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GLOBAL SENSITIVITY ANALYSIS OF SOUND TRANSMISSION LOSS OF DOUBLE WALL WITH POROUS LAYERS

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Abstract. This study examines the acoustic performance of a double-wall system with a porous layer and conducts a global sensitivity analysis of sound transmission loss. The authors use the transfer matrix method to predict sound transmission, which provides cost-effective modeling of complex acoustic interactions and detailed high-frequency information. The method employs transfer matrices to represent sound wave propagation in each layer, considers material characteristics and layer thickness, and incorporates interface matrices for boundary conditions. The poroelastic layer is modeled using the Biot-Allard approach with nine parameters. Morris and Sobol methods are applied for global sensitivity analysis, identifying significant parameters. The investigation focuses on eleven parameters, including foam properties and layer thicknesses. The findings indicate the impact of geometric parameters at lower frequencies and foam properties at higher frequencies. This study is the first to optimize sound transmission in double-wall systems with porous layers using sensitivity analysis methods, offering insights for system behavior and design.

1 INTRODUCTION

Double-wall structures with porous layers are widely employed in various industries due to their good acoustic and thermal insulation capabilities. Different approaches are utilized to characterize the sound transmission loss of multilayer systems depending on the frequency range of interest. At low frequencies, the finite element method is commonly used to study the system's response [1, 2, 3, 4], while for medium and high frequencies, more efficient methods such as the Transfer Matrix Method (TMM) [5] and Statistical Energy Method (SEA)[6] are employed. The SEA models the structure as a set of subsystems coupled through energy transfer pathways, while the TMM analyzes the transmission of sound waves through multilayer systems using transfer matrices [5,7,8].

Sensitivity analysis plays a crucial role in assessing model robustness and optimizing performance. Two types of sensitivity analysis, local and global, are commonly used. Local sensitivity analysis examines small perturbations around a nominal value, while global sensitivity [9,10] analysis explores the effect of each input parameter across its entire range of variability. For global sensitivity, qualitative methods classify input parameters in order of relevance as the Morris method [11], while quantitative methods provide exact proportions of output variation attributed to each parameter as Sobol indices [12].

Although sensitivity analysis has been applied to various models, few studies have exclusively focused on porous material models . Some publications [13] have employed quantitative methods like Sobol indices and FAST to quantify the sensitivity of porous models to parameter variations. Flow resistivity has been identified as a crucial parameter affecting acoustic performance. Other parameters can also significantly impact the vibroacoustic behavior, depending on the frequency range.

This work aims to predict sound transmission in a double-wall structure with poroelastic layers and investigate the sensitivity of acoustic indicators to uncertainties in geometric, mechanical, and acoustic parameters. The transfer matrix method (TMM) is utilized to model the system's vibroacoustic response to diffuse field excitation. Global sensitivity analysis techniques, including the Morris method and Sobol indices, are employed to identify influential parameters. A case study is presented, starting with the Morris method to eliminate non-influential parameters, and subsequently utilizing Sobol indices to rank uncertain parameters by importance. All uncertain variables are assumed to follow a uniform distribution. To the best of our knowledge, this work is the first to apply the Morris method and Sobol indices to optimize sound transmission in a double-wall system with porous layers using sensitivity analysis.

2 TMM FOR DOUBLE WALL WITH A POROUS LAYER

2.1 General formulation

The Transfer Matrix Method (TMM) is a efficient and simple technique used to predict the acoustic behavior of laterally infinite multilayered systems. It employs transfer matrices to represent plane wave propagation in different layers and interface matrices to account for boundary conditions. In this study, we apply the TMM to forecast the sound transmission loss of a double-wall structure consisting of plate, fluid, poroelastic, and plate layers (Figure 1). The system is subjected to an incident plane acoustic wave of angle θ, and is surrounded by two external fluids (Fluids 1 and 2). The problem under consideration is two-dimensional, with the incidence plane denoted as (x_1, x_3).

