Seismic Performance of Buildings with a Class of Adaptive Nonlinear Hybrid Systems

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Abstract. A class of hybrid control system which combines a passive nonlinear base isolator with an active controller is analyzed. The passive component of the system has a hysteretic behaviour. A single control force is applied on the structural base by means of a new adaptive control strategy, not requiring knowledge of the system parameters and the excitation and able to handle the typical nonlinearities associated with base isolators. A numerical study is performed to assess the improvement of the behaviour of buildings equipped with this control system.

Key words: Passive control, Active control, Hybrid system, Adaptive control, Base isolation, Hysteretic isolator

1 Introduction

The combination of passive and active systems, resulting in the so-called hybrid systems, has been increasingly considered in recent years for vibration control of civil engineering structures. One of the hybrid systems with most interesting perspectives for buildings consists of the combination of base isolators with active systems applying feedback control forces on the base (Reinhorn, 1987). The base isolation can reduce by itself both the inter-storey drift and the absolute accelerations of the structure. Thus the structure tends to behave like a rigid body, the price paid being a significant absolute displacement of the base which can occur for certain excitation frequency conditions (Skinner et al., 1993). The application of active control forces on the base may attempt to reduce this displacement. From a practical point of view, this hybrid scheme is appealing since it is possible to achieve the above mentioned attempt with a single force which, moreover, does not exceed some acceptable limits due to the high flexibility of the base isolators. From a theoretical point of view, the development of a control law to calculate the active force involves difficulties associated with the nonlinear behaviour of the base isolators and with the uncertainties in the models describing the structure–base isolator system and in the seismic excitation. A robust control law for uncertain linear base–isolated structures has been proposed by Kelly (Kelly et al., 1987) and recently by Schmitendorf (Schmitendorf et al., 1994). The nonlinearity of the iso-
lators has been considered in the work by Yang (Yang et al., 1992) assuming no uncertainties in the structure-base model. Some experimental works with small-scale hybrid systems have been recently reported in (Riley et al., 1992) and (Feng et al., 1993).

Rodellar and Ryan (Rodellar et al., 1993) have addressed the problem of stabilizing a class of uncertain nonlinear mechanical systems that can be decomposed into two coupled subsystems with feedback control on one of them. The control strategy is adaptive, which implies that it does not require a priori knowledge of either the system parameters or the external excitation. Within the framework of this class of system, Rodellar (Rodellar et al., 1994) have proposed a hybrid control system for building structures, in which the base is the subsystem with control and the structure the other one. In the last case, the control system has been presented and the stability has been proved. The main objective of this paper is to assess the effectiveness of this hybrid system.

In subsequent sections, the main issues of the objective and formulation of the control strategy are summarized, some aspects concerning its implementation are discussed and a numerical study is performed to evaluate the improvement of the seismic behaviour of buildings equipped with hysteretic active base isolators.

2 Adaptive control system

Consider the building structure with a hybrid control system illustrated in Figure 1. The passive component consists of a base isolator, while the active component applies control forces on the structural base.

When the parameters of the isolator are well tuned to the characteristics of the structure, good performance of the structure, with a reduced inter-storey drift, may be expected. Nevertheless, this desirable inter-storey behaviour can be associated with unacceptably high absolute base displacements. Therefore, the main purpose of the active control forces is to reduce these absolute base displacements. However, since the base and the stories are coupled, the application of forces at the base level can produce a negative effect, increasing the inter-storey drift as compared to that of the structure with purely passive control. Thus the active control law has to limit this effect as well. In this context, both passive and active elements are envisaged as cooperative systems: the active part is introduced to complement the base isolation by acting to the base absolute motion, thus producing a resistance scheme not attainable by purely passive means when the structure is subjected to seismic excitations (Haudivi and Kelly, 1990).

The objective of the active control strategy is essentially to drive the base absolute response asymptotically to an arbitrarily small prescribed neighborhood of the equilibrium position, while keeping the inter-storey drift within acceptable bounds. With the objective of being a complement for the base isolation system, we include the practical condition of setting on the active controller only when the above-mentioned base response is out of this neighborhood, which can be defined

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Fig. 1: Building structure with hybrid control

depending on the performance of the purely base isolation system.

