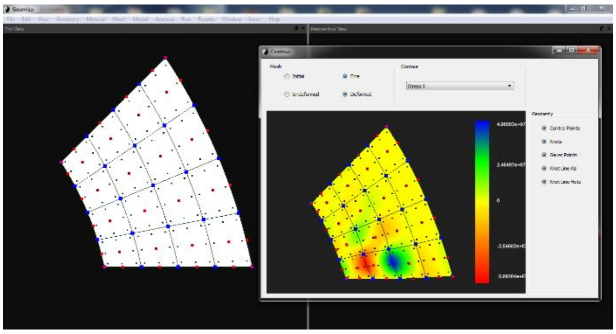
2.4 Material non-linearity

Basic material models, such as bilinear, are a set of rules that determine the elastic behavior, when the material yields (yield criterion), the behavior in the inelastic range after yielding (flow rule) and rules regarding the behavior under cyclic load reversals. Based on these, materials are categorized into elastic–perfectly plastic, elastic-perfectly plastic with isotropic hardening, and elastic-perfectly plastic with kinematic hardening [4]. For the bilinear model, the behavior prior yielding is described by Hooke's law. The yield point is defined by the yield stress σ_y or strain ε_y. When $\sigma>\sigma_y$, the slope of the monotonic σ-ε curve is the tangent modulus, E_T. The tangent post–yield modulus E_T is a function of the initial E and is defined with the hardening ratio b, as $E_T=b\cdot E$. The material is elastic-perfectly plastic when b=0. Instead of b, the post yield stiffness is often determined with hardening ratio H, the slope of the plastic strain σ-ε_{pl} curve ($\sigma=H\cdot\varepsilon_{pl}$, $\varepsilon_{pl} = \varepsilon - \varepsilon_y$). When the material yields ($\varepsilon \geq \varepsilon_y$, $\varepsilon_{pl}\neq 0$), a small strain increment $\Delta\varepsilon$ produces a stress increment, $\Delta\sigma=E_T\cdot\Delta\varepsilon$, and the total strain is equal to $\varepsilon=\varepsilon_{el}+\varepsilon_{pl}$.

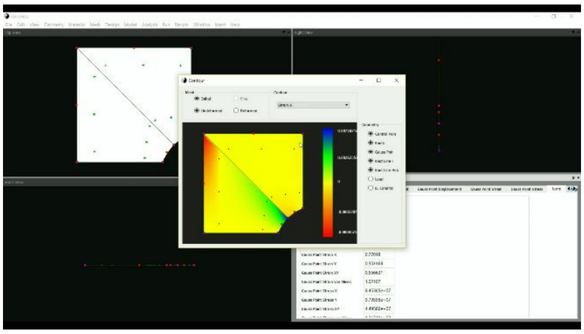
Co-financed by Greece and the European Union

3. THE DUAL NATURE OF GEOMISO TNL

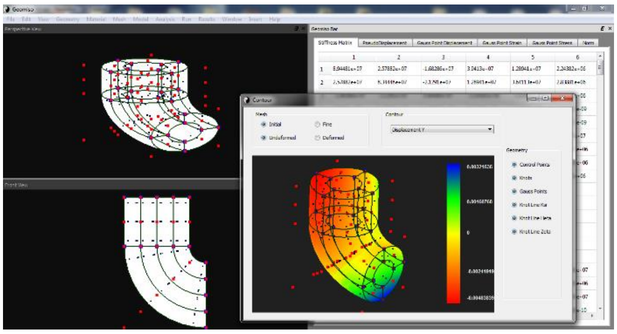
Geomiso TNL is not just a plug-in, but a new software having a dual CAD/CAE nature. All of the geometrical, connectivity, material, quadrature, refinement, constraint and load data, must be given as inputs to its hybrid user interface, in order to calculate stiffness matrix, pseudodisplacements, and contour plots of displacement, strain and stress fields. The geometrical data include control point variables, polynomial orders, number of univariate basis functions and knot value vectors for each of the three parametric directions (ξ, η, ζ).