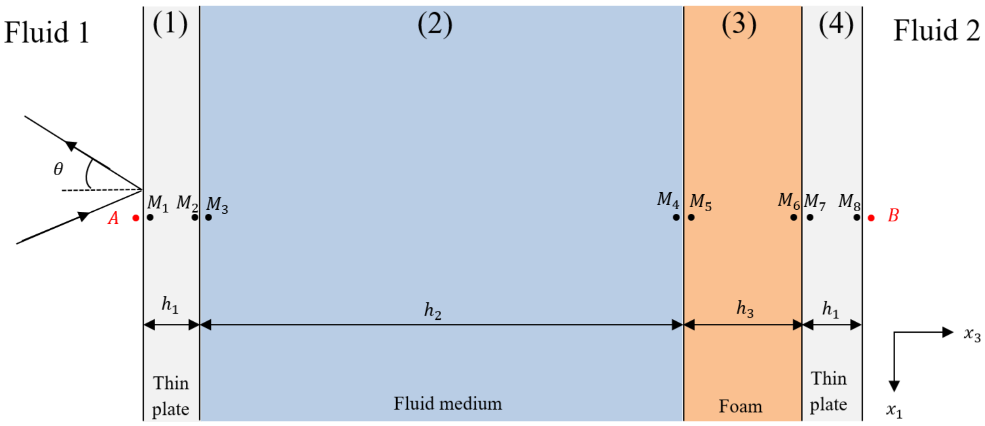


Figure 1: A double-wall with a porous layer subjected to an incident plane wave

Each layer in the system is represented by a transfer matrix $[T]$ that describes the propagation of sound waves within that layer [5]:

$$\vec{V}(M_n) = [T]\vec{V}(M_{n+1}) \tag{1}$$

The transfer matrix $[T]$ depends on both the thickness h_i of each layer i with $i = 1,2,3,4$ and the physical characteristics of the layer. It describes the sound wave propagation between two points, M_n and M_{n+1} with $n = 1,2,3,4,5,6,7$ located on the front and back faces of the layer, respectively. The vector $\vec{V}$ contains the variables that describe the acoustic field at a given point M within the layer.

2.2 Construction of the transfer matrix for each layer

2.2.1. Thin elastic plate

In the case of a thin elastic plate, only bending waves are considered for wave propagation. The harmonic equation (Equation (2)) of motion describes the relationship between the normal stress σ_{33}^s and normal velocity v_3^s of the plate at two points, M_1 and M_2. Equation (3) describes the continuity of velocities.

$$Z^s(\omega)v_3^s(M_2) = \sigma_{33}^s(M_2) - \sigma_{33}^s(M_1) \tag{2}$$

$$v_3^s(M_2) = v_3^s(M_1) \tag{3}$$

Equations (2) and (3) can be written in matrix form using the transfer matrix $[T_s]$ and the acoustic field vector $\overrightarrow{V^s}$.

$$\begin{pmatrix} \sigma_{33}^s(M_1) \\ v_3^s(M_1) \end{pmatrix} = \begin{bmatrix} 1 & -Z^s(\omega) \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \sigma_{33}^s(M_2) \\ v_3^s(M_2) \end{pmatrix} = [T_s]\overrightarrow{V^s}(M_2) \tag{4}$$

The mechanical impedance $Z^s(\omega)$ captures the characteristics of the plate, incorporating factors such as bending stiffness D, mass density per unit area m, wave number k, and incidence angle θ. Damping can be introduced through a complex Young's modulus.

$$Z^s(\omega) = j\omega m\left(1 - \frac{D(k\sin\theta)^4}{\omega^2 m}\right) \tag{5}$$

2.2.2. Fluid layer

The sound pressure p^f and velocity v_3^f in the fluid layer is considered as a superposition of a progressive and a regressive wave in x_3 direction:

$$p^f(x_3,t) = \left[A_0 e^{ik_3 x_3} + B_0 e^{-ik_3 x_3}\right] e^{-i\omega t} \tag{6}$$

$$v_3^f(x_3,t) = \frac{1}{Z^f}\left[A_0 e^{ik_3 x_3} - B_0 e^{-ik_3 x_3}\right] e^{-i\omega t} \tag{7}$$

where $k_3 = \frac{\omega}{c^f}$ is the component along x_3 direction of the wave number vector, c^f the speed of sound in the considered medium. The amplitude of the progressive and the regressive wave are denoted respectively A_0 and B_0. $Z^f = \rho_f c^f$ is the acoustic impedance depending on the density ρ_f and the celerity c^f of the fluid.