The dynamic behaviour of the structure is described by means of a model composed of the building and the base. It is assumed that the structure behaves linearly due to the effect of the base isolation. The motion of the structure is described by displacements of the $n$ degrees of freedom with respect to an inertial frame, while the displacement of the structural base is described by a single degree of freedom with the above mentioned frame. The dynamic seismic ground motion, characterized by

A vector $D$ which represents the horizontal
displacement $d(t)$ and velocity $v(t)$. A single horizontal control force $u(t)$ acts on the structural base. Thus, the equations of motion are

$$
M \ddot{d} + C \dot{d} + KD = CJ \dot{d}_b + KJd_b
$$

$$
-\dot{J}^T C \dot{d} - J^T KD - c_b v - k_b d + f(d_b, \dot{d}_b, d, v) = u
$$

where $M$, $C$ and $K$ are the mass, damping and stiffness matrices of the structure, respectively. The vector $J$ expresses the rigid body motion according to the degrees of freedom of the model (in this case it is the unit vector). $m_b$, $c_b$ and $k_b$ are the mass, damping and stiffness of the base. The last two parameters correspond to the elastic and damping forces which appear on the base due to the linear effects of the isolator; $f$ is an additional horizontal force produced on the structural base by nonlinearity in the isolator.

The following assumptions complete the description of the system:

1. The matrices $M$, $C$ and $K$ are positive definite with $M$ and $K$ symmetric.

2. The displacement and the velocity of the seismic ground motion are bounded so that the following holds:

$$
|c_b v(t) + k_b d(t)| \leq \nu
$$

for almost all $t$, $\nu$ being an unknown scalar.

3. The function $f$ is such that, for some known continuous function $\gamma'$, the following holds for some (unknown) scalar $\alpha'$:

$$
|f(d_b, \dot{d}_b, d(t), v(t))| \leq \alpha' \gamma'(d_b, \dot{d}_b)
$$

for almost all $t$ and all $d_b$ and $\dot{d}_b$.

Fix $\lambda > 0$ and $\varepsilon > 0$ (design parameters). Define

$$
d_b(t) = d(t) + \dot{d}_b(t)
$$

and

$$
\gamma(d_b, \dot{d}_b, D, \dot{D}) = \left[ \lambda \gamma'(d_b, \dot{d}_b)^2 + \dot{d}_b^2 + \dot{d}_b^2 + \sum_{i=1}^{n} (D_i^2 + \dot{D}_i^2) \right]^{1/2}
$$

The control strategy is formulated as
\[ u(t) = -k(t) \left[ p_b(t) + \gamma \left( d_b(t), \dot{d}_b(t), D(t), \dot{D}(t) \right) s_\lambda(p_b(t)) \right] \]
\[ \dot{k}(t) = \bar{k} \left[ |p_b(t)| + \gamma \left( d_b(t), \dot{d}_b(t), D(t), \dot{D}(t) \right) \right] d_\lambda(p_b(t)) \]
\[ k(0) = k_0 \quad \text{(initial condition)} \]

In equations (6), \( s_\lambda, d_\lambda \) are functions defined as
\[ s_\lambda(p_b) = \begin{cases} |p_b|^{-1}p_b, & \text{if } |p_b| \geq \lambda \\ \lambda^{-1}p_b, & \text{if } |p_b| < \lambda \end{cases} \]
\[ d_\lambda(p_b) = \begin{cases} |p_b| - \lambda, & \text{if } |p_b| \geq \lambda \\ 0, & \text{if } |p_b| < \lambda \end{cases} \]

The rationale for this control strategy lies in its capability of assuring a form of practical stability. We omit here a detailed stability analysis, which can be found in (Rodellar et al., 1994). We only summarize the main result in terms of its physical interpretation.

By substituting the equations (6) into the equations (1), it can be considered that the global controlled system is characterized by the set of state variables \((d_b, \dot{d}_b, D, \dot{D}, k)\). In this case, a Lyapunov-type stability analysis can show that, for \( \lambda > 0 \) and for any initial condition, the following properties are satisfied:

1. \( \lim_{t \to \infty} k(t) \) exists and is finite, i.e., the monotone gain function does not grow unbounded.
2. The state of the base, characterized by coordinates \((d_b, \dot{d}_b)\), tends asymptotically to a ball of any prescribed radius \( \lambda \) centered at zero.
3. The state of the structure, characterized by the vectors \( D \) and \( \dot{D} \), tends asymptotically to a ball (centered at zero) with radius proportional to \( \lambda \), however the proportionality constant depends on unknown bounds on the system uncertainties and so cannot be calculated a priori.