(a) Plane stress



(b) Plane strain



(c) 3D

Figure 3: (a) The control net, mesh of 16 elements and contour plot of stresses X for a part of an annulus (plane stress problem). Control points are shown in red, while knots in blue. (b) The coarse mesh for a plate with a hole (plane strain problem) with two one element patches and contour plot of strains X. (c) The mesh for a pipe (3D problem) consists of 8 elements, while the control net of 45 control points.

Geomiso TNL improves engineering productivity and accelerates development time, as it merges geometric design with mesh generation into a single procedure, eliminates geometric errors, significantly increases the accuracy of the numerical results, and drastically reduces the required computational cost and time, when compared with commercial finite element software packages.

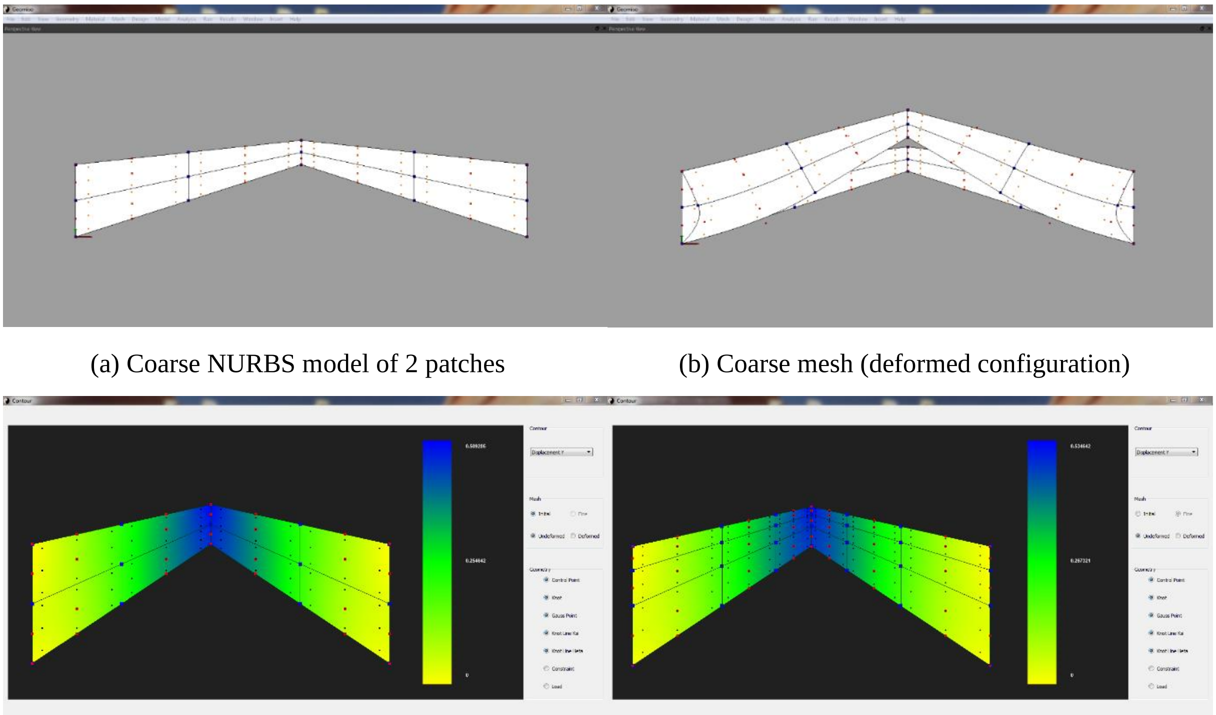
European Union
European Regional Development Fund
HELLENIC REPUBLIC
MINISTRY OF ECONOMY & DEVELOPMENT
SPECIAL SECRETARY FOR ERDF & CF
MANAGING AUTHORITY OF EPAnEK
EPAnEK 2014-2020
OPERATIONAL PROGRAMME
COMPETITIVENESS
ENTREPRENEURSHIP
INNOVATION
ΕΣΠΑ 2014-2020
ανάπτυξη - εργασία - αλληλεγγύη
Partnership Agreement 2014 - 2020
Co-financed by Greece and the European Union

4. APPLICATIONS

We present four applications of Geomiso TNL to linear and non-linear problems in solid mechanics. The numerical results, which are compared with FEA results and the exact solution, indicate that Geomiso TNL is a more efficient alternative to finite element software packages.