Consider that x_3 is equal to zero on the right boundary of the fluid layer, the pressure $p^f(0)$ and the velocity $v_3^f(0)$ at this boundary can be written:

$$p^f(0) = A_0 + B_0\ ;\ v_3^f(0) = \frac{1}{Z^f}[A_0 - B_0] \tag{8}$$

Using these last Equations (8) combined with Equations (6) and (7), the pressure $p^f(-h_2)$ and velocity $v_3^f(-h_2)$ on the left-hand side of the layer, where $x_3 = -h_2$, can be deduced from $p^f(0)$ and $v_3^f(0)$ in the following matrix form:

$$\begin{pmatrix} p^f(-h_2) \\ v_3^f(-h_2) \end{pmatrix} = \begin{bmatrix} \cos(k_3 h_2) & iZ^f \sin(k_3 h_2) \\ i\frac{1}{Z^f}\sin(k_3 h_2) & \cos(k_3 h_2) \end{bmatrix} \begin{pmatrix} p^f(0) \\ v_3^f(0) \end{pmatrix} = \left[T_f\right]\overrightarrow{V^f}(0) \tag{9}$$

where $\left[T_f\right]$ is the transfer matrix of the fluid layer, $\overrightarrow{V^f}$ is the associated acoustic fields vector and h_2 is the thickness of the fluid layer.

2.2.3. Porelastic layer

The propagation of acoustic waves in a poroelastic layer involves two compression waves and a shear wave for each phase. In total, there are six waves denoted as ϕ_1^S, ϕ_2^S, ψ_2^S ϕ_1^F, ϕ_2^F and ψ_2^F, which are defined by six independent acoustic quantities. These quantities include the velocity components of the solid phase v_1^S and v_3^S, the velocity component of the fluid phase v_3^F, the stress tensor components of the solid phase σ_{33}^S and σ_{13}^S, and the stress component of the fluid phase σ_{33}^F. To simplify the representation, a state vector $\overrightarrow{V^p}$ is introduced, defined at any point M with an abscissa of x_3 in the poroelastic layer.

$$\overrightarrow{V^p}(x_3) = \left[v_1^S\ v_3^S\ v_3^F\ \sigma_{33}^S\ \sigma_{13}^S\ \sigma_{33}^F\ \right]^T \tag{10}$$

By analyzing the material, we can determine a matrix $[\Gamma]$ that represents its characteristics. This matrix establishes a connection between the values of velocities and stresses at $x_3 = 0$ (one end) and $x_3 = h_3$ (opposite end) of the poroelastic layer, where h_3 is the thickness of the layer.

$$\overrightarrow{V^p}(0) = [\Gamma(0)][\Gamma(h_3)]^{-1}\overrightarrow{V^p}(h) \tag{11}$$

$$\left[T_p\right] = [\Gamma(0)][\Gamma(h_3)]^{-1} \tag{12}$$

The transfer matrix $\left[T_p\right]$ characterizes the poroelastic layer and its components are provided in reference [5]. In practical applications, to simplify calculations and avoid matrix inversion, the origin of the x_3 axis can be shifted, allowing the use of the transfer matrix $\left[T_p\right] = [\Gamma(-h_3)][\Gamma(0)]^{-1}$.