It is important to emphasize the uncertainty of knowledge about the system that is required a priori. From the control law, it is apparent that parameters of the stiffness need not be known to the designer. Also the external seismic excitation is unknown, assuming only that it is bounded by unknown constant as in (2). Regarding the nonlinear force \( f \) produced by the isolators, the control strategy allows it to be unknown but bounded, modulo arbitrary scaling, by a known continuous function as in (3). This function enters into the control law in the definition of the function \( \gamma \) in (5). The adaptive nature
of the control law, associated with the time-varying gain $k(t)$, guarantees the above stability properties be assured for any realization of the unknown parameters satisfying assumptions 1–3.

For implementation of the control law, the absolute displacement and velocity responses of the base and the structure are required as feedback information. With this information, equations (6) and (7) are used to calculate the value of the control $u(t)$. The parameters (positive-valued) $\lambda$, $k_0$ and $k$ are open to choice by the designer. $\lambda$ is the most significant of these parameters, since it defines the guaranteed stability ball and has a primary influence in achieving the control objective.

3.1 MAIN OBJECTIVES

The purpose of this study is to evaluate the improvement in the behaviour of a base isolated building structure when an active control force is applied to its base. This force is calculated by the adaptive law defined in the previous section. A specially interesting aspect of the study of the excitation coincides with the fact that produces large displacements of the base and considerable amplifications of the structural response. This occurs when either bad design of the isolation system provides it with an unsuitable frequency or when the frequency characteristics of the expected earthquakes are not correctly predicted and an unanticipated earthquake occurs.

The three main components of a hybrid system whose influence on the global structural behaviour are to be considered here are: the structure, the base isolation and the adaptive control law.

The building structure. In order to study the behaviour of a wide range of building structure types, a frequency analysis is made by considering a single degree of freedom model, varying its stiffness and computing its maximum response. A ten storey building, modelled as a ten degrees of freedom shear building but with fixed characteristics is also considered and an analysis of its maximum response is made.

The base isolation system. This component of the hybrid control system is in this case hysteretic. Its effect on the seismic behaviour of the building is evaluated by calculating the structural response for different values of the parameters that define the hysteretic base isolator.

The adaptive control law [see equation (6)] that defines the state of the base asymptotically tends. A study is made of the effect of the variation of $\lambda$ on the controlled response of the system.
The dynamic response parameters considered to evaluate the behaviour of the system are:

- The absolute displacement of the isolated base.
- The relative displacement of the highest point of the structure relative to the base and the inter-storey drift ratio.
- The absolute acceleration of the structure.
- The control force applied to the base.

The reasons for selecting these response parameters are: a) The displacement of the base is an essential factor in evaluating both the base isolation device and the control law, as the main objective of the control strategy employed is precisely to control this displacement. This factor is also required for the design of the devices that connect the building to its foundation. b) The inter-storey drift conditions the stress produced in the columns and beams of the structure and provides a measure of the damage suffered. c) The absolute acceleration determines the comfort level of the equipment inside the building. d) The control force conditions the characteristics of the actuator to be used.

3.2 Description of the model

The building structure. As explained before, a ten degrees of freedom model and a single degree of freedom one are used to simulate the isolated building structure. In the first case, the mass of each of the ten levels of the building, as well as that of the base, is $6 \times 10^3$ kg. The stiffness of the columns varies by $5 \times 10^3$ N/m between levels from $9 \times 10^5$ N/m at the bottom level to $4.5 \times 10^8$ N/m at the top. The damping ratio is fixed as 0.05 for all the modes of vibration. For the case of the first level of the model with two degrees of freedom.