4.1 Multipatch NURBS model

Figure 4 shows a multi-patch NURBS model of a Cook-type cantilever steel beam under concentrated load P=+100 kN at the middle of its span, designed in Geomiso TNL. This plane stress problem can be handled by assuming either two parent patches with the same parametrization Ξ=H={0,0,0,0.5,1,1,1} or one single parent patch consisted of two subsidiary patches with Ξ={0,0,0,0.25,0.5,0.5,0.75,1,1,1} and H={0,0,0,0.5,1,1,1}. The two subsidiary patches match geometrically and parametrically on the interval face, where the continuity is C^0.



(a) Coarse NURBS model of 2 patches

(b) Coarse mesh (deformed configuration)

(c) Contour plot of d_Y (coarse mesh)

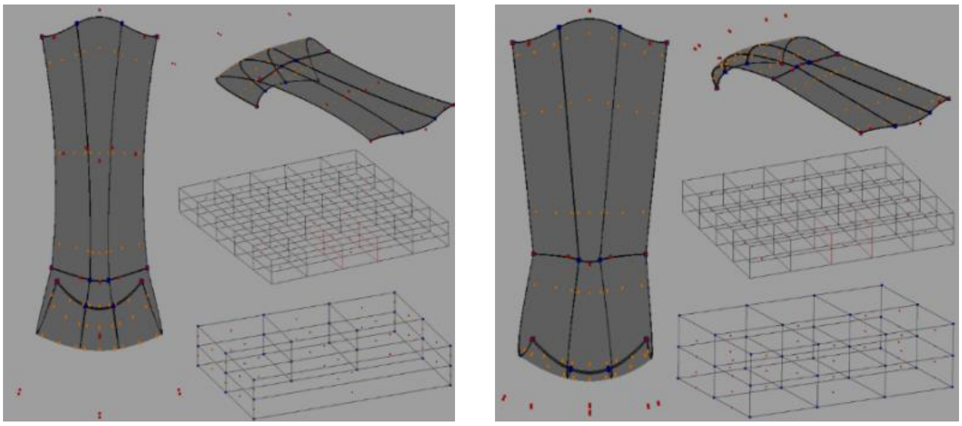
(d) Contour plot of d_Y (fine mesh)

Figure 4: Quadratic NURBS model of a Cook's cantilever beam (designed in Geomiso TNL). (a) Initial coarse mesh of 2 patches and 8 elements. Control points are shown in red, while knots in blue. (b) Deformed configuration of the coarse mesh. Gauss points are shown in orange. (c) Contour plot of displacement d_Y in the direction of the point load (coarse mesh, IGA results, Geomiso TNL). (d) Contour plot of d_Y (h-refined mesh of 2 patches and 18 elements, Ξ=H={0,0,0,0.5,0.8,1,1,1}, IGA results, Geomiso TNL).

European Union
European Regional Development Fund
HELLENIC REPUBLIC
MINISTRY OF ECONOMY & DEVELOPMENT
SPECIAL SECRETARY FOR ERDF & CF
MANAGING AUTHORITY OF EPAnEK
EPAnEK 2014-2020
OPERATIONAL PROGRAMME
COMPETITIVENESS ENTREPRENEURSHIP INNOVATION
ΕΣΠΑ 2014-2020
ανάπτυξη - εργασία - αλληλεγγύη
Partnership Agreement 2014 - 2020
Co-financed by Greece and the European Union

4.2 T-spline-based isogeometric analysis of a stadium roof

Figure 5 shows a part of a stadium roof consisted of three NURBS patches with the same parameterization merged into a T-mesh. Local refinement is applied at the interval faces of the first two patches and the third one.



(a) C^0 merged T-mesh (b) C^1 merged T-mesh

Figure 5: T-mesh (a) C^0 merged (8 elements) and (b) C^1 merged (12 elements) in index, parameter and Cartesian space. Control points are shown in red, knots in blue and Gauss points in orange.