2.3 Coupling conditions

2.3.1. Thin plate-fluid interface

At the interface between the elastic plate and the acoustic fluid, the continuity equations express the following conditions:

$$v_3^s(M_2) = v_3^f(M_3) \quad (13)$$

$$\sigma_{33}^s(M_2) = -p^f(M_3) \quad (14)$$

Equations (13) and (14) can be written in matrix form to represent the continuity of normal velocity and stress at the fluid-structure interface as follows:

$$[I_{sf}]\overrightarrow{V^s}(M_2) + [J_{sf}]\overrightarrow{V^f}(M_3) = \vec{0} \quad (15)$$

$$[I_{sf}] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } [J_{sf}] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad (16)$$

2.3.2. Fluid-porous interface

At the interface between the acoustic fluid and the porous medium, the continuity equations can be described as follows:

$$v_3^f(M_4) = (1-\phi)v_3^S(M_5) + \phi v_3^F(M_5) \quad (17)$$

$$-(1-\phi)p^f(M_4) = \sigma_{33}^S(M_5) \quad (18)$$

$$0 = \sigma_{13}^S(M_5) \quad (19)$$

$$-\phi p^f(M_4) = \sigma_{33}^F(M_5) \quad (20)$$

Where ϕ is the porosity of the porous medium. Equation (17) introduces the continuity of normal velocities between the porous medium and the acoustic fluid. Equation (18) establishes the continuity between the normal stress of the solid phase in the porous medium and the pressure of the acoustic fluid. Equation (19) assumes no shear in the solid phase of the porous material. Equation (20) presents the continuity between the normal stress in the fluid phase of the porous medium and the pressure in the fluid at the interface. These equations can be written in matrix form to represent the continuity conditions between the acoustic fluid and the porous medium.

$$[I_{fp}]\,\overrightarrow{V^f}(M_4) + [J_{fp}]\overrightarrow{V^p}(M_5) = \vec{0} \quad (21)$$

$$[I_{fp}] = \begin{bmatrix} 0 & -1 \\ (1-\phi) & 0 \\ 0 & 0 \\ \phi & 0 \end{bmatrix} \text{ and } [J_{fp}] = \begin{bmatrix} 0 & (1-\phi) & \phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (22)$$

2.3.3. Porous-thin plate interface

For a thin plate in contact with a porous layer, the continuity equations at the interface can be expressed as follows:

$$v_3^S(M_6) = v_3^s(M_7) \quad (23)$$

$$v_3^F(M_6) = v_3^s(M_7) \quad (24)$$

$$\sigma_{33}^S(M_6) + \sigma_{33}^F(M_6) = \sigma_{33}^s(M_7) \quad (25)$$

$$\sigma_{13}^S(M_6) = 0 \quad (26)$$

Equations (23) and (24) represent the continuity of the normal velocity at the porous-plate interface. Equation (25) represent the continuity of the stresses. Finally, Equation (26) reflects absence of shear at the interface. These equations can be written in matrix form as follows:

$$[I_{ps}]\overrightarrow{V^{p}}(M_6) + [J_{ps}]\overrightarrow{V^{s}}(M_7) = \vec{0} \tag{27}$$

$$[I_{ps}] = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \text{ and } [J_{ps}] = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{28}$$

2.4 Computation of the transmission loss *TL* of the multilayer

The application of the TMM leads to the following matrix problem to solve:

$$[D]\vec{V} = \vec{0} \tag{29}$$

Where T is the transmission coefficient, R the reflection coefficient and $[D]$ is the assembled global transfer matrix given by:

$$[D] = \begin{bmatrix} [T\ 0] & [0] & [0] & [0] & [0] & [-(1+R)\ 0] \\ [I_{fs}] & [J_{fs}][T_s] & [0] & [0] & [0] & [0] \\ [0] & [I_{sf}] & [J_{sf}][T_f] & [0] & [0] & [0] \\ [0] & [0] & [I_{fp}] & [J_{fp}][T_p] & [0] & [0] \\ [0] & [0] & [0] & [I_{ps}] & [J_{ps}][T_s] & 0 \\ [0] & [0] & [0] & [0] & [I_{sf}] & [J_{sf}] \end{bmatrix} \tag{30}$$

and

$$\vec{V} = \left[\overrightarrow{v^{f}}(A)\ \ \overrightarrow{v^{s}}(M_2)\ \ \overrightarrow{v^{f}}(M_4)\ \ \overrightarrow{v^{p}}(M_6)\ \ \overrightarrow{v^{s}}(M_8)\ \ \overrightarrow{v^{f}}(B)\right]^T \tag{31}$$