The base isolation system. For the purpose of assessing the seismic behaviour of building structures with the described adaptive hybrid system, the hysteretic isolator of Figure 2 is considered. For the force $f$ due to the nonlinear behaviour of the isolator, we need to specify the model describing the class of nonlinearities. One way of formulating a hysteretic models defined by means of differential equations. In this paper, by the way of example, Wen’s uniaxial model is adopted (Wen, 1976) and (Nagata, 1989). The hysteretic force $f$ is expressed by means of the following equation:

$$ f = f_y z $$

where $f_y$ is the yielding force, $z$ is an auxiliary variable defined by the differential equation.
and $x$ is the displacement of the base relative to the ground, i.e., $x = d_d - d$. The parameters $A$, $\nu_1$, $\nu_2$ and $m$ allow the class of materials, ranging from elastic to elasto-plastic ones (Wen, 1976).

To implement the control law, the force $f$ is required to be bounded in the form expressed in (3). Therefore, we consider that the force $f$ is bounded by a constant. This implies that the function $\gamma'$ in (3), also appearing in the control law in (5), is just equal to 1. It simplifies the implementation of the control law and, as ultimately verified through the numerical simulations, gives satisfactory results.

Some of the characteristics of the hysteretic isolator, namely $A$, $\nu_1$ and $m$, are fixed at 1, 0.5 and 1, respectively, while the value of coefficient $\nu_2$, which decisively influences the size of the hysteretic cycle is varied with the aim of evaluating the structural behaviour for different hysteretic devices. The yielding force and displacement are fixed at $f^Y = 5 \times 10^5$ N and $d^Y = 0.0245$ m and the damping ratio at 0.2. The base isolation system has an additional stiffness of $k_b = 0.2 \times 10^8$ N/m for the model with ten degrees of freedom and $k_b = 0.1185 \times 10^8$ N/m for the single degree of freedom model.

**Definition of the seismic motion.** The numerical tests use two types of seismic accelerations $a(t)$: in some cases it is considered to be sinusoidal with constant amplitude and in others it is the recorded accelerogram of real earthquakes.

When $a(t)$ is defined as sinusoidal with constant amplitude, it is given by $a(t) = A \sin \theta t$, where $A$ is the amplitude in m/s$^2$, $\theta$ is the frequency in rad/s and $t$ is the time in seconds.

For $a(t)$ representing actual records, three accelerograms corresponding to the earthquakes of El Centro (1940), Mexico City (1985) and Miyagioki (1978) are
used. These accelerograms are shown in Figures 3, 4 and 5, respectively. These particular earthquakes have been chosen because they cover three different predominant frequency ranges of interest.

3.3 FREQUENCY ANALYSIS

A frequency study is carried out in this section, considering the single degree of freedom model subjected to the action of the earthquakes El Centro (1940) and Mexico City (1985) and by varying its natural period from 0.1 s to 0.3 s. Results are included for the passive and hybrid cases, which allow a comparison of the effects of these two systems in reducing the maximum seismic response of the model.

In the plots of Figure 6 it is supposed that the base isolation system is suitable for the characteristics of an earthquake like El Centro (1940). This figure shows graphical representations of the maximum absolute displacement of the base as a function of the natural period of the structure. It indicates good behaviour of this displacement in the passive case and a substantial reduction in the hybrid case. This is in agreement with the objective of the adaptive control law. It also indicates that, in the case of the Mexico City earthquake, the passive case has bad behaviour reaching unacceptable absolute displacement values at the base, whereas the hybrid case has much lower values throughout the analyzed range of periods. This is the advantage of applying an active control force to the base: the displacement of the base is reduced in cases where the base isolation does not
Fig. 4. Accelerogram of Mexico City 1985 earthquake

Fig. 5. Accelerogram of Miyagioki 1978 earthquake
Fig. 6. Plot of the maximum absolute base displacement against the period