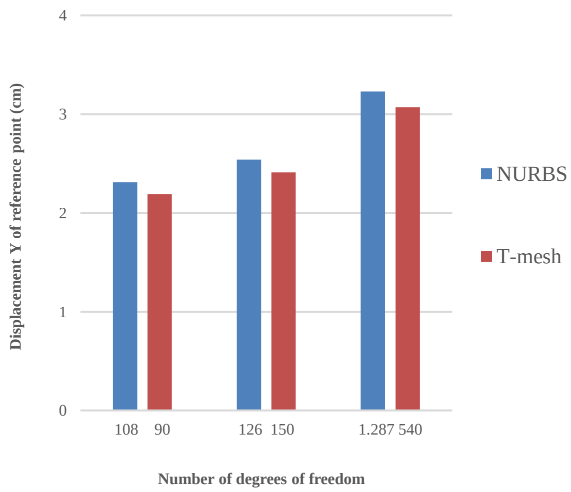


Figure 6: Comparison of NURBS and T-spline. The numerical results were obtained with Geomiso TNL. With local refinement, T-splines achieves the same accuracy of the denser NURBS mesh with much less degrees of freedom, proving the superiority of T-splines over NURBS.

Co-financed by Greece and the European Union

4.3 Rectangular steel plate under tensile load

A steel (E=200.000 MPa, ν=0,3, f_y=500 MPa) plate, with length 100 mm, rectangular cross–section 2x50 mm^2, its left edge fixed and its right edge under uniform tensile surface load, is presented in Figure 7.

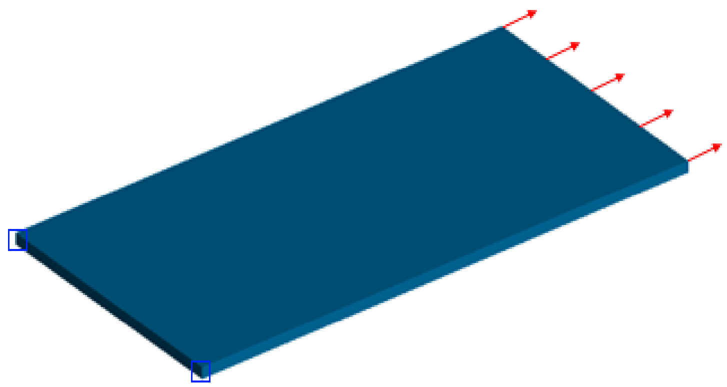


Figure 7: Steel plate under tensile load, designed in Geomiso TNL.

We compare the results obtained with Geomiso TNL, means the displacement d_X and the equivalent plastic strain of a reference point, with the FEA results and the analytical solution. The equivalent plastic strain for the elastic-perfectly plastic steel is described by the von Mises yield criterion for non-linear material. The reference point is at the middle of both width and thickness on the free edge. We use 24 meshes (6 linear, 6 quadratic, 6 cubic, 6 quadric - NxMxL: 1x1x1, 2x1x1, 4x2x1, 8x4x1, 16x8x1, 32x16x1) for this non-linear isogeometric analysis of the steel plate with Geomiso TNL.

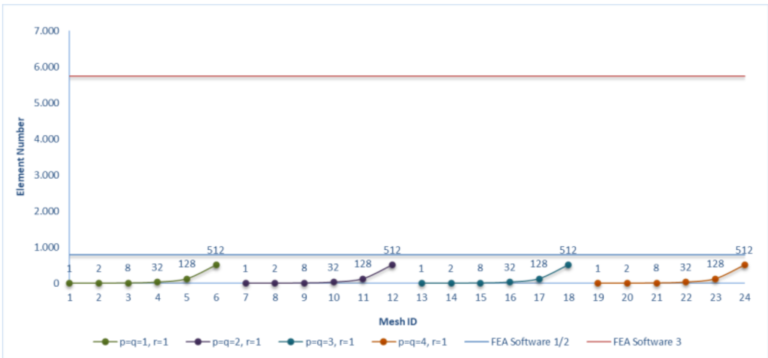


Figure 8: Geomiso TNL achieves accurate results with significantly fewer elements in comparison with commercial FEA software packages. 24 meshes are used in this non-linear isogeometric analysis of the steel plate with Geomiso TNL and are compared with three FEA meshes (one for each of the three commercial FEA software packages) and the analytical solution.

Co-financed by Greece and the European Union

The surface load – displacement response is depicted in Figure 9, while the surface load – equivalent von Mises plastic strain curve in Figure 10.