Finally, the sound transmission loss is computed using the following equation :

$$TL = -10\log\left[\frac{\int_{\theta_{min}}^{\theta_{max}}\int_0^{2\pi}\tau(\theta,\varphi)\sin\theta\cos\theta\, d\varphi d\theta}{\int_{\theta_{min}}^{\theta_{max}}\int_0^{2\pi}\sin\theta\cos\theta\, d\varphi d\theta}\right] \tag{32}$$

with $\tau(\theta,\varphi)$, calculated from TMM, represents the transmission coefficient of the multilayer panel for a given angle of incidence θ varying from θ_{min} to θ_{max}.

3 GLOBAL SENSITIVITY ANALYSIS

3.1 Morris method

The Morris method uses the mean and standard deviation of elementary effects to measure sensitivity. It categorizes input factors into three groups based on their effects: negligible effects (low average and low standard deviation), linear effects with no interaction high average and low standard deviation), and non-linear effects with interactions (high standard deviation). In a computational model, the output variable Y is determined by a deterministic function of M inputs represented by the vector $\vec{X}$. Sampling starts with a random point and each subsequent sample varies only one factor at a time. Elementary effects are calculated by measuring the variation in the output caused by changing one parameter while keeping others constant. Morris introduced this method for sensitivity analysis [14].

$$E_j^{(i)} = \frac{Y\left(X_1, \dots, X_j \pm \Delta, \dots, X_M\right) - Y\left(X_1, \dots, X_j, \dots, X_M\right)}{\Delta_i} \; ; i = 1, \dots, r \tag{33}$$

where the pre-defined value Δ_i is the trajectory step. Each parameter X_j is associated with one elementary effect yielded by one trajectory, and a set of r trajectories create the finite distribution of these elementary effects. The total number of simulations needed is equal to $r \times (M + 1)$. Two sensitivity indices can be estimated for each input factor, the mean of the absolute values of the effects μ_j^* and the standard deviation of the effects σ_j:

$$\mu_j^* = \frac{1}{r}\sum_{i=1}^{r}\left|E_j^{*(i)}\right| \tag{34}$$

$$\sigma_j = \sqrt{\frac{1}{r-1}\sum_{i=1}^{r}\left(\left|E_j^{*(i)}\right| - \mu_j^*\right)^2} \tag{35}$$

where $E_j^{*(i)}$ is the normalized elementary effect of $E_j^{(i)}$ associated to the parameter X_j . Since certain elementary effects might eliminate each other in non-monotonic models, Campolongo et al. [15] recommend using the average of absolute elementary effects rather than the standard average. The technical purpose of this approach, as highlighted by Campolongo et al. [15], is to identify parameters that are deemed "unimportant" or negligible.

3.2 Sobol indices

3.2.1. Decomposition of Sobol

The Sobol' decomposition method, introduced by I. M. Sobol [10], is widely used to analyze the impact of input variables on the system's output. This method quantitatively assesses the relative importance of each input variable by measuring its contribution to the output variance. By fixing a variable X_i, and observing the decrease in the output variance, the method determines if X_i is a significant contributor to the variance, indicating its importance. The Sobol' decomposition dissolves a function f into its constituent parts based on the contributions of each input variable to the output variance [12].

$$Y = f(\vec{X}) = f(X_1, \dots, X_M) = f_0 + \sum_{i=1}^{M} f_i(X_i) + \sum_{1 \le i \le j \le M} f_{ij}(X_i, X_j) + \cdots + f_{1,2,\dots,M}(X_1, \dots, X_M) \tag{36}$$

Where the constant f_0 and conditional expectations f_i , $f_{ij}, \dots, f_{1,2,\dots,M}$ are expressed as follows:

$$f_0 = \mathbb{E}(Y) \tag{37}$$

$$f_i(X_i) = \mathbb{E}(Y|X_i) - f_0 \tag{38}$$

$$f_{ij}\left(X_i, X_j\right) = \mathbb{E}\left(Y|X_i, X_j\right) - f_0 - f_i - f_j \tag{39}$$