behave in the expected way.
A similar frequency study analyses the maximum relative displacement of the structure respecting to the base for the earthquakes El Centro and Mexico City (Figure 7). It may be noted that for high frequencies the hybrid response is higher than that for the passive case. This is due to the application of the force to the base, since the control of the displacement of the base limits the energy dissipation capacity of the isolation. However, this worsening of the behaviour does not occur in all cases. As shown in Figure 7, for higher periods of the structure the hybrid system reduces the relative displacement. This proves that whenever the passive system fails, the hybrid system provides an improvement to the response.
A comparison of the passive and hybrid cases using the absolute maximum acceleration of the structure is shown in Figure 8. This figure shows a worsening of the response of the hybrid case to El Centro earthquake and a notable improvement to that of Mexico City.
Figure 9 shows the maximum active control force applied to the base. The plots indicate that the force required to control the displacement of the base for the Mexico City earthquake is greater than that required when the action is the El Centro one. As mentioned earlier, the characteristics of the isolation are suitable
for an earthquake of the same type as El Centro and the good behaviour of the base isolation allows the system to be controlled with forces that are not very high. On the other hand, in the Mexico City earthquake the isolation produces higher base displacements and so the control forces to be applied are much greater.

3.4 Maximum responses

The model with ten degrees of freedom with base isolation subjected to the earthquakes of El Centro (1940), Mexico City (1985) and Miyagioki (1978) is now considered in the numerical analysis. Figure 10 compares the passive and hybrid cases considering the maximum inter-storey drift ratio as response parameter. This figure shows that, with the exception of the passive case for the Mexico City earthquake, the predominant behaviour of the isolated structure is that of a rigid body and that the hybrid case continues to show this behaviour even when the passive system fails. The other factor analyzed is the absolute maximum acceleration for each level, shown in Figure 11, where a slight increase in the absolute acceleration is noted in the hybrid case.

The next comparison is for the behaviour of the structure with ten degrees of freedom with passive and hybrid isolation systems and the building with a fixed
base when subjected to the El Centro earthquake. In this case the stiffness of the base isolation system was $k_b = 0.5 \times 10^5$ N/m, higher than that used in the previous examples. Figure 12 shows that the inter-storey drift percentage is lower than in the case of the passive system or the fixed base building. The same occurs in Figure 13, which makes the same comparison using the maximum absolute acceleration for each level. These figures confirm the observation made in the previous section that in some cases the relative displacement response and the absolute acceleration response of a building with a hybrid control system behaves even better than the one corresponding to a purely passive system.

3.5 Time variation of the response

A study of the time history of the response was made for the model with ten degrees of freedom with passive and hybrid isolation systems. The excitation was sinusoidal with amplitude of $3.5 \text{ m/s}^2$, frequency $10 \text{ rad/s}$ and a duration of 10 seconds. Figure 14 shows the time history of the absolute displacement of the base for the passive and hybrid cases. In the hybrid case the considerable reduction in the displacement of the base is obvious, what complies with the objective of
the control strategy. The time variation of the relative displacement is shown in Figure 15. During the first moments of the earthquake, higher values appear in the hybrid model, which then tend to approach the zero as stated by the control law. The absolute accelerations are shown in Figure 16, where the hybrid model has greater values for this acceleration than the passive one during the first seconds of the earthquake. However, starting from a certain time instant, the hybrid case tends to behave in a similar way as the passive one. The behavior of the control law is seen in Figure 17, which shows the time history of the control force. It can be observed that there are many time intervals when this force is null as the response of the system is within the desired ball centered in zero. It is obvious that whenever the response exceeds the specified bounds, calculations are made of the control forces until the response is returned to within the required ball.

3.6 Influence of the Characteristics of the Base Isolation System

To evaluate the influence of the parameters characterizing the base isolation device on the response of the structure, the model with ten degrees of freedom is subjected to a sinusoidal excitation with amplitude of $3.5 \text{ m/s}^2$, frequency of $10 \text{ rad/s}$ and
duration of 10 seconds. The parameter of the hysteretic isolator whose influence is analyzed is \( \nu_2 \), which defines the size of the hysteretic cycle. Figure 18 shows the variation in the absolute displacement of the base as a function of \( \nu_2 \) for both the passive and hybrid cases. Note that the hybrid case is independent of the hysteretic cycle and that, for all values of \( \nu_2 \) used in this analysis, the hybrid case has lower response values. In addition, the passive case tends towards a certain value of the absolute base displacement as the hysteretic cycle has a limit for the dissipation of energy that is reached at high values of \( \nu_2 \). Figure 19 shows the variation in the
Fig. 11. Variation of the maximum absolute acceleration with the floor level, $k_b = 0.2 \times 10^8$ N/m

displacement of the highest point of the structure respecting the base as a function of $\nu_2$. In this case the hybrid system has response values that are reasonably higher than the passive system and that do not depend on the variation in $\nu_2$.