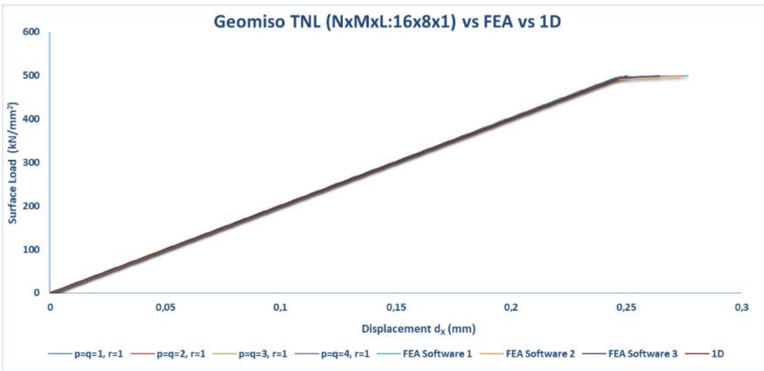


Figure 9: Displacement d_x of the reference point is plotted versus the surface load. Geomiso TNL vs. FEA and exact solution (NxMxL: 16x8x1, pxqxr: 1x1x1, 2x2x1, 3x3x1, 4x4x1).

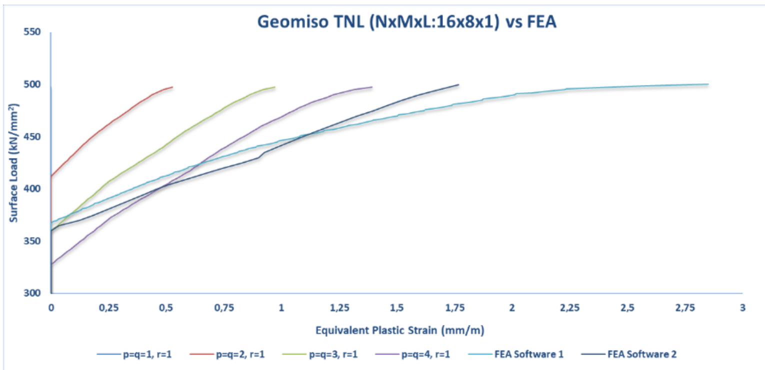
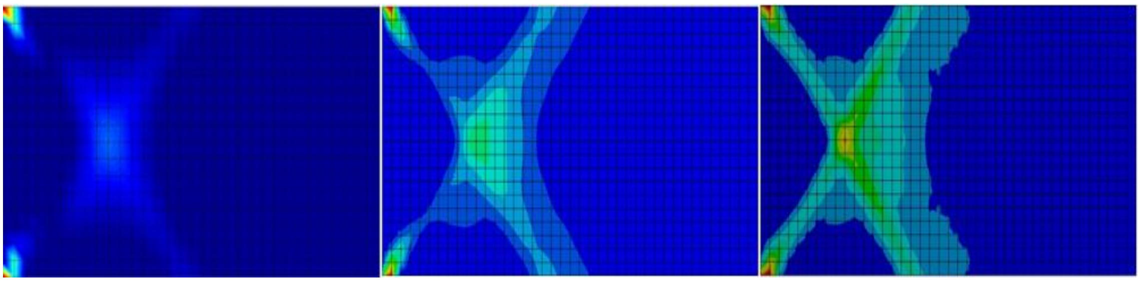


Figure 10: Equivalent plastic strain of the reference point is plotted versus the surface load. Geomiso TNL vs. FEA (NxMxL: 16x8x1, pxqxr: 1x1x1, 2x2x1, 3x3x1, 4x4x1).



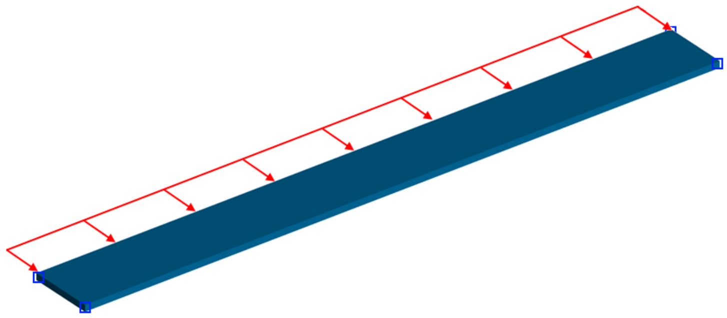
(a) Geomiso TNL (b) FEA software 1 (c) FEA software 2

Figure 11: Contour plots of equivalent plastic strains. Geomiso TNL vs. FEA software.