The total output variance $V(Y)$ can be written as:

$$V(Y) = V[\mathbb{E}(Y|X_i)] + V\left[\mathbb{E}\left(Y|X_i, X_j\right)\right] - V[\mathbb{E}(Y|X_i)] - V\left[\mathbb{E}\left(Y|X_j\right)\right] + \dots + V\left[f_{1,2,\dots,M}(X_1, \dots, X_M)\right] \quad (40)$$

3.2.2. Indices of Sobol

Indices of Sobol are defined in several categories: first-order, second order, ... M-order (M being the number of random variables) and total sensitivity indices. The first-order sensitivity index of Y to the parameter X_i, denoted S_i, considers only the contribution due to the parameter X_i alone without interactions and is defined as [14]:

$$S_i = \frac{V\left(\mathbb{E}(Y|X_i)\right)}{V(Y)} \quad (41)$$

The total sensitivity index S_{Ti} combines the contribution due to the parameter X_i alone, which corresponds to the index S_i, and the contribution due to the interaction of X_i with the other parameters:

$$S_{Ti} = 1 - \frac{V\left(\mathbb{E}(Y|X_{\sim i})\right)}{V(Y)} \quad (42)$$

The term $V\left(\mathbb{E}(Y|X_{\sim i})\right)$ represents the variance of the conditional expectation knowing all parameters except X_i.

4 CASE STUDY

4.1 Description of the uncertain parameters

The given double infinite wall system consists of two PVC walls separated by an air gap and surrounded by a fluid domain. To improve sound transmission reduction, a poroelastic layer (polymer foam) is introduced between the walls bonded to the second PVC wall. The system is subjected to a diffuse excitation field with a power spectral density of 1 Pa^2/Hz. The mechanical properties of the PVC are: Young's modulus $E_p = 2.8 \times 10^9$ Pa, Poisson ratio $\nu_p = 0.35$, density $\rho_p = 1460$ kg/m^3 and loss factor $\eta_p = 0.04$. The walls are separated by an air gap with its density $\rho_c = 1.213$ kg/m^3 and its sound velocity $c_c = 342.2$ m/s. The fluid domain surrounding the walls is a semi-infinite space filled with air. These parameters are fixed to their nominal values. Uncertainty in the mechanical and acoustic properties of the materials was not considered due to its negligible impact compared to other sources of uncertainty in the system. Table 1 presents uncertain parameters and their corresponding bounds. The study focuses on 11 parameters, including 9 for the polymer foam's material properties and their uncertainty ranges [13]. Only the thickness variation of plates and acoustic fluid is considered.

Table 1. Uncertain parameters for the double-wall with a porous layer

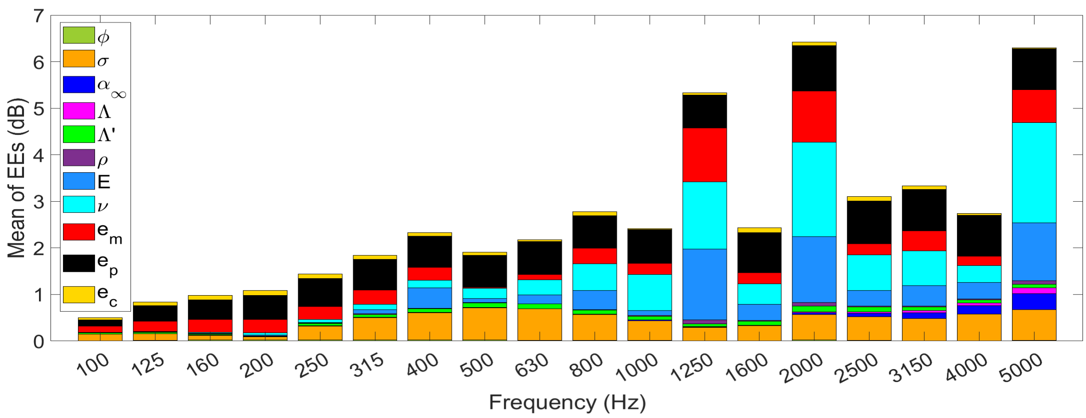
Domain	X_i	Parameter	Variable	Nominal value	Unit	Lower bound	Upper bound
Porous	X_1	Porosity	ϕ	0.97	-	0.96	0.98
	X_2	Flow resistivity	σ	166 000	N.s/m^4	119 000	212 000
	X_3	Tortuosity	α_∞	1.8	-	1.5	2.1
	X_4	Viscous length	Λ	6×10^{-5}	m	4.1×10^{-5}	7.9×10^{-5}