3.7 Influence of Parameter $\lambda$ on the Response

Recall that the parameter $\lambda$ defines the radius of the ball centered in zero to which the controlled response of the hybrid system asymptotically tends. The single degree of freedom model with base isolation has been used again and $\lambda$ has been varied between 0.1 and 1.5. The analysis has been carried out for five different sinusoidal actions, all with amplitude of $3.5\,\text{m/s}^2$ and duration of 10 seconds, but with different frequencies: 3, 4, 8 and 15 rad/s. Figure 20 shows the absolute
displacement of the base; observe that there is a certain value of $\lambda$ after which the hybrid system behaves in the same way as the passive one. This guarantees that, at limit, the hybrid and passive systems have the same behaviour. Figure 21 shows the variation in the relative displacement as a function of $\lambda$. Starting from a a certain value of $\lambda$, the zero-centred ball is so large that it always contains the response and the hybrid model behaves the same as the passive one. The maximum values of the absolute acceleration are shown in Figure 22. These values increase up to a certain value for $\lambda$ and then behave in the same way as in the passive case. Figure 23 shows the variation in the maximum control force applied to the base; the effect of the condition imposed by the control law is clearly seen: as of a certain value of $\lambda$ it is no longer necessary to apply any control force to the base. Figures 20–23 show that the pre-set value for $\lambda$ gives as demanding or as loose a control, as desired. Obviously, for high values of $\lambda$ no force is required to be applied.
Fig. 13. Variation of the maximum absolute acceleration with the floor level, $k_b = 0.5 \times 10^8$ N/m, El Centro 1940 earthquake

to the base, as the response is always within the defined bounds. In this case the hybrid system behaves in the same way as a passive one.

4 Conclusions

The hybrid control system in this paper combines a nonlinear base isolator with an active feedback controller applying forces on the base. The active controller is a complement designed to improve the effect of the passive isolation system. A significant reduction of the absolute base displacement is obtained as primary objective, with a controlled relative displacement and absolute acceleration of the structure that may exhibit slight increases with respect to the pure passive case. However, the global structural behaviour improves when the hybrid system is used. This can be particularly significant for excitations having predominant frequencies
Fig. 14. Time history of the absolute base displacement for the ten degrees of freedom model subjected to a sinusoidal ground acceleration.
Fig. 16. Time history of the top floor absolute acceleration for the ten degrees of freedom model subjected to a sinusoidal ground acceleration.

Fig. 17. Time history of the active control force for the ten degrees of freedom model subjected to a sinusoidal ground acceleration.
Fig. 18. Plot of the maximum absolute base displacement against the parameter $\nu_2$ of the hysteretic isolator for the ten degrees of freedom model subjected to a sinusoidal ground acceleration in the range in which the purely passive base isolated structure has its maximum response.

Two important features present in hybrid systems have been taken into account: (i) uncertainties in knowledge of the structure, the isolators and the excitation and (ii) nonlinear behaviour in the isolation devices. Regarding the uncertainties, the main remark is that no knowledge of the parameters of the base and structure model is required for the design and the implementation of the control law because of its adaptive nature. Concerning the nonlinearities in the base isolation, in this paper a hysteretic base isolator has been considered in the examples.

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Fig. 19. Plot of the maximum top floor displacement relative to the base against the parameter $\nu_2$ of the hysteretic isolator for the ten degrees of freedom model subjected to a sinusoidal ground acceleration.

References


Fig. 20. Variation with $\lambda$ of the maximum absolute base displacement for the single degree of freedom model subjected to various sinusoidal ground accelerations.

Fig. 21. Variation with $\lambda$ of the relative structural displacement for the single degree of freedom model subjected to various sinusoidal ground accelerations.
Fig. 22. Variation with $\lambda$ of the maximum absolute structural acceleration for the single degree of freedom model subjected to various sinusoidal ground accelerations

Fig. 23. Variation with $\lambda$ of the maximum active control force for the single degree of freedom model subjected to various sinusoidal ground accelerations