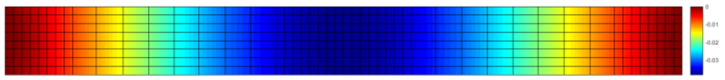
Co-financed by Greece and the European Union

4.4 Rectangular steel beam under pure bending

A steel (E=200.000 MPa, v=0,3, f_y=500 MPa) beam, with length 5.000 mm and rectangular cross-section 5x500 mm^2, under pure bending, is presented in Figure 12. Both edges are fixed, while on top of it, a uniform surface load is applied along thickness. Plastic hinges appear on both edges and its mid-point.



(a)



(b)

Figure 12: Steel beam under pure bending, designed in Geomiso TNL.
(a) Geometry designed in Geomiso TNL. (b) Contour plot of displacements d_Y.

We compare the results from Geomiso TNL, means displacement d_Y and equivalent plastic strain of two reference points (edges, mid-point), with FEA results and analytical solution. We use 17 meshes (4 linear, 5 quadratic, 4 cubic, 4 quadric - NxMxL: 1x1x1, 6x1x1, 12x2x1, 24x4x1, 48x8x1) for this non-linear isogeometric analysis with Geomiso TNL.

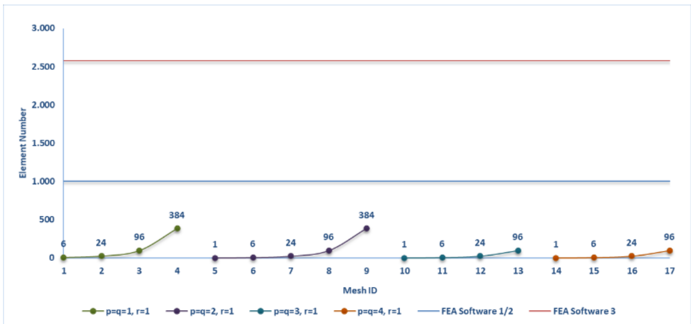


Figure 13: Geomiso TNL achieves accurate results with significantly fewer elements in comparison with FEA software. 17 meshes are used in this non-linear isogeometric analysis with Geomiso TNL and are compared with three FEA meshes (one for each of the three commercial FEA software packages) and the analytical solution.

Co-financed by Greece and the European Union

The surface load – displacement d_Y response is depicted in Figure 14, while the surface load – equivalent plastic strain curve in Figure 15 (edges) and Figure 16 (mid-point) respectively.

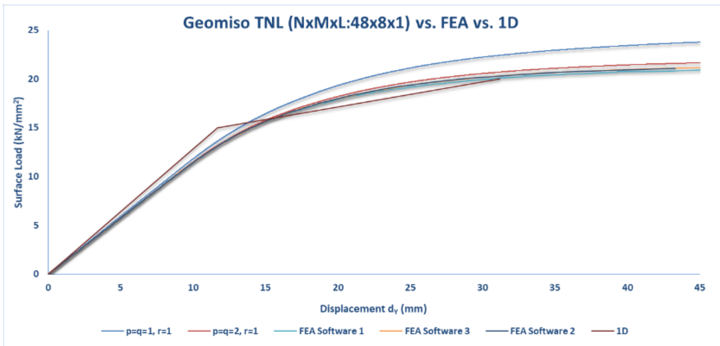


Figure 14: Displacement d_Y of the reference point (mid-point) is plotted versus the surface load. Geomiso TNL vs. FEA and exact solution (NxMxL: 48x8x1, pxqxr: 1x1x1, 2x2x1).