	X_5	Thermal length	Λ'	1.8×10^{-4}	m	1.37×10^{-4}	2.23×10^{-4}
	X_6	Density	ρ	39.5	kg/m^3	39,1	39.9
	X_7	Young modulus	E	205500	Pa	170000	241000
	X_8	Poisson ratio	ν	0.45	-	0.435	0.465
	X_9	Thickness of the foam	e_m	0.03	m	0.028	0.033
Plates	X_{10}	Thickness of the plates	e_p	0.002	m	0.0018	0.0022
Fluid	X_{11}	Thickness of fluid medium	e_c	0.20	m	0.19	0.22

4.2 Global sensitivity analysis using Morris method

The Morris method was used to conduct a sensitivity analysis of the 11 uncertain parameters in the double wall structure, assuming they followed uniform distributions. This involved conducting 600 model simulations with 50 trajectories. The analysis results are shown in Figure 2, presenting the average and standard deviation of the elementary effects for each parameter.

(a)



(b)

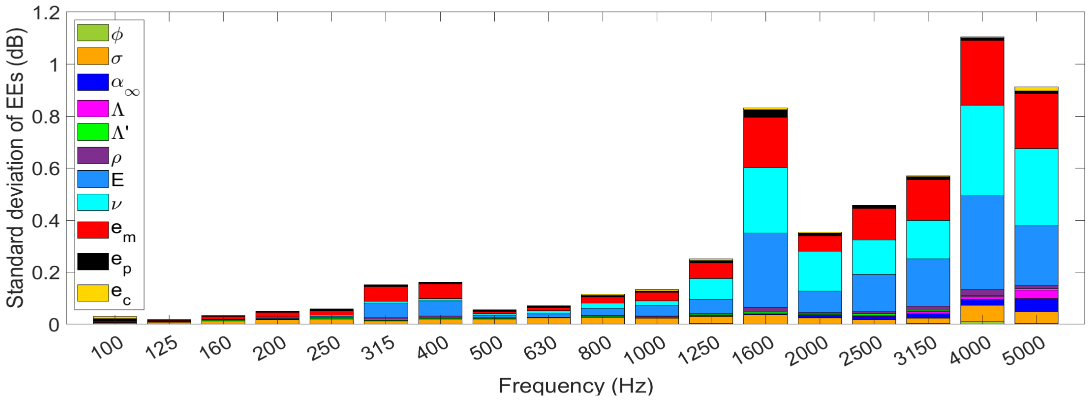


Figure 2: Elementary Effects of the double-wall with a porous layer: (a) the mean and (b) the standard deviation

Fig.2.a shows that in the lower frequency range (100 Hz to 315 Hz), the sound transmission loss response is highly sensitive to the system's geometrical parameters: plate thickness, foam thickness, and cavity thickness. The flow resistivity of the foam is also influential in this range. As the frequency exceeds 315 Hz, the foam's mechanical properties, including Young's modulus and Poisson's ratio, become more important. The foam's tortuosity only affects higher frequencies (4000 Hz to 5000 Hz). Fig.2.b reveals that the foam thickness, Young's modulus, and Poisson's ratio of the foam exhibit the highest standard deviation values. These parameters are sensitive to non-linear effects and/or interactions, indicating that small variations in them can significantly impact the sound transmission loss response of the double wall structure. Reducing the uncertainty associated with these parameters is crucial. We have been able to eliminate four least influence parameters as porosity, foam density, viscous length, and thermal length.