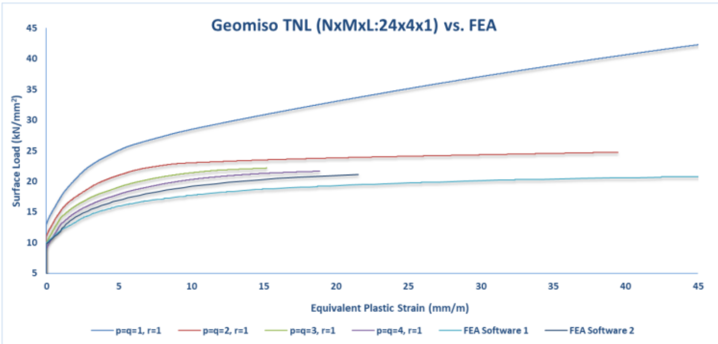


Figure 15: Equivalent plastic strain of the reference point (edges) is plotted versus the surface load. Geomiso TNL vs. FEA. (NxMxL: 24x4x1, pxqxr: 1x1x1, 2x2x1, 3x3x1, 4x4x1).

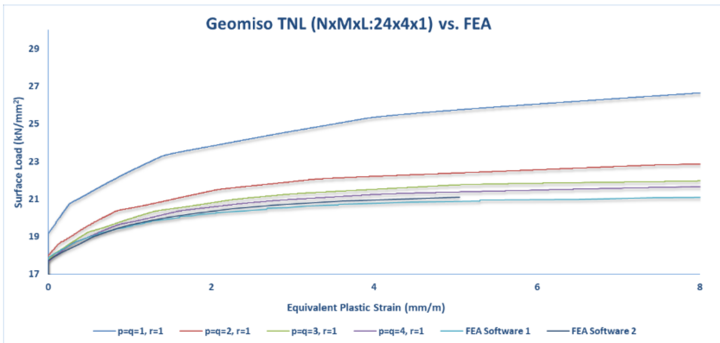


Figure 16: Equivalent plastic strain of the reference point (mid-point) is plotted versus the surface load. Geomiso TNL vs. FEA (NxMxL:24x4x1, pxqxr: 1x1x1, 2x2x1, 3x3x1, 4x4x1).

Co-financed by Greece and the European Union

5 CONCLUSIONS

The main conclusion to be drawn in this paper is that the new software Geomiso TNL is a more efficient alternative to commercial finite element software packages, as it facilitates the geometry modeling within analysis, preserves geometry at all refinement levels ensuring that its detailed features can be retained without excessive mesh refinement, eliminates geometric errors and offers superior quality numerical results. This is the first time such a hybrid software has been developed. Applications to linear and non-linear plane strain, plane stress and 3D problems in solid mechanics have proved successful. Moreover, a number of single and multiple patch meshes have been designed and analyzed. The accuracy of the displacement, strain and stress fields is particularly noteworthy. Superior accuracy of isogeometric analysis over finite element analysis has been shown in all applications, and indications of significantly increased robustness and stability level in analysis have been noted. It is demonstrated that Geomiso TNL represents improvements over traditional finite element programs, as higher accuracy is accomplished with considerably reduced computational cost.

ACKNOWLEDGEMENTS

This research has been co-financed by the European Union and Greek national funds through the Operational Program Competitiveness, Entrepreneurship and Innovation, under the call RESEARCH – CREATE – INNOVATE (project code: T1EDK-04288).

REFERENCES

[1] Hughes, T.J.R., Cottrell, J.A. and Bazilevs, Y. *Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement*. Comput. Methods Appl. Mech. Engrg. (2005) 194:4135–4195.

[2] Karatarakis, A., Karakitsios, P. and Papadrakakis, M. *GPU accelerated computation of the isogeometric analysis stiffness matrix*. Comput. Methods Appl. Mech. Engrg. (2014) 269:334-355.

[3] Bazilevs, Y., Calo, V.M., Cottrell, J.A., Evans, J.A, Hughes, T.J.R., Lipton, S., Scott, M.A. and Sederberg, T.W. *Isogeometric analysis using T-splines*. Comput. Methods Appl. Mech. Engrg (2010) 199:229-263.

[4] Elguedj, T., Bazilevs, Y., Calo, V.M. and Hughes, T.J.R. *$\bar{B}$ and $\bar{F}$ projection methods for nearly incompressible linear and non-linear elasticity and plasticity using higher-order NURBS elements.* Comput. Methods Appl. Mech. Engrg. (2008) 197:2732-2762.