4.3 Global sensitivity analysis using Sobol indices

Using the Sobol method, a sensitivity analysis was conducted on the 7 most important parameters identified by the Morris method. Figure 3 displays the first order S_i and total S_{Ti} Sobol indices for these parameters across a frequency range of 100 Hz to 5000 Hz per third band. A sample size of $N = 10^4$ was used, resulting in a computational cost equivalent to 90000 model simulations ($(M + 2) \times N$) for each frequency.

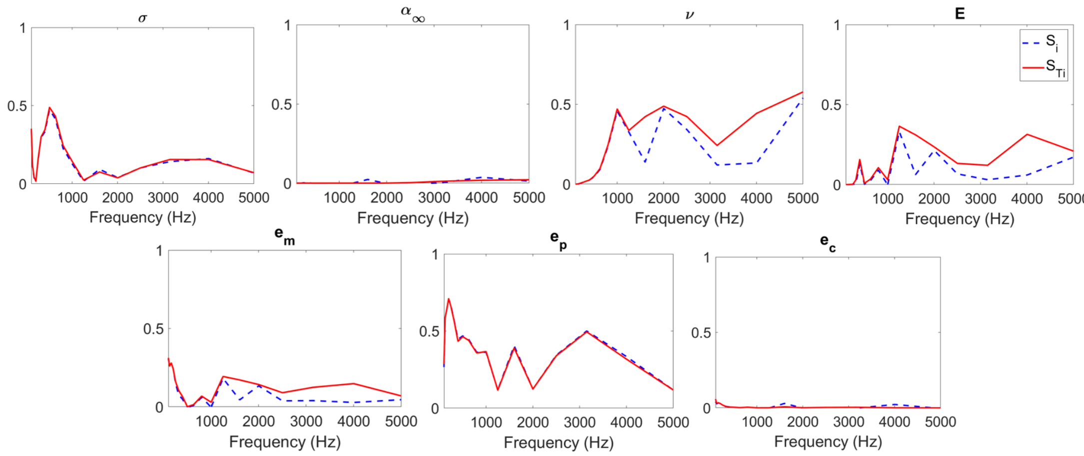


Figure 3: Sobol indices of the seven most influential parameters

The study exposes that tortuosity and cavity thickness had negligible impact on sound transmission loss. Instead, in the lower frequency range (100 Hz to 315 Hz), plate thickness, foam thickness, and resistivity were the most influential parameters. Between 315 Hz and 800 Hz, plate thickness became the most influential, followed by resistivity, while foam thickness became less important. At higher frequencies (800 Hz to 5000 Hz), Poisson's ratio and Young's modulus were the primary contributors. These parameters were sensitive to interactions, particularly the coupling effects between the plate and porous material. Foam thickness also had some influence due to interactions in this frequency range. Plate thickness remained significant at high frequencies but to a lesser extent. Resistivity, tortuosity, plate thickness, and cavity thickness were found to be insensitive to interactions.

5 CONCLUSION

This study examined the acoustic performance of a double wall system with a poroelastic layer to reduce sound transmission. The matrix transfer method was used to calculate sound transmission loss of the multilayer. Morris and Sobol methods were employed for global sensitivity analysis to identify influential parameters. The study focused on 11 parameters, including porosity, resistivity, tortuosity, viscous length, thermal length, density, Young's modulus, Poisson's ratio of the foam, and plate, cavity, and foam thicknesses. Morris analysis indicated that geometric parameters like plate, foam, and cavity thickness were critical in the lower frequency range. In the higher frequency range, foam properties such as Young's modulus and Poisson's ratio became more significant. Sobol analysis further explored parameter interactions and highlighted the interaction between plate thickness and foam at higher frequencies. It is important to note that parameter hierarchy varies with frequency. This study underscores the significance of considering uncertain parameters and utilizing sensitivity analysis to optimize the design of double wall systems with poroelastic layers for noise reduction in diverse applications.

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